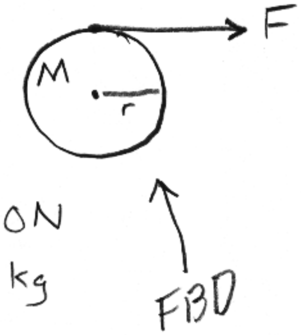
Newton's 2nd Law... again

$\boxed{\Sigma\tau = I\alpha}$ $\Sigma\tau = F\cdot r\sin\theta$

$\Sigma F\cdot r\sin\theta = I\alpha$ ← Function for dynamics

EX: Solid cylinder $I = \frac{1}{2}mr^2$



F = 20N
M = 3kg
r = 2m

What is the angular acceleration?

$$\Sigma\tau = I\alpha$$
$$-F\cdot\cancel{r} = \frac{1}{2}Mr^{\cancel{2}}\alpha$$
$$-F = \frac{1}{2}Mr\alpha$$
$$\alpha = \frac{-2F}{Mr} = \boxed{-6.67\,rad/s^2}$$

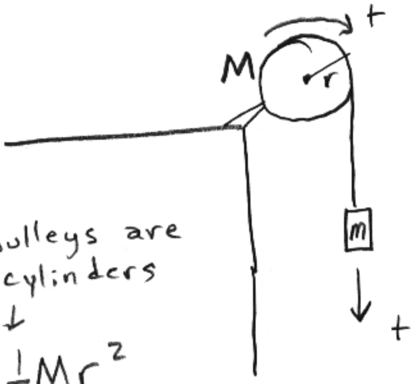
If it starts from rest what is $\Delta\theta$ after $t = 2s$?

$\alpha = -6.67\,rad/s^2$
$t = 2s$
$\omega_0 = 0$

$$\Delta\theta = \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2$$
$$\Delta\theta = \frac{1}{2}(-6.67)(2)^2 = \boxed{-13.34\,rad}$$

Easy ☺

With Pulleys!



- What is the acceleration of the block?

  ☆ The worlds of linear and angular motions must collide!!

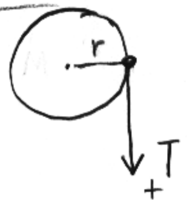
1) Draw FBD for all objects with mass.
2) Write 2nd Law for all FBD's
3) Choose positive direction
4) Relate α to $a \rightarrow \alpha = \frac{a}{r}$
5) Combine equations and solve.

Most pulleys are solid cylinders

$\downarrow$

$$I = \frac{1}{2}Mr^2$$

Rotational



$$\Sigma \tau = I\alpha$$

$$T \cdot r = I\frac{a}{r}$$

$$T \cdot \cancel{r} = \frac{1}{2}Mr^{\cancel{2}}\frac{a}{\cancel{r}}$$

$$T = \frac{1}{2}Ma$$

linear

$$\Sigma F = ma$$

$$mg - T = ma$$

$$mg - \frac{1}{2}Ma = ma$$

$$mg = ma + \frac{1}{2}Ma$$

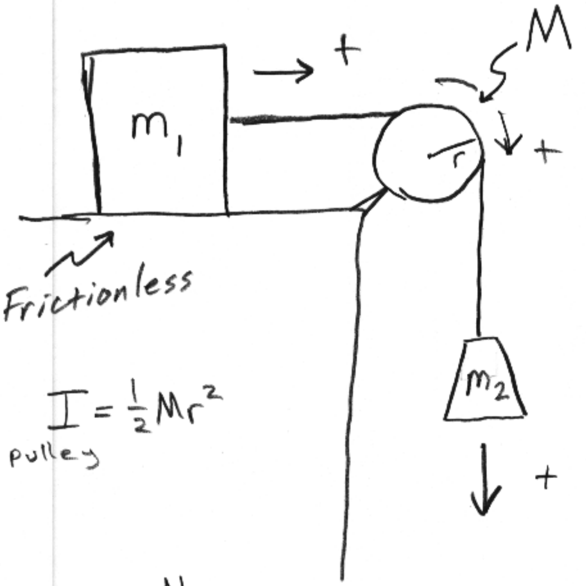
$$mg = a\left(m + \frac{1}{2}M\right)$$

$$\boxed{a = \frac{mg}{\left(m + \frac{1}{2}M\right)}}$$

- We used to treat pulleys as being massless, now since the pulley has mass (M) it has resistance to rotation.

- This is all assuming that the string does not slip and the axle is frictionless!

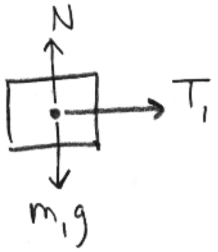
Try 1 More...

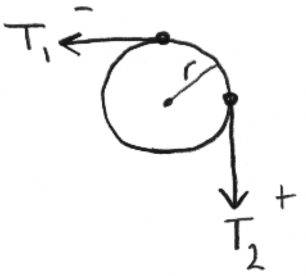


$I = \frac{1}{2}Mr^2$
pulley

What is the acceleration of m_2?

- We need 3 FBDs
- (a) will be shared by m_1 and m_2 and will also be the (a) at the rim of the pulley!
- Notice that tensions are different





$\Sigma F = m_1 a$

$\boxed{T_1} = m_1 a$

$\Sigma \tau = I\alpha$

$\boxed{T_2 r - T_1 r} = I\alpha$ $\quad \alpha = \frac{a}{r}$

$(T_2 - T_1) r = \boxed{\frac{1}{2}Mr^2}\alpha$

Need to sub

$(T_2 - T_1)\cancel{r} = \frac{1}{2}M\cancel{r^2}\boxed{\frac{a}{\cancel{r}}}$

$T_2 - T_1 = \frac{1}{2}Ma$

$\Sigma F = m_2 a$

$m_2 g - T_2 = m_2 a$

$\boxed{T_2} = m_2 g - m_2 a$

$m_2 g - m_2 a - m_1 a = \frac{1}{2}Ma$ ← *Alot of factoring get as on same side

$m_2 g = \frac{1}{2}Ma + m_2 a + m_1 a$

$m_2 g = a\left(\frac{1}{2}M + m_2 + m_1\right)$

$a = \dfrac{m_2 g}{\left(\frac{1}{2}M + m_2 + m_1\right)}$ ← Net force ← total mass

looks familiar :)