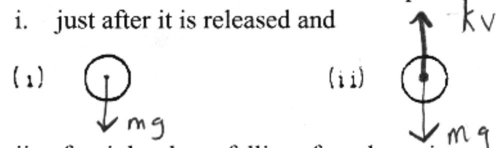
1975M1. A sphere of mass m is released from rest. As it falls, the air exerts a retarding force on the sphere that is proportional to the sphere's velocity (FR = -kv). Neglect the buoyancy force of the air.

a. On the circles below draw vectors representing the forces acting on the sphere
   i. just after it is released and



   ii. after it has been falling- for a long time and reached terminal velocity. Give each vector a descriptive label

b. Determine the terminal velocity of the sphere

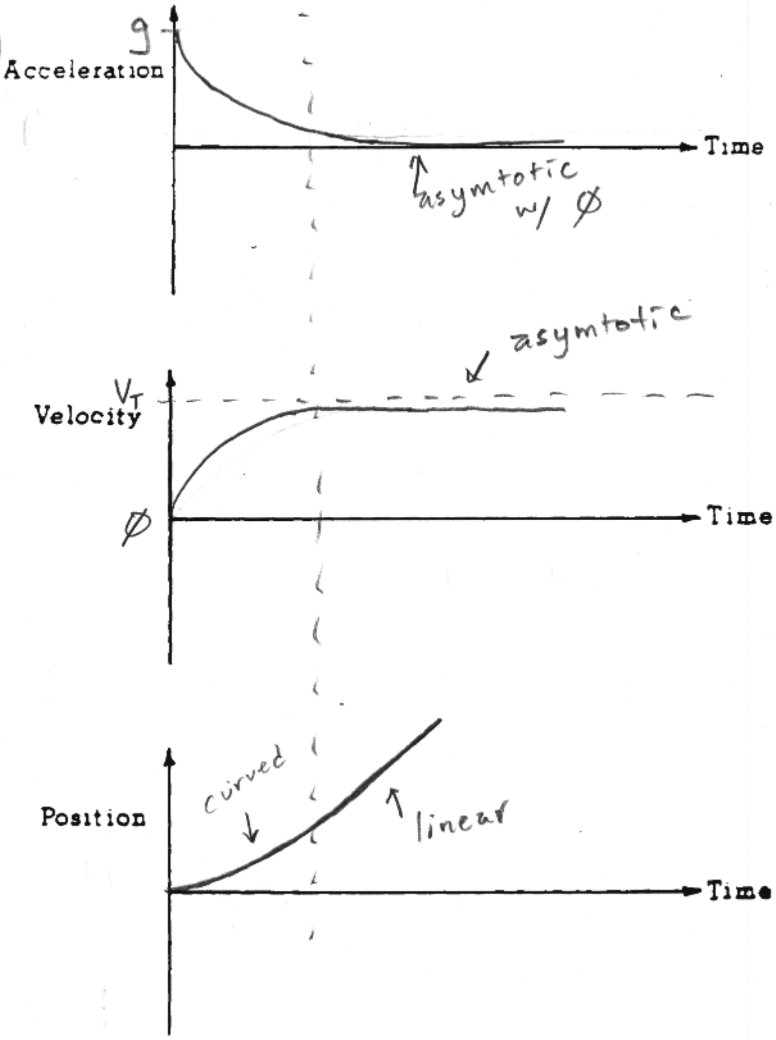
$\Sigma F = ma$ $\quad a = \emptyset$ $\quad \Sigma F = \emptyset$

$mg - kv = \emptyset$

$$\boxed{V_T = \frac{mg}{k}}$$

2b → c. Draw the following three graphs for the sphere's motion clearly showing significant features of the motion just after the sphere is released as well as after a long time.

Write and solve a differential equation for the velocity of the ball in respect to time. (ON back)



If we call down positive

i. Acceleration as a function of time ii. Velocity as a function of time iii. Position as a function of time.

$$\Sigma F = ma$$

$$\Sigma F = m\frac{dv}{dt}$$

$t = \emptyset$

$V_0 = \emptyset$

$$\boxed{mg - kv = m\frac{dv}{dt}}$$

differential equation

$$\boxed{\begin{aligned} &\underline{U\text{-sub}} \\ &U = mg - kv \\ &\frac{dU}{dv} = -k \\ &dv = \frac{dU}{-k} \end{aligned}}$$

$$dt\,(mg - kv) = m\,dv$$

$$dt\,U = m\frac{dU}{-k}$$

$$\int_0^t \frac{-k\,dt}{m} = \int_{U_0}^{U} \frac{dU}{U}$$

Rule

$$\boxed{\int \frac{dx}{x} = \ln|x|}$$

$$e^{\ln(x)} = x$$

$$\boxed{\begin{aligned} &U_0 = mg - kv_0 \\ &v_0 = \emptyset \\ &U_0 = mg \end{aligned}}$$

$$\frac{-kt}{m} = \ln\left|\frac{U}{U_0}\right|$$

$$e^{\left(-\frac{kt}{m}\right)} = e^{\ln\left|\frac{U}{U_0}\right|}$$

$$e^{-\frac{kt}{m}} = \frac{U}{U_0}$$

$$U_0 e^{-\frac{kt}{m}} = U$$

$$mg\left(e^{-\frac{kt}{m}}\right) = mg - kv$$

$$mg\left(e^{-\frac{kt}{m}}\right) - mg = -kv$$

$$-mg\left(e^{-\frac{kt}{m}}\right) + mg = kv$$

$$\boxed{\frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right) = V_{(t)}}$$

$$V_{(t)} = V_T\left(1 - e^{-\frac{kt}{m}}\right)$$