Physics B AP Review Packet: Mechanics Name: KEY

Position

Location of a particle in space.
(x) or (x,y) or (x,y,z)

Distance

The total length of the path traveled by an object.
Does not depend upon direction.

Displacement

The change in position of an object.
Depends only on the initial and final positions, not on path.
Includes direction.
Represented by Δx.

Problem: Distance versus Displacement

1. A hiker hikes 25 miles due north and then all the way back to the starting point.

a) How far does the hiker hike? ***Show your work.***

$\boxed{50 \text{ miles}}$

b) What is the hiker's displacement? ***Show your work.***

$\boxed{0 \, m}$

Average speed

$s_{ave} = d / \Delta t$

Average Velocity

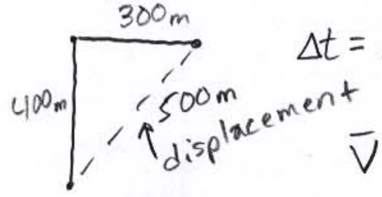
$v_{ave} = \Delta x / \Delta t$

Problem: Average Velocity (1988)

62. A truck traveled 400 meters north in 80 seconds, and then it traveled 300 meters east in 70 seconds. The magnitude of the average velocity of the truck was most nearly

(A) 1.2 m/s [✓ (B) 3.3 m/s] (C) 4.6 m/s
(D) 6.6 m/s (E) 9.3 m/s

Show your work:



$\Delta t = 150_s$

$\bar{V} = \frac{\Delta X}{\Delta t} = 3.3 m/s$

$c = \sqrt{400^2 + 300^2} = 500_m$

Acceleration

A change in velocity.
Acceleration can be speeding up, slowing down, or turning.
The SI unit for acceleration is m/s^2.
If the sign of the velocity and the sign of the acceleration is the same, the object speeds up.
If the sign of the velocity and the sign of the acceleration are different, the object slows down.

Uniformly Accelerated Motion

$a_{ave} = \Delta v / \Delta t$

Problem: Acceleration (1993)

1. In which of the following situations would an object be accelerated?

I. It moves in a straight line at constant speed.
II. It moves with uniform circular motion.
III. It travels as a projectile in a gravitational field with negligible air resistance.

(A) I only
(B) III only
(C) I and II only
[✓ (D) II and III only]
(E) I, II, and III

Explain your answer:

Change in velocity includes direction and speed

Kinematic Equations

$v = v_o + at$
$x = x_o + v_ot + 1/2 \, at^2$
$v^2 = v_o^2 + 2a(\Delta x)$

Problem: Kinematic Equations (1984)

65. A body moving in the positive x direction passes the origin at time *t* = 0. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At *t* = 1 second, the body is given a constant acceleration of 6 meters per second squared in the negative *x* direction. The position *x* of the body at *t* = 11 seconds is

(A) +99 m (B) +36 m [✓ (C) -36 m]
(D) -75 m (E) -99 m

Show your work:

2 part

part 1

$t = 1s$
$\bar{V} = 24 m/s$
$X = \bar{V} \cdot t = 24_m$

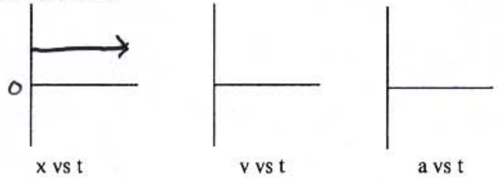
part 2

$X_o = 24_m$ $V_o = 24 m/s$
$t = 10_s$
$a = -6 m/s^2$
$X = X_o + V_o t + \frac{1}{2} a t^2$

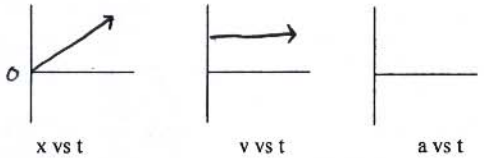
$X = -36m$

Problem: Sketch the Kinematic Graphs

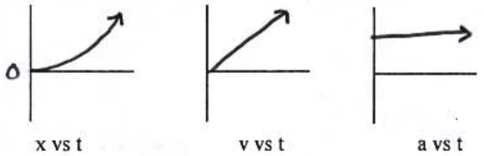
Stationary particle



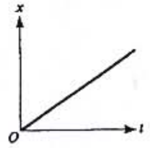
Particle moving with constant velocity



Particle moving with constant non-zero acceleration



Problem: Kinematic Graphs (1988)

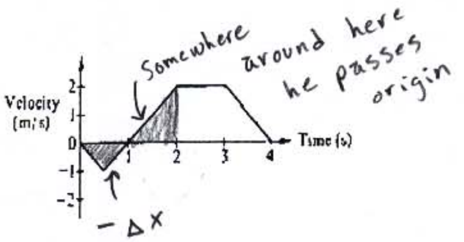


1. The displacement x of an object moving along the x-axis is shown above as a function of time *t*. The acceleration of this object must be

[✓ (A) zero]
(B) constant but not zero
(C) increasing
(D) decreasing
(E) equal to g

Explain your answer:

A This show constant velocity



Somewhere around here he passes origin

$-\Delta X$

Problem: Kinematic Graphs (1984)

3. The graph shows the velocity versus time for an object moving in a straight line. At what time after time = 0 does the abject again pass through its initial position?

(A) Between O and 1 s
(B) 1 s
[✓ (C) Between 1 and 2 s]
(D) 2 s
(E) Between 2 and 3 s

Show your work:

Use area for displacement

$0-1_s \quad \Delta X = -0.5m$

$1-2_s \quad \Delta X = +1m$

Free Fall

Occurs when an object falls unimpeded.
Gravity accelerates the object toward the earth.
$g = 9.8\ m/s^2$ downward.
$a = -g$ if up is positive.
acceleration is down when ball is thrown up
EVERYWHERE in the balls flight.

Problem: Free Fall (1993)

5. An object is released from rest on a planet that has no atmosphere. The object falls freely for 3.0 meters in the first second. What is the magnitude of the acceleration due to gravity on the planet?

(A) $1.5\ m/s^2$ (B) $3.0\ m/s^2$ [✓ (C) $6.0\ m/s^2$]
(D) $10.0\ m/s^2$ (E) $12.0\ m/s^2$

Show your work:

$V_0 = 0 \qquad \Delta X = \cancel{V_0 t} + \frac{1}{2}at^2$

$\Delta X = 3m$

$t = 1_s \qquad \frac{2\Delta X}{t^2} = a = 6 m/s^2$

Projectile Motion

An example of 2-dimensional motion.
Something is fired, thrown, shot, or hurled near the earth's surface.
Horizontal velocity is constant.
Vertical velocity is accelerated.
Air resistance is ignored.

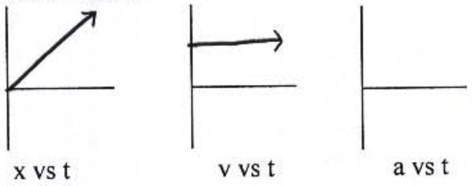
Trajectory of Projectile

The trajectory of a projectile is the path it follows.
It is defined by a parabola.
The RANGE of the projectile is how far it travels horizontally.
The MAXIMUM HEIGHT of the projectile occurs halfway through its range, provided the projectile is fired over level ground.
Acceleration points down at $9.8\ m/s^2$ for the entire trajectory.
Velocity is tangent to the path for the entire trajectory.
The vertical velocity changes while the horizontal velocity remains constant.

Problem: Graphs for 2D Projectiles

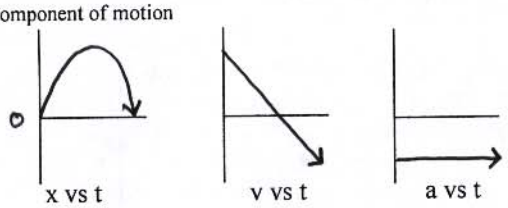
x-component of motion

No a_x



y-component of motion

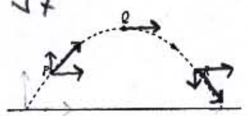
$a_y = -g$



Problems: Projectile Motion (1993)

Questions 64-66

Speed is V_x and V_y combined.
the vectors



A ball is thrown and follows the parabolic path shown above. Air friction is negligible. Point *Q* is the highest point on the path. Points *P* and *R* are the same height above the ground.

64. How do the speeds of the ball at the three points compare?
    - (A) $v_P < v_Q < v_R$
    - (B) $v_R < v_Q < v_P$
    - (C) $v_Q < v_R < v_P$
    - [✓ (D) $v_Q < v_P = v_R$]
    - (E) $v_P = v_R < v_Q$

Explain your choice:

65. Which of the following diagrams best shows the direction of the acceleration of the ball at point P ?

Explain your choice:

$a = -g$

(A)


(B)


(C)


(D)

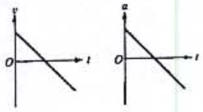

[✓ (E)]

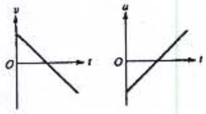

Problem: Graphs of Projectile Motion (1988)

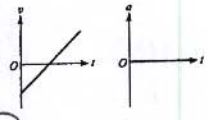
63. A projectile is fired with initial velocity v_0 at an angle θ with the horizontal and follows the trajectory shown above. Which of the following pairs of graphs best represents the vertical components of the velocity and acceleration, *v* and *a*, respectively, of the projectile as functions of time *t* ?

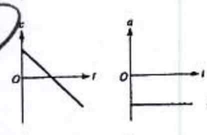
Explain your reasoning:

positive V goes to zero then negative V

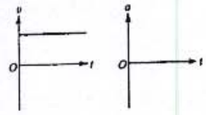
(A)


(B)


(C)


[✓ (D)]


← $-g$ (constant)

(E)


2D Motion problems

Resolve vectors into components.
Work as one-dimensional problems.

Horizontal Component of Velocity

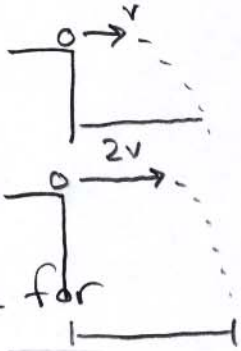
Not accelerated
Not influence by gravity
Follows equation:
$x = V_{o,x}t$

Problem: Horizontal Component (1988)

9. A diver initially moving horizontally with speed v dives off the edge of a vertical cliff and lands in the water a distance d from the base of the cliff. How far from the base of the cliff would the diver have landed if the diver initially had been moving horizontally with speed 2v ?

(A) d (B) $\sqrt{2d}$ [✓ (C) $2d$] (D) $4d$
(E) It cannot be determined unless the height of the cliff is known.

Show your work or explain your reasoning:



$d = vt$ time is same for both

$d = 2vt$

Vertical Component of Velocity

Accelerated by gravity (9.8 m/s^2 down)

$V_y = V_{o,y} - gt$

$y = y_o + V_{o,y}t - {}^1/_2gt^2$

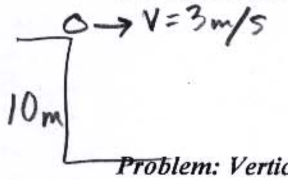
$V_y^2 = V_{o,y}^2 - 2g(y - y_o)$

Problem: Vertical Component (1988)

5. A 2-kilogram block rests at the edge of a platform that is 10 meters above level ground. The block is launched horizontally from the edge of the platform with an initial speed of 3 meters per second. Air resistance is negligible. The time it will take for the block to reach the ground is most nearly

(A) 0.3 s [✓ (B) 1.0 s] (C) 1.4 s
(D) 2.0 s (E) 3.0 s

Show your work:



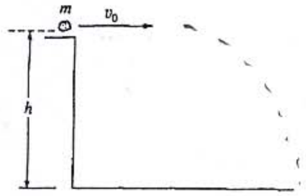
$\Delta y = V_{oy}t - \frac{1}{2}gt^2$

$h = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}}$

Problem: Vertical Component (1998)

Questions 59-60



A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0.

59. How much time does it take the rock to travel from the edge of the building to the ground?

(A) $\sqrt{hv_0}$

$t = \sqrt{\frac{2h}{g}}$

(B) h/v_0

(C) hv_0/g

(D) $2h/g$

[✓ (E) $\sqrt{2h/g}$]

Show your work:

$\Delta y = \cancel{V_{oy}t} - \frac{1}{2}gt^2$

$V_{oy} = 0$ $h = \frac{1}{2}gt^2$

Force

A force is a *push* or *pull* on an object.

Forces cause an object to accelerate…

- To speed up
- To slow down
- To change direction

Unit: Newton (SI system)

Newton's First Law

The Law of Inertia.

A body in motion stays in motion at constant velocity and a body at rest stays at rest unless acted upon by an external force.

Problem: Newton's 1st Law (1998)

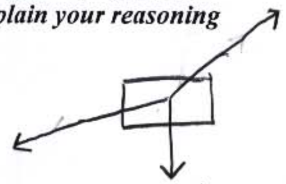
7. Three forces act on an object. If the object is in translational equilibrium, which of the following must be true? moving at constant speed

I. The vector sum of the three forces must equal zero.
II. The magnitudes of the three forces must be equal
III. All three forces must be parallel

[✓ (A) I only]
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

$\Sigma F = 0$

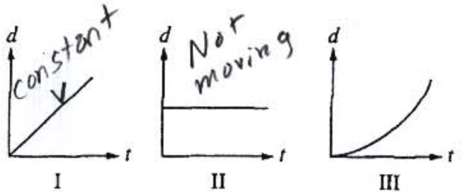
Explain your reasoning



Problem: Newton's 1st Law (1998)

Questions 43-44

Three objects can only move along a straight, level path. The graphs below show the position *d* of each of the objects plotted as a function of time *t*.



44. The sum of the forces on the object is zero in which of the cases?

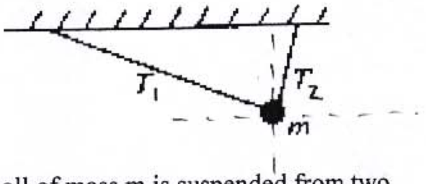
(A) II only
(B) III only
[✓ (C) I and II only]
(D) I and III only
(E) I, II, and III

Explain your reasoning

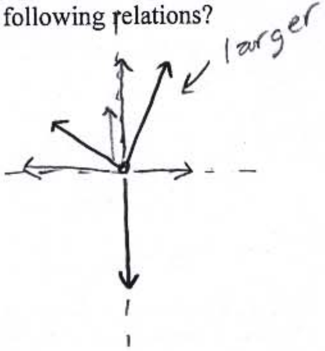
I no "a"

II no "a"

Problem: Newton's 1st Law (1984)



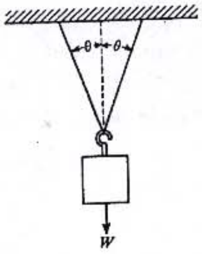
5. A hall of mass m is suspended from two strings of unequal length as shown above. The tensions T_1 and T_2 in the strings must satisfy which of the following relations?
   - (A) $T_1 = T_2$
   - (B) $T_1 > T_2$
   - [✓ (C) $T_1 < T_2$]
   - (D) $T_1 + T_2 = mg$
   - (E) $T_1 - T_2 = mg$



Show your work:

Problem: Newton's 1st Law (1988)

58. When an object of weight W is suspended from the center of a massless string as shown above, the tension at any point in the string is



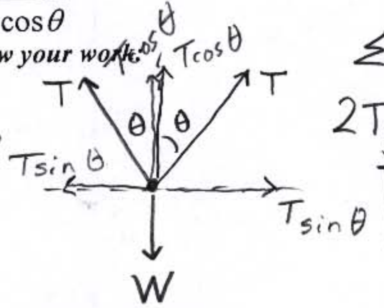
- (A) $2W\cos\theta$
- (B) $\dfrac{W\cos\theta}{2}$
- (C) $W\cos\theta$
- [✓ (D) $\dfrac{W}{2\cos\theta}$]
- (E) $\dfrac{W}{\cos\theta}$

Show your work:

$\Sigma F_x = 0$

$T\sin\theta - T\sin\theta = 0$

$T\sin\theta = T\sin\theta$



$\Sigma F_y = 0$

$2T\cos\theta - W = 0$

$\boxed{T = \dfrac{W}{2\cos\theta}}$

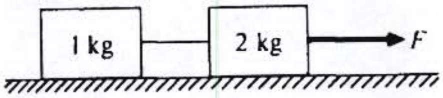
Newton's Second Law

$\Sigma F = ma$

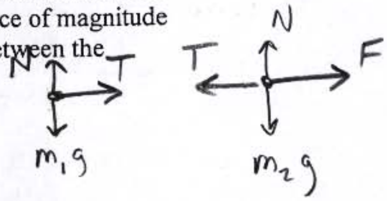
General Procedure for Solving Second Law Problems

Step 1: Draw the problem
Step 2: Free Body Diagram
Step 3: Set up equations
$\Sigma \mathbf{F} = ma \quad \Sigma F_x = ma_x \quad \Sigma F_y = ma_y$
Step 4: Substitute
Make a list of givens from the word problem.
Substitute in what you know.
Step 5: Solve

Problem: Second Law (1984)



11. When the frictionless system shown above is accelerated by an applied force of magnitude F, the tension in the string between the blocks is
    - (A) $2F$
    - (B) F
    - (C) $\frac{2}{3}F$
    - (D) $\frac{1}{2}F$
    - [✓ (E) $\frac{1}{3}F$]



$\Sigma F = m_1 a$ $\qquad$ $\Sigma F = m_2 a$

$T = m_1 a$ $\qquad$ $F - T = m_2 a$

$\cancel{T} + F - \cancel{T} = (m_1 + m_2)a$

Show your work:

$T = 1\dfrac{F}{3}$ $\qquad$ $\dfrac{F}{3} = a$

$T = \frac{1}{3}F$

Problem: Second Law (1993)

2. A ball falls straight down through the air under the influence of gravity. There is a retarding force F on the ball with magnitude given by F = bv, where t is the speed of the ball and b is a positive constant. The magnitude of the acceleration a of the ball at any time is equal to which of the following?
   - (A) $g - b$
   - [✓ (B) $g - \dfrac{bv}{m}$]
   - (C) $g + \dfrac{bv}{m}$
   - (D) $\dfrac{g}{b}$
   - (E) $\dfrac{bv}{m}$

$\Sigma F = ma$

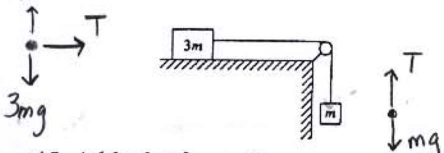
$mg - bv = ma$

$a = \dfrac{mg - bv}{m}$

$\boxed{a = g - \dfrac{bv}{m}}$

Show your work:

Problem: Second Law (1993)



$\Sigma F = ma$

$mg - T = ma$

$\Sigma F = 3ma$

$T = 3ma$

45. A block of mass 3m can move without friction on a horizontal table. This block is attached to another block of mass m by a cord that passes over a frictionless pulley, as shown above. If the masses of the cord and the pulley are negligible, what is the magnitude of the acceleration of the descending block?

(A) Zero [✓ (B) g/4] (C) g/3
(D) 2g/3 (E) g

Show your work:

$\cancel{T} + mg - \cancel{T} = 3ma + ma$

$\cancel{m}g = 4\cancel{m}a$

$a = \frac{g}{4}$

Newton's Third Law

For every action there exists an equal and opposite reaction.
If A exerts a force **F** on B, then B exerts a force of **-F** on A.

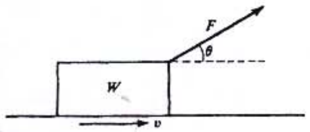
Weight

W = mg

Normal Force

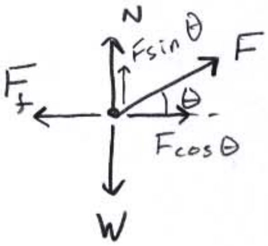
Force that prevents objects from penetrating each other
Reaction to other forces
Commonly a reaction to gravity

Problem: Normal Force Flat (1988)



4. A block of weight *W* is pulled along a horizontal surface at constant speed *v* by a force *F*, which acts at an angle of θ with the horizontal, as shown above. The normal force exerted on the block by the surface has magnitude

(A) W - Fcos θ
[✓ (B) W - Fsin θ]
(C) W
(D) W + Fsin θ
(E) W + Fcos θ



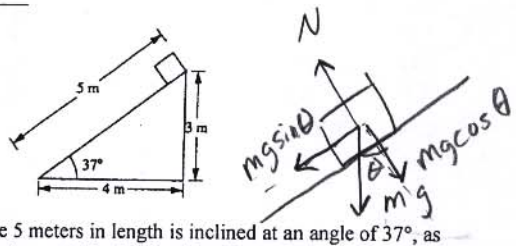
Show your work

$\Sigma F_y = 0$

$N + F\sin\theta - W = 0$

$\boxed{N = W - F\sin\theta}$

Problem: Normal Force Ramp (1993)



A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.

62. The magnitude of the normal force exerted on the block by the plane is most nearly

(A) 10 N (B) 12N [✓ (C) 16 N]
(D) 20 N (E) 33 N

Show your work

$N = mg\cos\theta =$

Problem: Elevators and Normal Force (PAB)

2. A 50-kg middle school student stands on a scale in an elevator that is moving downward, but slowing with an acceleration of magnitude 2.0 m/s^2. What does the scale read (in N)?

a) 300 [✓ b) 400] c) 500
d) 600 e) 700

Show your work

$\Sigma F = ma$

$N - mg = ma$

$\downarrow \quad a = -2.0 m/s^2$

$N = ma + mg$

$N = 50(-2) + 50(10)$

Friction

The force that opposes a sliding motion.
Static friction exists before sliding occurs
Kinetic friction exists after sliding occurs
In general *Kinetic friction* <= *Static friction*

Calculating Static Friction

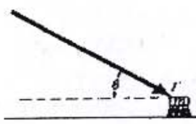
$f_s \le \mu_s N$

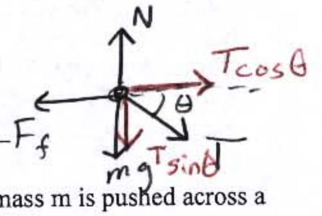
Static friction increases as the force trying to push an object increases… up to a point!

Calculating Kinetic Friction

$f_k = \mu_k N$

Problems: Friction on Flat Surface (1984)





61. A push broom of mass m is pushed across a rough horizontal floor by a force of magnitude T directed at angle θ as shown above. The coefficient of friction between the broom and the floor is ~~μ~~ μ. The frictional force on the broom has magnitude
    - [✓ (A) $\mu(mg + T\sin\theta)$]
    - (B) $\mu(mg - T\sin\theta)$
    - (C) $\mu(mg + T\cos\theta)$
    - (D) $\mu(mg - T\cos\theta)$
    - (E) μmg

Show your work

Find N

$\Sigma F_y = 0$

$F_f = \mu N$

$N - T\sin\theta - mg = 0$

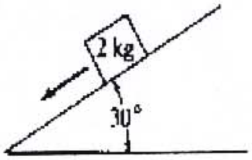
$\boxed{F_f = \mu(mg + T\sin\theta)}$

$N = mg + T\sin\theta$

Problems: Friction on Ramp(1984)

Questions 6-7

A 2-kilogram block slides down a 30° incline as shown above with an acceleration of 2 meters per second squared.

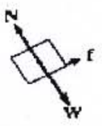


6. Which of the following diagrams best represents the gravitational force **W**, the frictional force **f**, and the normal force **N** that act on the block?

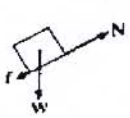
(A)

(B)

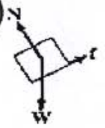
(C)



(D)

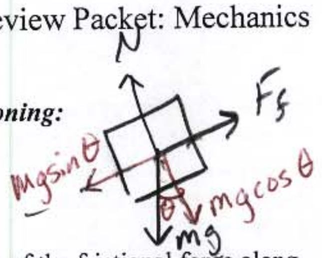


[✓ (E)]



1,

Explain your Reasoning:



7. The magnitude of the frictional force along the plane is most nearly
   (A) 2.5 N (B) 5N [✓ (C) 6 N] (D) 10 N (E) 16 N

Show your work:

$\Sigma F = ma$

$mg\sin\theta - F_f = ma$

$F_f = mg\sin\theta - ma$

$F_f = 6N$

Uniform Circular Motion

An object moves at uniform speed in a circle of constant radius.

Acceleration in Uniform Circular Motion

Turns object; doesn't speed it up or slow it down.

Acceleration points toward center of the circle.

Called centripetal acceleration.

Centripetal Acceleration

$a_c = v^2/r$

Force in Uniform Circular Motion

Any force or responsible for uniform circular motion is referred to as a *centripetal force*. Centripetal force can arise from one force, or from a combination of sources.

$F_c = \Sigma F = ma_c$

$F_c = \Sigma F = m\,v^2/r$

Centripetal forces always arise from other forces.

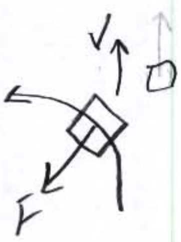
Since speed of object remains constant, kinetic energy remains constant, and work is zero.

Friction, *tension*, *normal force*, *gravity* and the *magnetic force* are common forces that can act centripetally to cause uniform circular motion.

Problem: Centripetal Force (1993)

46. A car initially travels north and then turns to the left along a circular curve. This causes a package on the seat of the car to slide toward the right side of the car. Which of the following is true of the net force on the package while it is sliding?
    - (A) The force is directed away from the center of the circle.
    - (B) The force is directed north.
    - (C) There is not enough force directed north to keep the package from sliding.
    - (D) There is not enough force tangential to the car's path to keep the package from sliding.
    - [✓ (E) There is not enough force directed toward the center of the circle to keep the package from sliding.]

Explain your reasoning:



package wants to keep moving North

Problem: Centripetal Force (1988)

2. The horizontal turntable shown below rotates at a constant rate. As viewed from above, a coin on the turntable moves counterclockwise in a circle as shown. Which of the following vectors best represents the direction of the frictional force exerted on the coin by the turntable when the coin is in the position shown?

(A) ←
(B) →
(C) ↙
[✓ (D)] ↓
(E) ↑

Explain your reasoning:

Force always toward center.

Universal Law of Gravity

$F_g = -Gm_1m_2/r^2$

The negative sign simply means the force is attractive.
Most orbit problems can be solved by setting the gravitational force equal to the centripetal force.

$Gm_1m_2 / r^2 = m_1v^2 / r$

(a good starting point for most problems!)

Problem: Orbit (1988)

61. A satellite of mass M moves in a circular orbit of radius R at a constant speed v. Which of the following must be true?

I. The net force on the satellite is equal to mv^2/R and is directed toward the center of the orbit.

II. The net work done on the satellite by gravity in one revolution is zero.

~~III. The angular momentum of the satellite is a constant.~~

skip

(A) I only (B) III only (C) I and II only
(D) II and III only (E) I, II, and III

Explain your reasoning:

$\Sigma F_c = \frac{mv^2}{R}$

$G\frac{mM}{R^2} = \frac{mv^2}{R}$

$F_g = G\frac{mM}{R^2}$

$v = \sqrt{\frac{GM}{R}}$

Problem: Orbit (1998)

40. What is the kinetic energy of a satellite of mass *m* that orbits the Earth, of mass M, in a circular orbit of radius R ?

(A) Zero [✓ (B) $\frac{1}{2}\frac{GMm}{R}$] (C) $\frac{1}{4}\frac{GMm}{R}$
(D) $\frac{1}{2}\frac{GMm}{R^2}$ (E) $\frac{GMm}{R^2}$

$K = \frac{1}{2}mv^2$

$K = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2$

$K = \frac{1}{2}\frac{mMG}{R}$

Show your work:

Problem: Law of Gravity and Weight (1998)

39. An object has a weight W when it is on the surface of a planet of radius *R*. What will be the gravitational force on the object after it has been moved to a distance of *4R* from the center of the planet?

(A) $16W$
(B) $4W$
(C) W
(D) $\frac{1}{4}W$
[✓ (E)] $\frac{1}{16}W$

Show your work ;

$W = G\frac{mM}{R^2}$

$W = G\frac{mM}{(4R)^2} = \frac{1}{16}$

Problem: Law of Gravity and Accel. (1993)

48. The planet Mars has mass 6.4×10^{23} kilograms and radius 3.4×10^6 meters. The acceleration of an object in free-fall near the surface of Mars is most nearly

(A) zero (B) 1.0 m/s2 (C) 1.9 m/s2
[✓ (D)] 3.7 m/s2 (E) 9.8 m/s2

Show your work:

$F_g = G\frac{mM}{R^2}$

$F_g = mg$
on surface

$\cancel{m}g = G\frac{\cancel{m}M}{R^2}$

$g = G\frac{M}{R^2} = 6.67\times10^{-11}\left(\frac{6.4\times10^{23}}{(3.4\times10^6)^2}\right)$

Restoring Force:

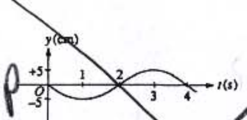
F = -kx (basic form of restoring force)
Restoring force is greatest at maximum displacement and zero at equilibrium

Period (T)
The length of time it takes for one cycle of periodic motion to complete itself.

Frequency (f):
How fast the oscillation is occurring.
Frequency is inversely related to period.
$f = 1/T$
The units of frequency is the Herz (Hz) where 1 Hz = 1 s^{-1}.

~~***Problem: Simple Harmonic Motion (1993)***~~

Skip

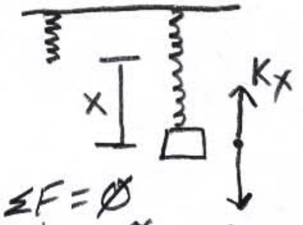


~~43. A particle oscillates up and down in simple harmonic motion. Its height y as a function of time t is shown in the diagram above. At what time t does the particle achieve its maximum positive acceleration?~~
~~(A) 1s (B) 2s (C) 3s (D) 4s~~
~~(E) None of the above, because the acceleration is constant~~
~~***Explain your reasoning:***~~

Springs
A common Simple Harmonic Oscillator.
$F_s = -kx$ (Hooke's Law)

Problem: Hooke's Law (1993)

3. An ideal spring obeys Hooke's law, F = - kx. A mass of 0.50 kilogram hung vertically from this spring stretches the spring 0.075 meter. The value or the force constant for the spring is most nearly
   - (A) 0.33 N/m
   - (B) 0.66 N/m
   - (C) 6.6 N/m
   - (D) 33 N/m
   - [✓ (E) 66 N/m]



Show your work:

$\Sigma F = ma$ $\Sigma F = 0$

$mg - kx = 0$ mg

$mg = kx$ $\boxed{k = \frac{mg}{x}}$

Potential Energy of a Spring
$U_s = ½ k x^2$

Pendulum
The pendulum can be thought of as an oscillator.
The displacement needs to be ***small*** for it to work properly.
Pendulum Forces: Gravity and tension

Potential Energy of a Pendulum
$U_g = mgh$

Work
The bridge between force and energy.
Work is a scalar.
$W = F \Delta x \cos \theta$
SI Unit: Joule (N m)

Counterintuitive Results
There is no work if there is no displacement.
Forces perpendicular to displacement don't work.
By doing positive work on an object, a force or collection of forces increases its **mechanical energy** in some way.
The two forms of mechanical energy are called **potential** and **kinetic energy**.

Problem: Work (B-1988)

6. A horizontal force F is used to pull a 5-kilogran block across a floor at a constant speed of 3 meters per second. The frictional force between the block and the floor is 10 newtons. The work done by the force F in 1 minute is most nearly
   (A) 0 J (B) 30 J (C) 600 J
   (D) 1,350 J [✓ (E) 1,800 J]

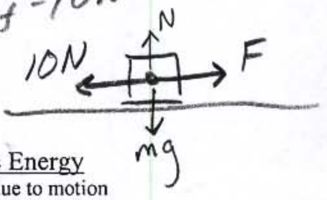
Show your work:

$t = 60s$

$d = v \cdot t = 180m$

$a = 0$

$F_f = 10N$



$F = 10N$

$W = F \cdot d = 10(180) = \boxed{1800J}$

Kinetic Energy
Energy due to motion
$K = ½ m v^2$

Problem: Kinetic Energy (B-1988)

3. Which of the following quantities is a scalar that is always positive or zero?
(A) Power (B) Work [✓ (C) Kinetic energy]
(D) Linear momentum (F) Angular momentum

State your reasoning:

The Work-Energy Theorem

$W_{net} = \Delta K$

Net work is used in this theorem. This is work due to ALL FORCES acting upon object.

When net work is positive, the kinetic energy of the object will increase (it will speed up).

When net work is negative, the kinetic energy of the object will decrease (it will slow down).

When there is no net work, the kinetic energy is unchanged (constant speed).

Work and graphs

The area under the curve of a graph of force *vs* displacement gives the work done by the force in performing the displacement.

Springs: stretching

Springs:compressing

Power

The *rate* of which work is done.

When we run upstairs, t is small so P is big.

When we walk upstairs, t is large so P is small.

$P = W/t$

work/time

$P = F V$

(force)(velocity)

SI unit for Power is the Watt.

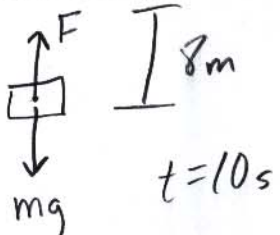
1 Watt = 1 Joule/s

Problem: Power (B-1998)

2. A student weighing 700 N climbs at constant speed to the top of an 8 m vertical rope in 10 s. The average power expended by the student to overcome gravity is most nearly

(A) 1.1 W
(B) 87.5 W
[✓ (C) 560 W]
(D) 875 W
(E) 5,600 W

Show your work:



$$P = \frac{W}{t} = \frac{F \cdot d}{t} = \frac{700N \cdot 8m}{10s}$$

How We Buy Energy…

The kilowatt-hour is a commonly used unit by the electrical power company.

Power companies charge you by the kilowatt-hour (kWh), but this not power, it is really energy consumed.

1 kW = 1000 W

1 h = 3600 s

1 kWh = 1000J/s • 3600s = 3.6×10^6J

Problem: Power (B-1998)

5. Units of power include which of the following?

I. Watt
II. Joule per second
III. Kilowatt-hour

(A) I only
(B) III only
[✓ (C) I and II only]
(D) II and III only
(E) I, II, and III

State your reasoning: $\text{Watt} = \frac{\text{Work}}{\text{time}} = \frac{(J)}{(s)}$

Kw·hr

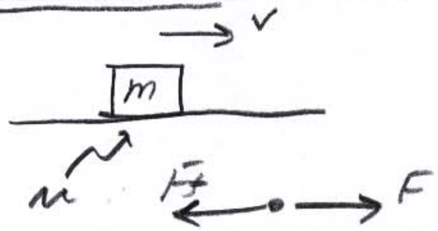
Power x time = Work or Energy

Problem: Power (B-1998)

9. A child pushes horizontally on a box of mass *m* which moves with constant speed υ across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

This is power

[✓ (A) μmgυ]
~~(B) mgυ~~
(C) υ/μmg
(D) μmg/υ
(E) μmυ^2



Show your work:

$$P = \frac{W}{t} = \frac{F \cdot d}{t} = F \cdot V \quad F_f = F$$

$$\boxed{P = \mu mg \cdot V}$$

Force Types

Conservative forces:

Work in moving an object is ***path independent***.

Work in moving an object along a closed path is zero.

Work done against conservative forces increases potential energy; work done by them decreases it.

Ex: gravity, springs

Non-conservative forces:

Work is ***path dependent***.

Work along a closed path is NOT zero.

Work may be related to a change in total energy (including thermal energy).

Ex: friction, drag

Potential energy

Energy an object possesses by virtue of its position or configuration.

Examples:

Gravitational Potential Energy

Spring Potential Energy

Potential energy is related to work done by CONSERVATIVE FORCES only.

$\Delta U_g = -W_g$ (gravity)

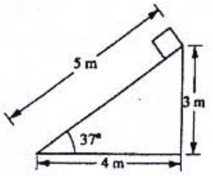
$\Delta U_g = -W_s$ (spring)

Gravitational potential energy close to earth's surface.

$W_g = -mgh$ (close to earth's surface)
$\Delta U = -W_g = mgh$

*Note: we calculate **changes** in potential energy only using this method. We assign the potential energy to be zero at some certain point, usually the surface of the earth.*

Problem: Work due to gravity (B-1993)



A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.

63. The work done on the block by the gravitational force during the 5-meter slide down the plane is most nearly
(A) 20 J [✓ (B) 60 J] (C) 80
(D) 100 J (E) 130 J

Show your work:

$W_g = -\Delta U_g$
$mgh =$

ADVANCED TOPIC

Gravitational potential energy changes far from earth's surface.

$U_g = -GM_em/r$ (close to earth's surface)
- G: Universal gravitational constant
- M_e: Mass of earth
- m: mass of another object
- r: distance from center of earth

U_g has been defined to be zero when an object is infinitely far from the earth, and it gets increasingly negative as an object approaches the earth.

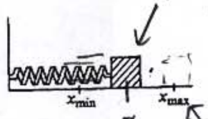
Note: This literal definition is impractical in most problems, but this is the equation that must be used to calculate ΔU when you a very far from the earth's surface.

Spring potential energy

Springs also can possess potential energy (U_s).
$U_s = ½ kx^2$
U_s is zero when a spring is in its preferred, or equilibrium, position where the spring is neither compressed or extended.

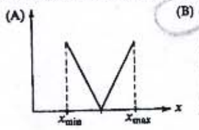
Problem: Conservation of Energy and Springs(B-1988)

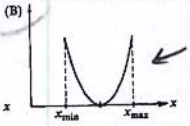
Questions 11-12



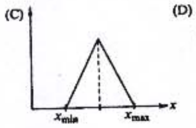
max speed
Ø
All potential

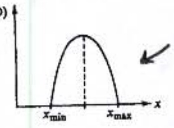
A block oscillates without friction on the end of a spring as shown above. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max} The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.

(A)


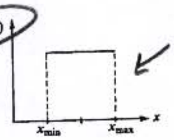
(B)


← potential

(C)


(D)


← kinetic

(E)


← E=K+U

Stays constant

11. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C
(D) D [✓ (E) E]
Explain your reasoning:

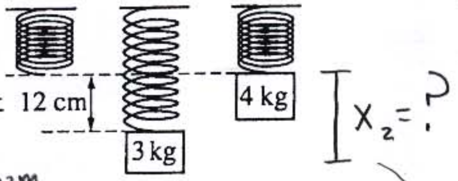
12. Which graph can represent the kinetic energy of the block as a function of x ?
(A) A (B) B (C) C
[✓ (D) D] (E) E
State your reasoning:

Law of Conservation of Mechanical Energy

$U + K = \text{Constant}$
$\Delta U + \Delta K = 0$
$\Delta U = -\Delta K$

Note: Sometimes conservation of energy problems can be best worked with potential energy alone, such as in the problem below.

Problem: Conservation of Energy (B-1998)



38. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown above. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown above, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?

~~(A)~~ 9 cm (B) 18 cm (C) 24 cm
[✓ (D) 32 cm] (E) 48 cm

Show your work:

Kx

mg

$\Sigma F = 0$

$Kx_1 = mg$

$K = \frac{mg}{x} = 250\,N/m$

$\Sigma F = 0$

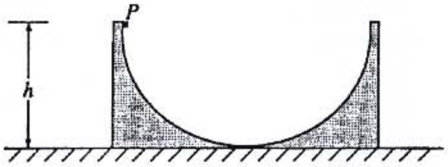
Kx

$mg - Kx_2 = 0$

$mg = Kx_2$

$x_2 = \frac{mg}{K} = \frac{\overset{mg}{40}}{250} = .16\,m$

Problem: Conservation of Energy (B-1993)



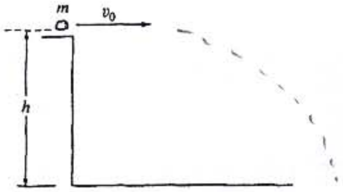
4. The figure above shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?

(A) It is equal to h/2 π.
(B) It is equal to h/4.
(C) It is equal to h/2.
(D) It is equal to h.
[✓ (E)] It is between zero and h; the exact height depends on how much energy is lost to friction.

Show your work:

Problem: Conservation of Energy (B-1998)

Questions 59-60



A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is υ_0.

59. How much time does it take the rock to travel from the edge of the building to the ground?

(F) $\sqrt{h\upsilon_0}$ h/υ_0

(G) $h\upsilon_0/g$ $2h/g$

[✓ (H) $\sqrt{2h/g}$]

Show your work:

$t = \sqrt{\frac{2h}{g}}$

60. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh (B) $\frac{1}{2}m\upsilon_0^2$

(C) $\frac{1}{2}m\upsilon_0^2 - mgh$ [✓ (D) $\frac{1}{2}m\upsilon_0^2 + mgh$]

(E) $mgh - \frac{1}{2}m\upsilon_0^2$

Show your work:

$K_0 + U_0 = K + \cancel{U}$

$\frac{1}{2}mv_0^2 + mgh = K$

Problem: Conservation of Energy (B-1993)

47. A block of mass m slides on a horizontal frictionless table with an initial speed v_0. It then compresses a spring of force constant k and is brought to rest. How much is the spring compressed from its natural length?

(A) $\frac{v_0}{2g}^2$

(B) $\frac{mg}{k}$

(C) $\frac{m}{k}v_0$

[✓ (D) $\sqrt{\frac{m}{k}}v_0$]

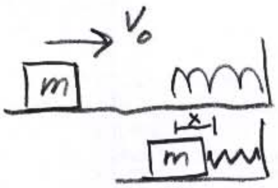
(E) $\sqrt{\frac{k}{m}}v_0$

$K_0 + \cancel{U_0} = \cancel{K} + U$

$\frac{1}{\cancel{2}} m v_0^2 = \frac{1}{\cancel{2}} k x^2$

$X = \sqrt{\frac{m v_0^2}{k}}$

Show your work:



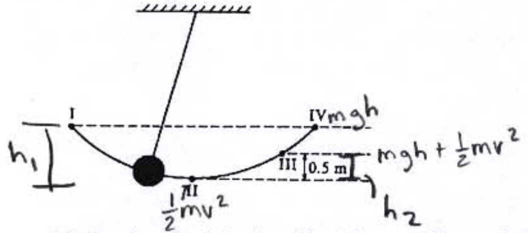
Conservation of Energy and Dissipative Forces.

Dissipative forces cause loss of mechanical energy by producing heat.

$W_{nc} = \Delta U + \Delta K$

Problem: Conservation of Energy and Pendulums (B-1988)

Questions 51-52



A ball swings freely back and forth in an arc from point I to point IV, as shown above. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is 1 meter above II. Air resistance is negligible.

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$v = \sqrt{2gh} = \sqrt{2(10)1}$

$V_{max} = 4.47 m/s$

51. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?

(A) At point II

~~(B) At some point~~ between II and III

[✓ (C) At point III]

(D) At some point between III and IV

(E) At point IV

State your reasoning:

52. The speed of the ball at point II is most nearly

[✓ (A) 3.0 m/s] (B) 4.5 m/s (C) 9.8 m/s

~~(D) 14~~ m/s (E) 20 m/s

Need to know the total E

Show your work:

$\cancel{m}gh_1 = \cancel{m}gh_2 + \frac{1}{2}\cancel{m}v^2$

$h_2 = 0.5 m$

$h_1 = 1 m$

$V = \sqrt{2(gh_1 - gh_2)}$

$V = 3.16 m/s$

Momentum

How hard it is to stop a moving object.

Related to both mass and velocity.

For one particle

$p = mv$

For a system of multiple particles

$\mathbf{P} = \Sigma \mathbf{p}_i = \Sigma m_i \mathbf{v}_i$

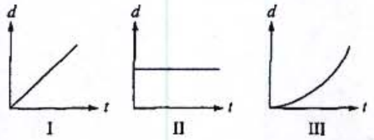
Units: N s or kg m/s

Momentum is a vector!

Problem: Momentum (1998)

Questions 43-44

Three objects can only move along a straight, level path. The graphs below show the position d of each of the objects plotted as a function of time t.



43. The magnitude of the momentum of the object is increasing ~~in which of~~ the cases?

(A) II only [✓ (B) III only] (C) I and II only

(D) I and III only (E) I, II, and III

Explain your reasoning:

Only one that shows changing velocity

Impulse (J)

The product of an external force and time, which results in a change in momentum

$J = F t$

$J = \Delta P$

Units: N s or kg m/s

Problem: Impulse (1984)

56. Two planets have the same size, but different masses, and no atmospheres. Which of the following would be the same for objects with equal mass on the surfaces of the two planets?

I. The rate at which each would fall freely

II. The amount of mass each would balance on an equal-arm balance

III. The amount of momentum each would acquire when given a certain impulse

(A) I only (B) III only

(C) I and II only

[✓ (D) II and II1 only]

~~(E) I, II, and III~~

Explain your reasoning:

$J = \Delta m v$

Problem: Impulse (1998)

57. A ball of mass 0.4 kg is initially at rest on the ground. It is kicked and leaves the kicker's foot with a speed of 5.0 m/s in a direction 60° above the horizontal. The magnitude of the impulse imparted by the ball to the foot is most nearly

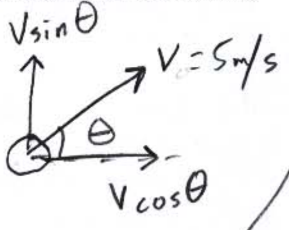
(A) 1 N·s

(B) $\sqrt{3}$ N·s

[✓ (C) 2 N·s]

(D) $\frac{2}{\sqrt{3}}$ N·s

(E) 4 N·s



$V_0 = 0$

Just a distractor

Show your work:

$J = \Delta m V$

$J = m(V \cancel{- V_0}) = 2 kg\cdot m/s$

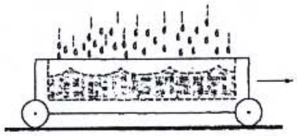
Law of Conservation of Momentum

If the resultant external force on a system is zero, then the momentum of the system will remain constant.

The sum of the momentums before a collision is equal to the sum of the momentums after a collision.

$$\Sigma P_b = \Sigma P_a$$

Problem: Conservation of Momentum (1998)



4. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of momentum

(B) increase because of conservation of mechanical energy

[✓ (C) decrease because of conservation of momentum]

(D) decrease because of conservation of mechanical energy

(E) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

Explain your reasoning:

$\overset{\uparrow}{m}\underset{\downarrow}{v} = p(\text{constant})$

Collisions

Follow ***Newton's Third Law*** which tells us that the force exerted by body A on body B in a collision is equal and opposite to the force exerted on body B by body A.

During a collision, external forces are ignored.

The time frame of the collision is very short.

The forces are ***impulsive*** forces (high force, short duration).

Collision Types

Elastic: P is conserved, K is conserved

Inelastic: P is conserved, K is NOT conserved

Perfectly Inelastic means the bodies stick together

Problem: Collisions (1993)

10. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?

~~(A)~~ The collision is elastic.

~~(B)~~ All of the initial kinetic energy of the less-massive object is lost.

~~(C)~~ The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.

[✓ (D)] The speed of the objects that are stuck together will be less than the initial speed of the less-massive object.

~~(E)~~ The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

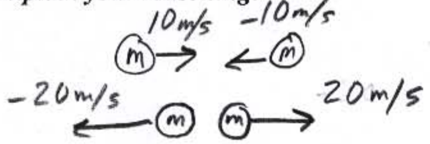
Explain your reasoning:

momentum will conserve

Problem: Collisions (1993)

11. Two objects having the same mass travel toward each other on a flat surface, each with a speed of 10 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 20 meters per second relative to the surface. Which of the following assessments of this report is most accurate?
(A) Momentum was not conserved, therefore the report is false.
[✓ (B) If potential energy was released to the objects during the collision, the report could be true.]
(C) If the objects had different masses, the report could be true.
(D) If the surface was inclined, the report could be true.
(E) If there was no friction between the objects and the surface, the report could be true.

Explain your reasoning:



Problem: Collision (1998)

3. A railroad car of mass *m* is moving at speed υ when it collides with a second railroad car of mass *M* which is at rest. The two cars lock together instantaneously and move along the track. What is the speed of the cars immediately after the collision?

(A) $\frac{\upsilon}{2}$
(B) $\frac{m\upsilon}{M}$
(C) $\frac{M\upsilon}{m}$
(D) $\frac{(m+M)\upsilon}{m}$
[✓ (E) $\frac{mv}{m+M}$]

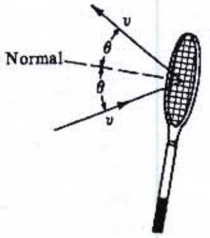
Show your work:

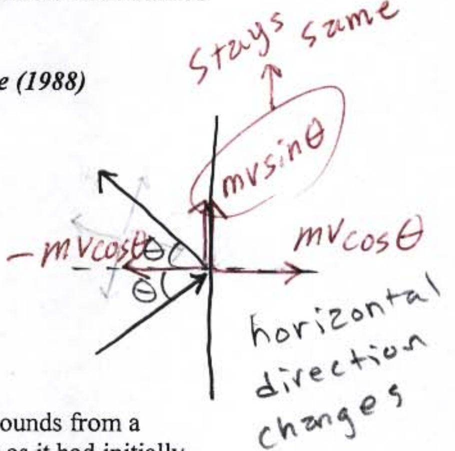
$P_0 = P$

$mv = (m+M)V_f$

$V_f = \frac{mV}{m+M}$

Problem: Momentum Change (1988)





7. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially, as shown above. The magnitude of the momentum change of the ball is
~~(A) 0~~ (B) *mv* (C) 2*mv*
(D) 2*mv* sin θ [✓ (E) 2*mv* cos θ]

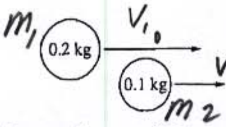
Show your work:

$J = \Delta mV$

$J = -mv\cos\theta - mv\cos\theta$

$J = -2mv\cos\theta$

Problem: Collision (1998)



41. Two objects of mass 0.2 kg and 0.1 kg, respectively, move parallel to the x-axis, as shown above. The 0.2 kg object overtakes and collides with the 0. 1 kg object. Immediately after the collision, the y-component of the velocity of the 0.2 kg object is 1 m/s upward. What is the y-component of the velocity of the 0.1 kg object immediately after the collision?
[✓ (A) 2 m/s downward]
(B) 0.5 m/s downward
(C) 0 m/s
(D) 0.5 m/s upward
(E) 2 m/s upward

split axis

Show your work:

$P_{0_y} = P_y$

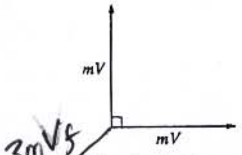
$0 = m_1V_1 + m_2V_2$

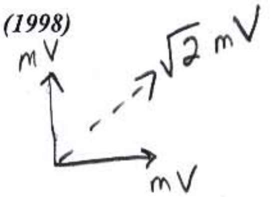
$V_2 = \frac{-m_1V_1}{m_2}$

$V_2 = \frac{-(0.2)(1)}{(0.1)} = -2m/s$

Explosion
Mathematically, handled just like an ordinary perfectly inelastic collision.
Momentum is conserved, kinetic energy is not.

Problem: Explosion (1998)





67. A stationary object explodes, breaking into three pieces of masses *m*, *m*, and 3*m*. The two pieces of mass *m* move off at right angles to each other with the same magnitude of momentum *m*V, as shown in the diagram above. What are the magnitude and direction of the velocity of the piece having mass 3*m* ?

	Magnitude	Direction
~~(A)~~	$\frac{V}{\sqrt{3}}$	↗
(B)	$\frac{V}{\sqrt{3}}$	↙
~~(C)~~	$\frac{\sqrt{2}\,V}{3}$	↗
[✓ (D)]	$\frac{\sqrt{2}\,V}{3}$	↙
(E)	$\sqrt{2}\,V$	↙

$$V_f = \frac{\sqrt{2}}{3}V$$

Show your work:

$$0 = 3mV_f + \sqrt{2}\,mV$$

$$-3\cancel{m}V_f = \sqrt{2}\,\cancel{m}V$$

<<ADVANCED TOPIC>>

Center of Mass

Where all the mass can be considered to exist
For uniform objects, the center of mass resides at geometric center.

For collection of points, use these equations

$x_{cm} = \Sigma m_i x_i / \Sigma m_i$
$y_{cm} = \Sigma m_i y_i / \Sigma m_i$
$z_{cm} = \Sigma m_i z_i / \Sigma m_i$

where x_{cm}, y_{cm}, and z_{cm} are the coordinates of the center of mass, and Σm_i is the total mass of the system.

Problem: Center of Mass (1998)

63. Two people of unequal mass are initially standing still on ice with negligible friction. They then simultaneously push each other horizontally. Afterward, which of the following is true?

(A) The kinetic energies of the two people are equal.
(B) The speeds of the two people are equal.
(C) The momenta of the two people are of equal magnitude.
(D) The center of mass of the two-person system moves in the direction of the less massive person.
(E) The less massive person has a smaller initial acceleration than the more massive person.

Explain your reasoning:

Problem: Center of Mass (1993)

8. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?

(A) Acceleration
(B) Velocity
(C) Kinetic energy
(D) Displacement from the center of mass
(E) Gravitational force

Show your work or explain your reasoning: