Moment of Inertia

Analogies thus far

linear	Angular	
ΔX	$\Delta\theta$	$\theta \cdot r = x$
v	ω	$\omega \cdot r = v$
a	α	$\alpha \cdot r = a$
ΣF	$\Sigma \tau$	$F \cdot r = \tau$
$F = ma$	$\tau = I\alpha$ ↑ what is this	

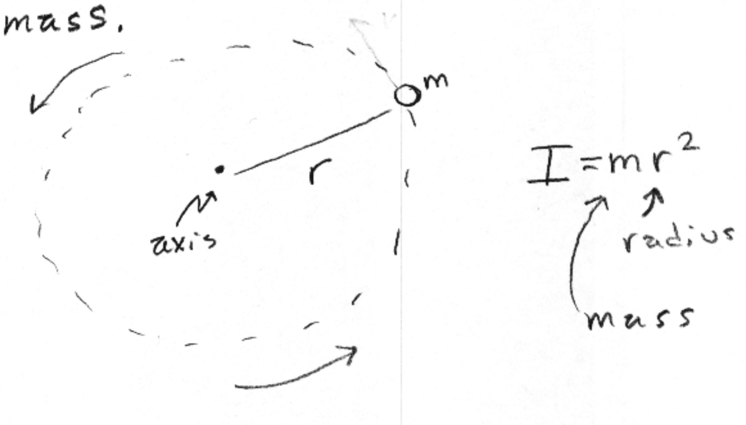
(I) symbol for moment of inertia

Moment of Inertia (I) is an angular analogy for mass.

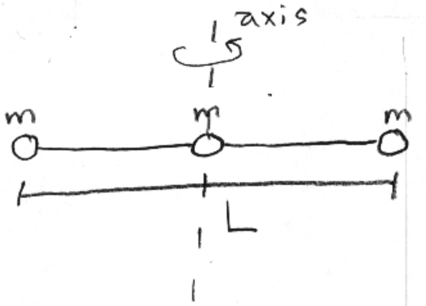
- Moment of inertia is an objects resistance to a change in rotation
- Just as mass resists a change in linear motion.

Equation for a point mass.

$$\boxed{I = mr^2}$$



For a series of masses.



- Determine where the axis is.
- (r) is the distance from that axis to (m)

$I = \Sigma mr^2$

-or-

$I = m_1 r_1^2 + m_2 r_2^2 \ldots.$

What is the moment of inertia of this?

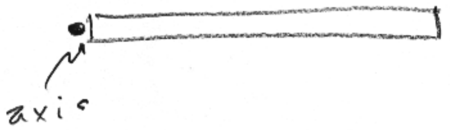
$$I = \Sigma mr^2$$

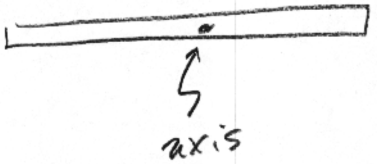
$$I = m\left(\tfrac{1}{2}L\right)^2 + m\left(\tfrac{1}{2}L\right)^2 + \cancel{m(0)^2}$$

$$I = \tfrac{1}{4}mL^2 + \tfrac{1}{4}mL^2 = \boxed{\tfrac{1}{2}mL^2}$$

To Estimate for moment of inertia think about how the mass is distributed from the axis of rotation.

- Which has the greater (I)?





Relationship to torque

$a = \alpha \cdot r$

$F = ma$

$F = m\alpha r$

$r \cdot F = m\alpha \cdot r \cdot r$

$\tau = F \cdot r$

$\tau = m\alpha r^2$

$\tau = mr^2\alpha$

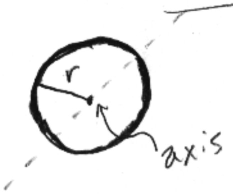
angular acceleration

$\tau = I\alpha$

resistance to rotation

Moment of Inertia for Common Shapes

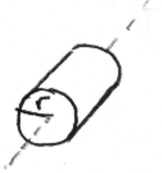
Ring



$I = mr^2$

Same as point mass!

Solid cylinder



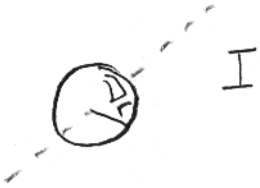
$I = \frac{1}{2}mr^2$

-or-

Disk

$I = \frac{1}{2}mr^2$

Solid sphere



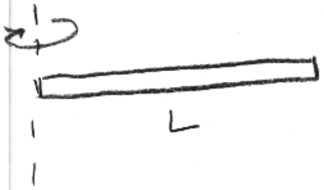
$I = \frac{2}{5}mr^2$

Hollow Sphere

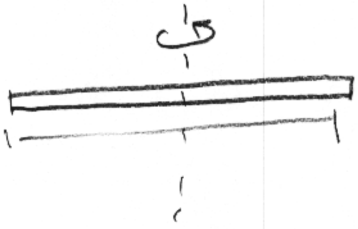
$I = \frac{2}{3}mr^2$

Thin Rod

$I = \frac{1}{3}mL^2$



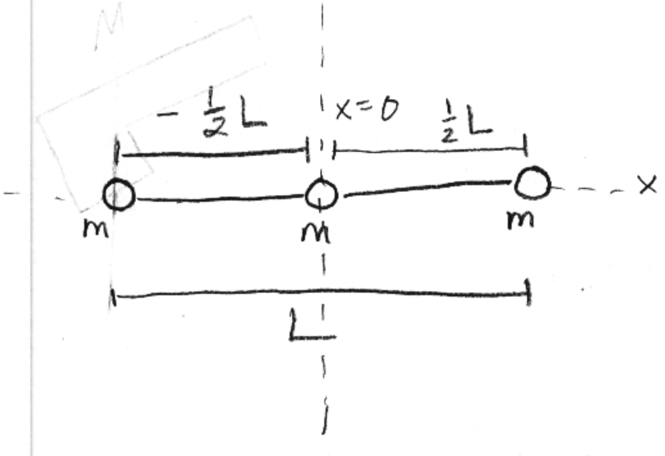
Thin Rod



$I = \frac{1}{12}mL^2$

Center of mass

★ Very different from moment of inertia! ★



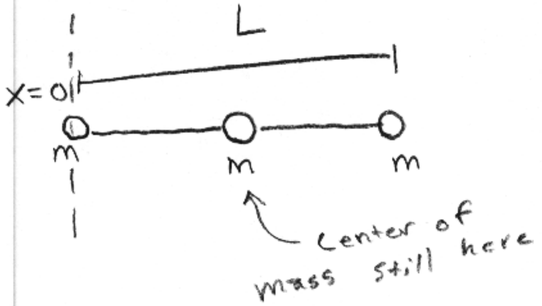
$$X_{cm} = \frac{\Sigma m x}{\Sigma m} = \frac{m_1 x_1 + m_2 x_2 \ldots}{m_1 + m_2 \ldots}$$

- Choose an origin point
- must use directions (You don't do that with I)

$$X_{cm} = \frac{m(\frac{1}{2}L) + m(-\frac{1}{2}L)}{m+m+m} = \frac{\frac{1}{2}mL - \frac{1}{2}mL}{3m} = 0$$

$$X_{cm} = 0\,m$$

Notice if we shift the origin.

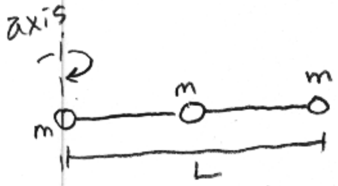


center of mass still here

$$X_{cm} = \frac{\cancel{m(0)} + m(\frac{1}{2}L) + mL}{m+m+m} = \frac{\frac{3}{2}\cancel{m}L}{3\cancel{m}}$$

$$\boxed{X_{cm} = \frac{1}{2}L}$$

Moment of Inertia (Notice the difference)

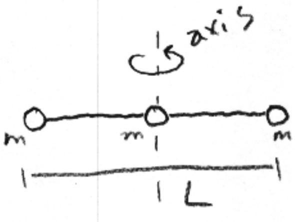


$$I = \Sigma m r^2$$

$$I = m(\frac{1}{2}L)^2 + mL^2$$

$$I = \frac{1}{4}mL^2 + mL^2$$

$$\boxed{I = \frac{5}{4}mL^2}$$



No direction or (r)

$$I = \Sigma m r^2$$

$$I = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{4}mL^2 + \frac{1}{4}mL^2$$

$$\boxed{I = \frac{1}{2}mL^2}$$