

KEY

NT6 WORK AND ENERGY

NT6A-WWT1: OBJECT CHANGING VELOCITY—WORK
A 2-kg object accelerates as a net force acts on it. During the 5 seconds this force acts, the object changes its velocity from 3 m/s east to 7 m/s west.



A student states:

"The change in kinetic energy of this object during these 5 seconds was 40 J, and thus the work done on this object by the net force during this period was also 40 J."

What, if anything, is wrong with this statement? If something is wrong, identify it, and explain how to correct it. If this statement is correct, explain why.

Answer: The student's answer is correct. The initial kinetic energy of the object was 9 Joules, and the final kinetic energy was 49 Joules. The change in kinetic energy was 40 Joules. Since the change in kinetic energy is equal to the net work done on the object, the net work must also be 40 Joules.

1

NT6A-CCT2: BICYCLIST ON A STRAIGHT ROAD—WORK

A bicyclist initially travels at a steady 8 m/s for 100 seconds on a straight level road, and then takes 40 seconds to slow to 5 m/s. Three students discussing this situation make the following contentions about the bicycle's kinetic energy:

Axel: "The bicycle is just going to slow down naturally. It doesn't take any work for something to slow down."

Bram: "I disagree. The speed of the bike decreased, so there is a change in kinetic energy. That means work was done on the bike."

Cassie: "I think Axel is right that no work was done, but I don't agree with his reason. There is no work being done here because there are no external forces being exerted."

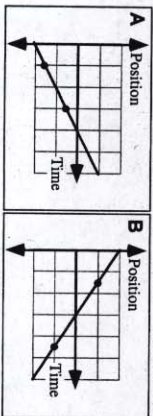
Which, if any, of these students do you agree with?

Axel _____ Bram _____ Cassie _____ None of them _____
Explain.

Answer: Bram is correct. The kinetic energy of the bicycle decreased, because its kinetic energy is one-half its mass times its velocity squared, and the velocity is decreasing. Since there is a negative change in kinetic energy for the bicycle, there must be a negative net work on the bicycle by the environment. (Since the system in question here is the bicycle, the forces acting on it include forces by the road, by the air (as wind resistance) and by the bicyclist.)

NT6A-WWT3: BOAT POSITION VS. TIME GRAPHS—WORK

Shown are graphs of the position versus time for two boats traveling along a narrow channel. The scales on both axes are the same for the graphs. In each graph, two points are marked with dots.



A student who is using these graphs to compare the net work done on the two boats between the two points says:

"I think that more net work was done on the boat in graph B because it moved farther during the interval between the points."

What, if anything, is wrong with this statement? If something is wrong, identify it, and explain how to correct it. If this statement is correct, explain why.

Answer: The student's contention is wrong. Since both graphs have a straight line for the motion of the sailboats, they moved at constant speeds, so there was no change in their kinetic energy and, consequently, no work was done on them.

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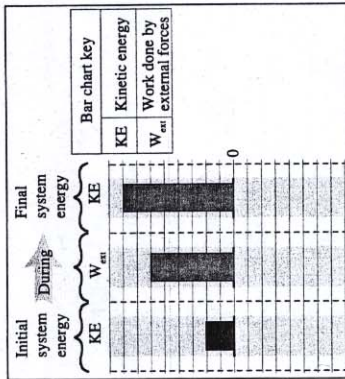
NT6A-BCT4: TUGBOAT CHANGING VELOCITY—WORK & KINETIC ENERGY BAR CHART

a) The velocity of a tugboat increases from 2 m/s to 4 m/s in the same direction as a force is applied to the tugboat for 20 seconds.

Fill in the missing bars for the work & kinetic energy bar chart for this process.



Explain.

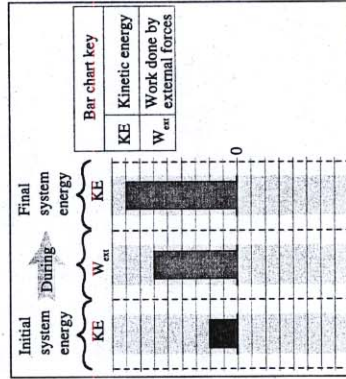


b) The velocity of a tugboat changes from 2 m/s to 4 m/s in the other direction as a force is applied to the tugboat for 20 seconds.

Fill in the missing bars for the work & kinetic energy bar chart for this process.



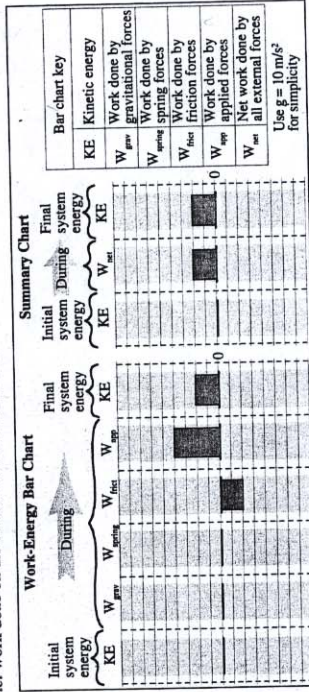
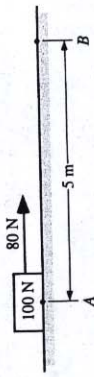
Explain.



NT6A-BCT6: BOX PULLED ON ROUGH SURFACE—WORK & KINETIC ENERGY BAR CHARTS

A 100 N box is initially at rest on a rough horizontal surface where the coefficient of static friction is 0.6 and the coefficient of kinetic friction is 0.4. A student decides to move the box by applying a horizontal force of 80 N to the box to the right as shown. The box starts at rest at point A.

Fill in the work-kinetic energy bar charts below for the box as it moves between points A and B. The chart at left below has columns for work done by four kinds of external force. In the summary chart at right below you should include the net work done on the box as it moves from A to B.



Explain.

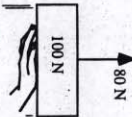
Answer: The applied force has to do enough work to both increase the KE of the box and to compensate for the negative work done by the frictional force. The applied force will do $(80 \text{ N})(5 \text{ m}) = 400 \text{ J}$ of work. The frictional force will do $(40 \text{ N})(5 \text{ m}) = -200 \text{ J}$ of work. So the final kinetic energy will be 200 J, which is the net work done.

Answers (a) and (b) will have the same bar chart graphs. In both cases, the speed of the tugboat doubles and so the kinetic energy quadruples. In both cases, the same net work will have been done on the tugboat.

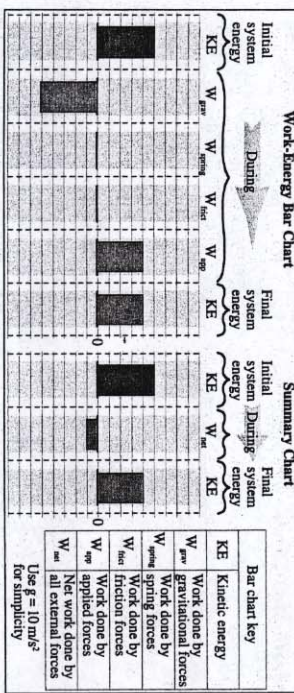
NT6A-BCT7: BOX MOVING UPWARD I—WORK & KINETIC ENERGY BAR CHARTS

A 100-N box is initially moving upward at 10 m/s. A student is applying a vertical force of 80 N upward with his hand as shown.

Fill in the work-kinetic energy bar charts below for the box as it moves upward a distance of 5 meters. The chart at left below has columns for work done by four kinds of external force. In the summary chart at right below you should include the net work done on the box as it moves upward 5 meters.



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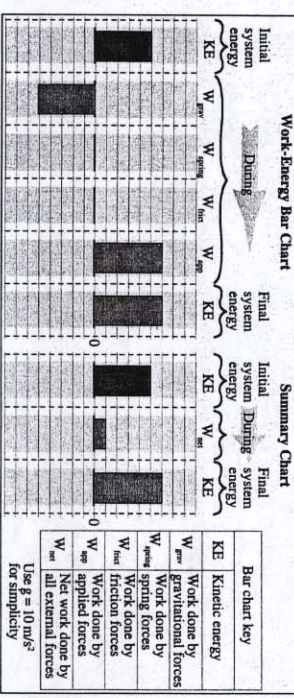
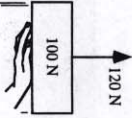
Explain.

Answer: The initial kinetic energy is $(0.5)(10\text{kg})(10\text{m/s})^2$, or 500J. The work done by the applied force is $+(80\text{N})(5\text{m}) = 400\text{J}$. There is no friction force or spring force for this problem. Since the weight is acting in the opposite direction to the displacement, the work done by the gravitational force is negative: $-(100\text{N})(5\text{m}) = -500\text{J}$. The net work done on the box is therefore $400\text{J} - 500\text{J} = -100\text{J}$, and this added to the initial kinetic energy gives the final kinetic energy, $500\text{J} - 100\text{J} = 400\text{J}$.

NT6A-BCT8: BOX MOVING UPWARD II—WORK & KINETIC ENERGY BAR CHARTS

A 100-N box is initially moving upward at 10 m/s. A student is applying a vertical force of 120 N upward with her hand as shown.

Fill in the work-kinetic energy bar charts below for the box as it moves upward a distance of 5 meters. The chart at left below has columns for work done by four kinds of external force. In the summary chart at right below you should include the net work done on the box as it moves upward 5 meters.

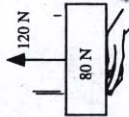


Explain.

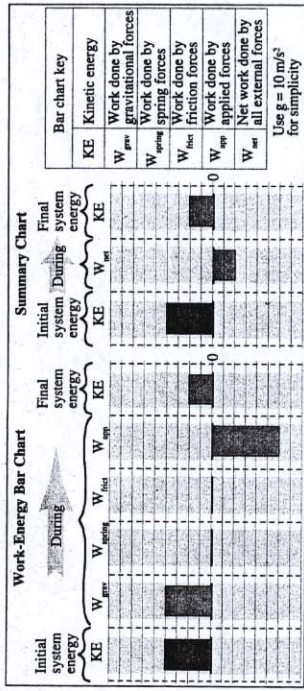
Answer: The initial kinetic energy is $(0.5)(10\text{kg})(10\text{m/s})^2$, or 500J. The work done by the applied force is $+(120\text{N})(5\text{m}) = 600\text{J}$. There is no friction force or spring force for this problem. Since the weight is acting in the opposite direction to the displacement, the work done by the gravitational force is negative: $-(100\text{N})(5\text{m}) = -500\text{J}$. The net work done on the box is therefore $600\text{J} - 500\text{J} = 100\text{J}$, and this added to the initial kinetic energy gives the final kinetic energy, $500\text{J} + 100\text{J} = 600\text{J}$.

NT6A-BCT9: BOX MOVING DOWNWARD—WORK & KINETIC ENERGY BAR CHARTS

An 80-N box is initially moving downward at 10 m/s. A student is applying a vertical force of 120 N upward on the box with her hand as shown.



Fill in the work-kinetic energy bar charts below for the box as it moves downward a distance of 5 meters. The chart at left below has columns for work done by four kinds of external force. In the summary chart at right below you should include the *net* work done on the box as it moves downward 5 meters.

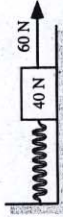


Explain.

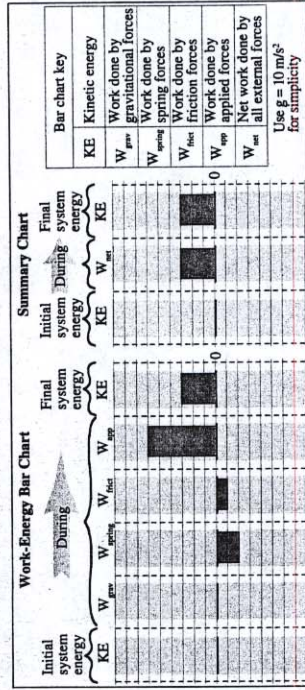
Answer: The initial kinetic energy is $(0.5)(8\text{kg})(10\text{m/s})^2$, or 400J. The work done by the applied force is negative because this force is in the opposite direction to the displacement, and is $(120\text{N})(5\text{m}) = -600\text{J}$. There is no friction force or spring force for this problem. Since the weight is acting in the same direction as the displacement, the work done by the gravitational force is positive: $(80\text{N})(5\text{m}) = 400\text{J}$. The net work done on the box is therefore $400\text{J} - 600\text{J} = -200\text{J}$, and this added to the initial kinetic energy gives the final kinetic energy, $400\text{J} - 200\text{J} = 200\text{J}$.

NT6A-BCT10: BOX ATTACHED TO SPRING—WORK & KINETIC ENERGY BAR CHARTS

A 40-N box is initially at rest on a rough horizontal surface. A spring with spring constant 10 N/m connects the box to the wall and is unstretched. A 60 N force is applied horizontally to the right as shown. The maximum static friction force between the box and the surface is 15 N, and there is a kinetic friction force of 10 N between the box and the surface when the box is moving.



Fill in the work-kinetic energy bar charts below for the box as it moves to the right a distance of 4 meters. The chart at left below has columns for work done by four kinds of external force. In the summary chart at right below you should include the *net* work done on the box as it moves to the right 4 meters.

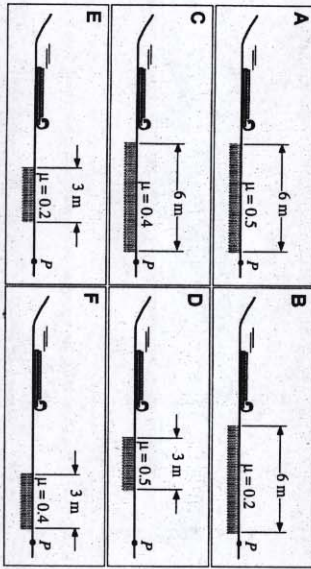


Explain.

Answer: The initial kinetic energy is zero since the box is initially at rest. There is no work done by the gravitational force since the displacement of the box is perpendicular to the weight. The spring exerts a force in a direction opposite the displacement, so the work done by the spring is negative, and will be $-(0.5)(10\text{N/m})(4\text{m})^2$, or -80J. (The spring force is increasing as the spring is stretched since $F_{\text{spring}} = -kx$, and the work done is the integral of the force times the displacement from zero to the final displacement.) The work done by the applied force is positive because this force is in the same direction as the displacement, and is $(60\text{N})(4\text{m}) = 240\text{J}$. The friction force acts in the opposite direction to the displacement, so it does negative work equal to $-(10\text{N})(4\text{m}) = -40\text{J}$. The net work done on the box is therefore $240\text{J} - 40\text{J} - 80\text{J} = 120\text{J}$, and this added to the initial kinetic energy (zero) gives the final kinetic energy, 120J.

NT6B-RT11: TOBOGGANS ON A HORIZONTAL SURFACE—SPEED

The figures below show identical toboggans that have traveled down a snowy hill. The toboggans all have the same speed at the bottom of the hill. Assume that the horizontal surfaces that they travel along are frictionless except for the shaded areas, where the coefficient of friction is given. These shaded areas have different lengths as shown.



Rank these situations on the basis of the speed of the toboggans as they reach point P.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The speed is the same but not zero for all these toboggans.

OR, The speed is zero for all these toboggans.

OR, We cannot determine the ranking for the speed of these toboggans.

Please explain your reasoning.

Answer: $E > B = F > D > C > A$. If we consider the toboggan as a system, the force of friction does negative work on the system which is equal to the net work. This work is equal to the change in kinetic energy for the toboggan. All toboggans start with the same kinetic energy, and so the toboggan with the greatest change in kinetic energy will have the smallest kinetic energy (and therefore speed) at point P. The work on the toboggan is the force of friction times the distance through which this force acts, and this work is negative and proportional to the product of $\mu \cdot d$. Therefore, the ranking of the speeds at point P is in inverse order to the ranking of the product $\mu \cdot d$.

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NT6B-LMCT12: BLOCK PUSHED ON INCLINE—WORK DONE

A block is pushed so that it moves up a ramp at constant speed.

Identify from choices (a)-(e) below the appropriate description for the work done by the specified force while the block moves from point A to point B.

- (a) is zero.
- (b) is less than zero.
- (c) is greater than zero.
- (d) could be positive or negative depending on the choice of coordinate systems.
- (e) cannot be determined.

(1) The work done on the block by the hand

Explain.

Answer c, greater than zero. The force by the hand has a component in the same direction as the displacement, so the work done is positive.

(2) The work done on the block by the normal force from the ramp

Explain.

Answer a, zero. The normal force is always perpendicular to the displacement, so the dot product of force and displacement is zero.

(3) The work done on the block by friction

Explain.

Answer b, less than zero. The angle between the friction force and the displacement is 180° , so the dot product of friction and displacement is negative.

(4) The work done on the block by the gravitational force

Explain.

Answer b, less than zero. The angle between the gravitational force and the displacement is greater than 90° , so the dot product of the gravitational force and displacement is negative.

(5) The net work done on the block

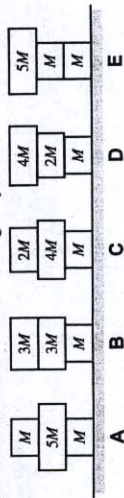
Explain.

Answer a, zero. The block is moving at a constant speed, so there is no change in kinetic energy. From the work-kinetic energy theorem, the net work done equals the change in kinetic energy.



NT6B-RT13: STACKED BLOCKS SETS—WORK TO ASSEMBLE

Shown below are five stacks, each containing three blocks. The masses of the blocks are given in the diagram in terms of M , the mass of the smallest block. Each block has the same height and has its center of mass at the center of the block. Originally, all the blocks were flat on the ground.



Rank the work required to assemble each stack.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ Least

OR, The work required to assemble each stack is the same but not zero.

OR, The work required to assemble each stack is zero.

OR, The ranking for the work required to assemble the stacks cannot be determined.

Explain your reasoning.

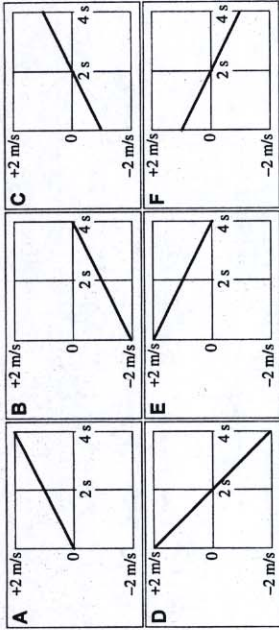
Answer: $E > D > B > C > A$.

The ranking depends on the mass of each block and the location of the center of mass of each block in the stack. No work is required for the lowest blocks since they are already at that height. Each block above the first one requires work equal to the weight of that block times the distance that it is raised. For stack A, block 5M requires $5Mgh$ and the top block requires $3Mgh + 3Mg(2h) = 9Mgh$; for C, $4Mgh + 2Mg(2h) = 8Mgh$; for stack D, $2Mgh + 4Mg(2h) = 10Mgh$; and for stack E, $Mgh + 5Mg(2h) = 11Mgh$.



NT6B-RT14: VELOCITY VS. TIME GRAPHS FOR IDENTICAL OBJECTS—WORK DONE

Shown below are graphs of velocity versus time for six identical objects that move along a straight, horizontal, frictionless surface. A single external force acts on each object.



Rank the work done on the objects by the external force during the 4-second interval shown.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The work done is the same but not zero for all these situations.

OR, The work done is zero for all these situations.

OR, We cannot determine the ranking for the work done for these situations.

Please explain your reasoning.

Answer: $A > C = D = F > B = E$.

From the work-kinetic energy theorem, the work done on the object is equal to the change in kinetic energy of the object. So if we rank the change in kinetic energy of the objects, the ranking of work done by the external agents will be the same. The change in kinetic energy will be proportional to the change in the square of the velocity. The change in the square of the velocity is the same for B and E, but is negative.

NT6B-CCT16: BLOCKS SLIDING DOWN FRICTIONLESS RAMPS—WORK BY THE NORMAL FORCE

Two identical blocks are released from rest at the same height. Block A slides down a steeper ramp than Block B. Both ramps are frictionless. The blocks reach the same final height indicated by the lower dashed line. Three students comparing the work done on the two blocks by the normal force state:



Amnika: "I think the normal force doesn't do any work on either block. The force on the block by the ramp is perpendicular to the ramp, and the displacement is parallel to the ramp. So the dot product is zero."

Bobae: "Work is force times displacement. The work done on Block A is negative, while the work done on Block B is positive, because the displacement for B is in the positive direction, while the displacement for A is in the negative direction."

Craig: "Since work is force times distance, and the distance the block travels is greater for Block B, the work done is greater for Block B."

Which, if any, of these students do you agree with?

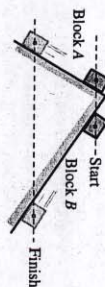
Amnika _____ Bobae _____ Craig _____ None of them _____
Please explain your reasoning.

Amnika is correct. Work is a dot product of force and displacement, and if the force is perpendicular to the displacement, the work done by that force is zero.

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NT6B-CCT17: BLOCKS SLIDING DOWN FRICTIONLESS RAMPS—WORK BY THE EARTH

Two identical blocks are released from rest at the same height. Block A slides down a steeper ramp than Block B. Both ramps are frictionless. The blocks reach the same final height indicated by the lower dashed line. Three students comparing the work done on the two blocks by the gravitational force (the weight of the blocks):



Asmita: "Work is the dot product of force and displacement, and the weight is the same since the blocks are identical. But Block B travels further, so more work is done on Block B by the gravitational force than on Block A."

Ben: "Both blocks fall the same vertical distance, so the work done is the same."

Cocheta: "By Newton's Third Law, the force exerted on the block by the earth is exactly cancelled by the force exerted on the earth by the block. The work done is zero."

Danne: "The dot product depends on the angle that the force makes with the displacement. If we put the displacement and force vectors tail-to-tail, the angle is smaller for Block B than for Block A, and so the work done is greater."

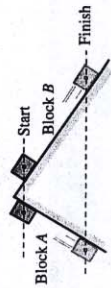
Which, if any, of these students do you agree with?

Asmita _____ Ben _____ Cocheta _____ Danne _____ None of them _____
Please explain your reasoning.

Ben is correct. Work is the dot (scalar) product of force and displacement, and the force in both cases here is straight down. The work done by the earth is equal to $mgL\cos\theta$, where θ is the angle that the ramp makes with the vertical and L is the distance traveled down the ramp. But $L\cos\theta$ is also the vertical distance between the dashed lines, which is the same for both blocks.

NT6B-QRT18: BLOCKS SLIDING DOWN FRICTIONLESS INCLINES—WORK AND ENERGY

Two identical blocks are released from rest at the same height at the same time. Block A slides down a steeper ramp than Block B. Both ramps are frictionless. The blocks reach the same final height, indicated by the lower dashed line.



- From the starting height to the final height, the work done on Block A by the normal force from the ramp
 - is zero.
 - is negative.
 - is positive.
 - could be positive or negative depending on the choice of coordinate systems.

Explain.

(a) The normal force is perpendicular to the displacement, so the work done by the normal force is zero.

- From the starting height to the final height, the work done on Block A by the gravitational force (the weight)
 - is zero.
 - is negative.
 - is positive.
 - could be positive or negative depending on the choice of coordinate systems.

Explain.

(c) The angle between the weight and the displacement is less than 90° , so the work done by the earth is positive.

- From the starting height to the final height, the work done on Block A by the gravitational force is
 - greater than the work done on Block B by the gravitational force.
 - less than the work done on Block B by the gravitational force.
 - equal to the work done on Block B by the gravitational force.

Explain.

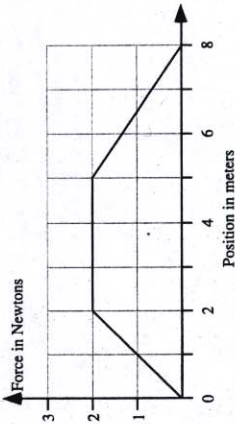
(c) Since the dot product of the weight and the displacement is equal to the negative of the weight times the change in height for both blocks, the work done on the two blocks is the same.

- The speed of Block A as it crosses the lower dashed line is
 - greater than the speed of Block B as it crosses the lower dashed line.
 - less than the speed of Block B as it crosses the lower dashed line.
 - equal to the speed of Block B as it crosses the lower dashed line.

Explain.

NT6C-WWT21: FORCE VS. POSITION GRAPH I—WORK DONE ON BOX

A 10-kg box initially at rest is pushed a distance of 8 m along a smooth horizontal floor. A graph of the applied horizontal force on the block as a function of displacement is shown below.



A student calculates that the work done by the applied force during the first 2 meters was 4 J and that the work done during the following 3 meters was 6 J.

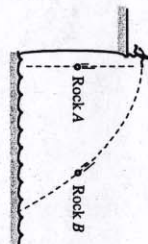
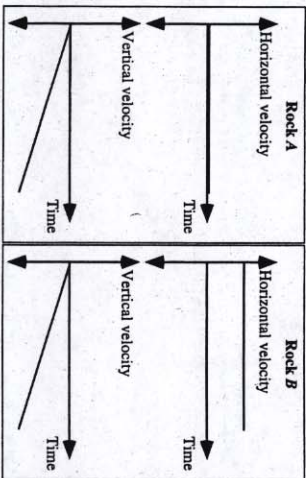
What, if anything, is wrong with this calculation? If something is wrong, identify it and explain how to correct it. If this calculation is correct, explain why.

Answer: For the first two meters, the work is given by the area under the curve and is therefore 2 J. $(1/2) * 2 \text{ m}^2 \text{ N}$



NT6D-CT26: DROPPED AND THROWN ROCK—KINETIC ENERGY

Rock A is dropped from the top of a cliff at the same instant that an identical Rock B is thrown horizontally away from the cliff. Each of the following graphs describes part of the motion of the rocks. Use a coordinate system in which up is the positive vertical direction and the positive horizontal direction is away from the cliff with the origin at the point the balls were released.



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1) Is the kinetic energy of the dropped Rock A at the start of the motion greater than, less than, or the same as the kinetic energy of the thrown Rock B?

Explain.

Rock A has less initial kinetic energy since Rock A has zero initial velocity and thus zero kinetic energy.

2) Is the kinetic energy of the dropped Rock A halfway down greater than, less than, or the same as the kinetic energy of thrown Rock B halfway down?

Explain.

Rock A has less kinetic energy since Rock A and Rock B have the same vertical velocity but Rock B also has a horizontal velocity, and a greater speed, thus it a larger kinetic energy.

NT6D-CCT29: SKATERS PUSHING OFF EACH OTHER—FORCE

Two skaters—a small girl and a large boy—are initially standing face-to-face but then push off each other. After they are no longer touching, the girl has more kinetic energy than the boy. Three physics students make the following commentions about the forces the boy and girl exerted on each other:

Aranna: "I think the boy pushed harder on the girl because he is bigger, so she ended up with more kinetic energy than he did."

Boris: "I disagree. They pushed equally hard on each other, but the girl moved farther while they were pushing on each other, so she ended up with more kinetic energy."

Carmen: "I think the girl had to push harder to get the boy moving since he is bigger, but that caused her to accelerate more as she recoiled."

Which, if any, of these three students do you agree with?

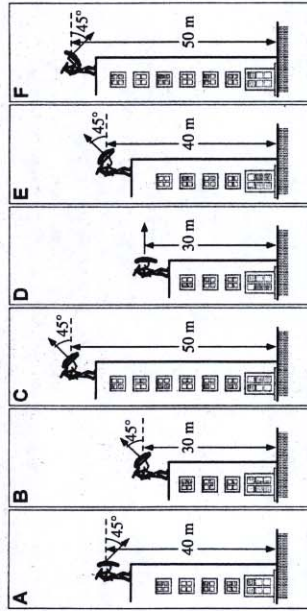
Aranna _____ Boris _____ Carmen _____ None of them _____

Explain.

Answer: Boris is correct. Newton's Third Law requires that they exert forces of equal magnitude on each other and they are in contact for the same time. Equal force on the smaller mass of the girl will give her a larger acceleration. The larger acceleration for the same time means that she moves farther. Equal force applied through a larger distance means more work was done on her and so she ends up with more kinetic energy.

NT6F-RT39: ARROWS SHOT FROM BUILDINGS—FINAL SPEED

In each case below, an arrow has been shot from the top of a building either up at a 45° angle, straight out horizontally, or down at a 45° angle. All arrows are identical and are shot at the same speed, and the heights of the buildings and the direction the arrows are shot are given. Ignore air resistance.



Rank these arrows on the basis of their speeds just before they hit the ground below.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The speed is the same for all these arrows but not zero.

OR, The speed is zero for all these arrows.

OR, We cannot determine the ranking for the speeds of the arrows.

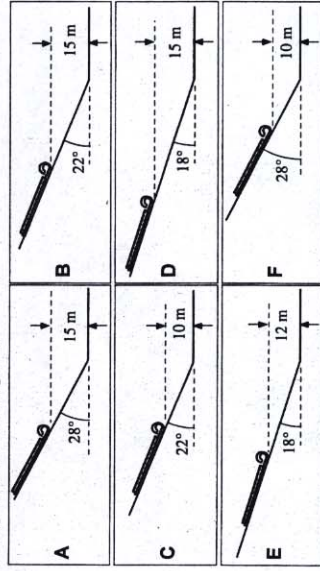
Please explain your reasoning.

Answer: $C = F > A = E > B = D$.

This is an application of conservation of energy. All have the same kinetic energy at the start because they all are fired at the same speed. All arrows have zero potential energy at end of their flight, so those with greatest potential energy at start will have greatest kinetic energy and speed at bottom.

NT6F-RT40: TOBOGGANS GOING DOWN SLIPPERY HILLS—SPEED AT BOTTOM

In each case below, a toboggan starts from rest and slides without friction down a snowy hill. The toboggans are all identical, and the starting heights (vertical distance above the flat bottom of the incline) and angles of the hills are given.



Rank these situations on the basis of the speed of the toboggan at the bottom of the incline.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The speed is the same for all these toboggans but it is not zero.

OR, The speed is zero for all these toboggans.

OR, We cannot determine the ranking for the speed of these toboggans.

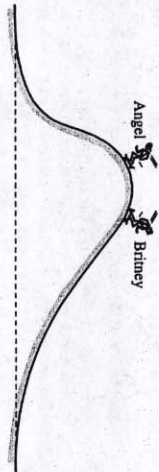
Please explain your reasoning.

Answer: $A = B = D > E > C = F$.

The speed depends on the starting height only. The initial potential energy of the toboggan will be proportional to the starting height, and without friction, all of this potential energy will be converted into kinetic energy at the bottom.

NT6F-CT141: SKATEBOARDERS ON A HILL—TIME, SPEED, KINETIC ENERGY, AND WORK

Starting from rest, Angel and Britney skateboard down a hill as shown. Angel rides down the steep side while Britney rides down the shallow side. Angel has more mass than Britney. Assume that friction and air resistance are negligible.



a) Is the speed at the bottom of the hill greater for Angel, greater for Britney, or the same for both skateboarders? Explain.

Answer—Same for both. Both skateboarders lose the same amount of height as they travel down the hill, and their change in potential energy (strictly speaking, the change in potential energy of the skateboard-earth system) is equal to their gains in kinetic energy. Both kinetic and potential energy terms are proportional to the mass, so it doesn't matter that the skateboarders have different masses.

b) Is the time it takes to get to the bottom of the hill greater for Angel, greater for Britney, or the same for both skateboarders? Explain.

Answer—Greater for Britney. Both start from rest, and Angel, who will have the greater acceleration, also has a shorter path.

c) Is the work done by the gravitational force on the skateboarder greater for Angel, greater for Britney, or the same for both skateboarders? Explain.

Answer—Greater for Angel. If the two skateboarders had the same mass, then the work done by the gravitational force would be the same for both. (Work depends only on displacement in direction of force, and the vertical displacement is the same for the two skateboarders.) But since the gravitational force is proportional to the mass, this force is greater on Angel, and more work will be done on her by the gravitational force.

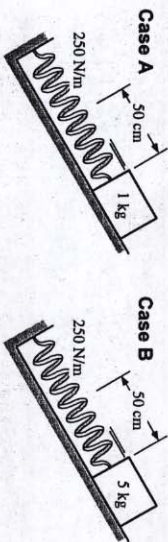
d) Is the work done by the normal force on the skateboarder greater for Angel, greater for Britney, or the same for both skateboarders? Explain.

Answer—Same for both. The displacement for each skateboarder at all points on the hill is parallel to the surface of the hill, and the normal force at all points is perpendicular to the hill, so the angle between the normal force and the displacement is 90° . Since work depends on the dot product of the force and displacement vectors, the work done by the normal force is zero for both skater.

e) Is the kinetic energy at the bottom of the hill greater for Angel, greater for Britney, or the same for both skateboarders? Explain.

Answer: Greater for Angel. They have the same speed, but Angel has a larger mass.

NT6F-CCT45: BLOCK WITH COMPRESSED SPRING ON A RAMP—HEIGHT UP RAMP
Two blocks are placed on a frictionless ramp and held against a spring that is compressed one-half meter. (The mass of the blocks and force constant of the springs are also given for each system.) The blocks are then released from rest, and the compressed springs cause the blocks to accelerate up the ramp while the springs are in contact with the blocks. At the instant shown, the blocks are just about to lose contact with the end of the springs. Three students are discussing how far the blocks will slide up the ramps.



Andy: "I think they will both travel the same distance up the inclines. The kinetic energy at the point shown in the diagram is equal to the initial potential energy stored in the compressed spring. This is the same for both cases since they both are compressed the same distance and have the same spring constants. The kinetic energy at the point shown is equal to the gravitational potential energy at the top. Since both the kinetic energy and the gravitational potential energy depend on the mass, the mass cancels out, leaving the same heights for each case."

Badu: "Since they both have the same energies when they are initially compressing the springs, they have to have the same energy at the top when they stop. So the lighter mass has to go higher."

Colin: "I think the block in Case B will go higher since it has more mass and its momentum should be larger at the point shown since they both have the same initial potential energy."

Which, if any, of these three students do you agree with?

Andy _____ Badu _____ Colin _____ None of them _____

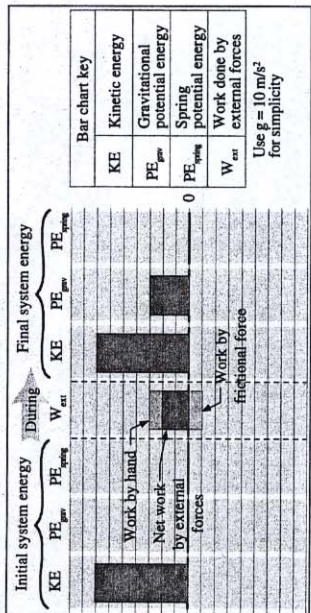
Explain.

Answer: Badu is correct. The initial potential energy stored in the spring is the same for the two cases, since this only depends on the spring constant and on how much the spring is compressed. At the instant shown, all of this spring potential energy has been converted to gravitational potential energy and kinetic energy. When the block reaches its highest point, it will be at rest, and all of the initial spring potential energy will have been converted to gravitational potential energy. Since gravitational potential energy is proportional to mass, and the final gravitational potential energies will be the same, block B will have less height so that the product mgh will be the same for both.

NT6F-BCT47: BLOCK PUSHED ON ROUGH RAMP—BASIC ENERGY BAR CHART

A block is pushed so that it moves up a rough ramp at constant speed from A to B.

Complete the qualitative energy bar chart below for the earth-block system before and after the block has moved from A to B. Put the zero point for the gravitational potential energy at A.



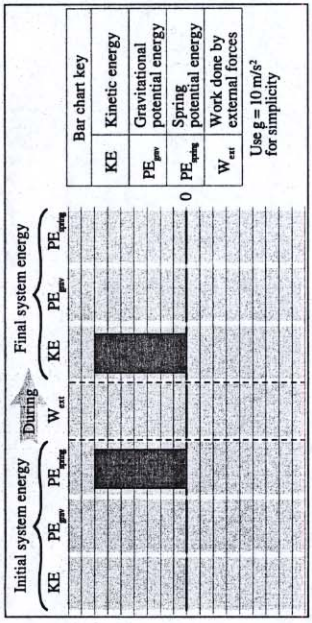
Explain.

Answer: Since the block is moving at constant speed, there is no change in the kinetic energy. The push on the block has a component in the same direction as the displacement from A to B, so positive work is done on the block from A to B by the force from the hand. Friction acts in the opposite direction to the displacement, so it does negative work on the block from A to B. Since the block is higher at B than at A, there is an increase in gravitational potential energy for the block-earth system.

NT6F-BCT51: SKATEBOARDER LAUNCHED BY A SPRING—BASIC ENERGY BAR CHART I

A circus performer on a skateboard is launched by a spring initially compressed a distance Δx as shown at right. His speed on the horizontal portion of the ramp is v , and he rises to a height H after he leaves the ramp. Ignore any effects due to friction.

Draw a qualitative energy bar chart for the earth-skateboarder-spring system as the skateboarder goes from the compressed spring position at rest to where he moves free of the spring on the horizontal surface. Put the zero point for the gravitational potential energy at the height of the skateboarder before launching.



Explain.

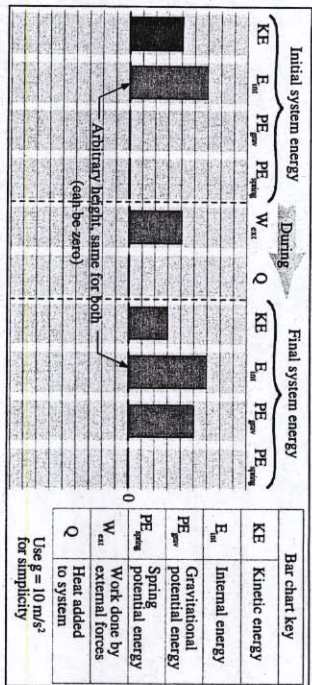
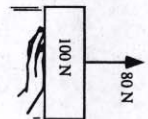
Answer: The performer is initially at rest and at the zero gravitational potential energy height, so the only initial system energy is the potential energy of the compressed spring. Since the spring is part of the system, the push on the performer is not external work; there is no external work done on our system. The final state is the performer just after he has lost contact with the spring. Here he has kinetic energy, and the gravitational potential energy is still zero. Since the spring is no longer compressed there is no spring potential energy. All of the initial system energy has been converted into kinetic energy.

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NT6F-BCT53: BOX MOVING UPWARD —ENERGY BAR CHART FOR THE EARTH-BOX SYSTEM

A 100-N box is initially moving upward at 4 m/s. A student is applying a vertical force of 80 N to the box as shown.

Complete the energy bar chart below for the earth-box system as the box moves upward a distance of 1 m with the zero point for the gravitational potential energy set at the initial height.



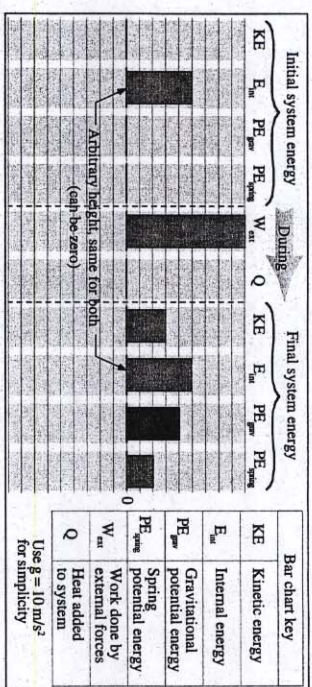
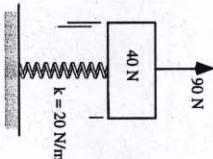
Explain.

Answer: The initial kinetic energy is $(0.5)(10\text{kg})(4\text{m/s})^2 = 80\text{J}$. There is no change in the internal energy of the earth-box system so the initial and final values of the internal energy are the same. The work done by the hand is positive since the force and displacement are in the same direction, and is equal to $(80\text{N})(1\text{m}) = 80\text{J}$. The initial gravitational potential energy is zero, and the final gravitational potential energy is $mgh_f = (100\text{N})(1\text{m}) = 100\text{J}$.

NT6F-BCT57: BOX ATTACHED TO A SPRING —ENERGY BAR CHART

A 40-N box is initially held at rest, is pulled a distance of 2 m upward by a constant upward vertical force of 90 N. The box is attached to a spring with a stiffness of 20 N/m that is initially not stretched or compressed.

Complete the energy bar chart for the earth-box-spring system for this process.

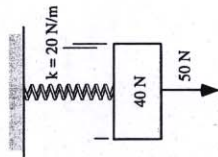


Explain.

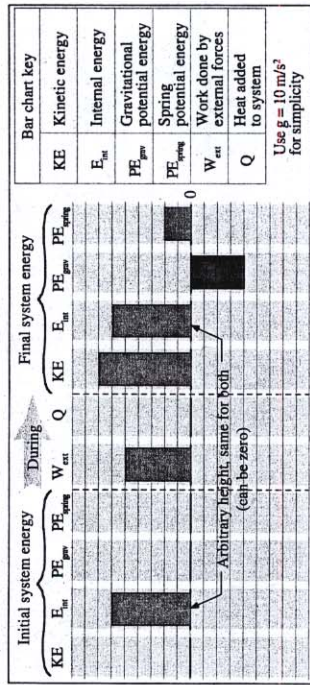
Answer: The final potential energy will be $mgh_f = (40\text{N})(2\text{m}) = 80\text{J}$ greater than the initial potential energy, so if we let 1 division be 20J, then the bar given for the final gravitational potential energy implies an initial gravitational potential energy of zero. The initial kinetic energy is zero since the box is initially at rest. There is no change in the internal energy of the earth-box-spring system so the initial and final values of internal energy are the same. The work done by the upward force is positive since the force and displacement are in the same direction, and is equal to $(90\text{N})(2\text{m}) = 180\text{J}$. The spring is initially at its rest position, so the initial spring potential energy is zero, and the final spring potential energy is $(0.5)(20\text{N/m})(2\text{m})^2 = 40\text{J}$.

NT6F-BCT57: BOX SUSPENDED BY A SPRING II—ENERGY BAR CHART

A 40-N box initially held at rest is pulled a distance of 2 m downward by a constant downward vertical force of 50 N. The box is attached to a fixed spring with a stiffness of 20 N/m that is initially not stretched or compressed.



Complete the energy bar chart for the earth-box-spring system for this process.



Explain.

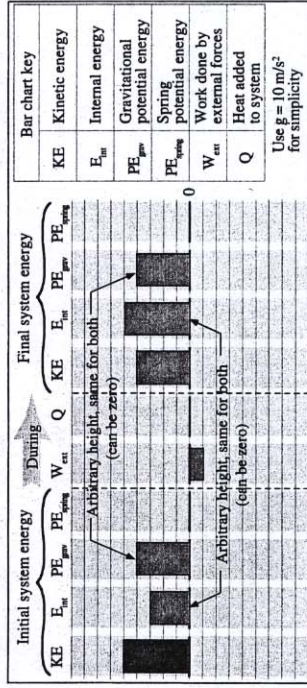
Answer: The final potential energy will be $mg\Delta h = (40\text{N})(2\text{m}) = 80\text{J}$ less than the initial potential energy, so if we let 1 division be 20J, then the bar given for the final gravitational energy is zero since the box is initially at rest. There is no change in the internal kinetic energy of the earth-box-spring system so the initial and final values of internal energy are the same. The work done by the downward force is positive since the force and displacement are in the same direction, and is equal to $(50\text{N})(2\text{m}) = 100\text{J}$. The spring is initially at its rest position, so the initial spring potential energy is zero, and the final spring potential energy is $(0.5)(20\text{N/m})(2\text{m})^2 = 40\text{J}$.

NT6F-BCT60: BALL THROWN UPWARDS I—ENERGY BAR CHART FOR BALL & EARTH

A ball is thrown straight upwards. After reaching the top of its trajectory, it falls back to the height it was released from, but during its flight 20% of the ball's initial kinetic energy was lost due to air resistance. Assume the ball's temperature does not change during this time.



Complete the energy bar chart below for the ball-earth system for this process.



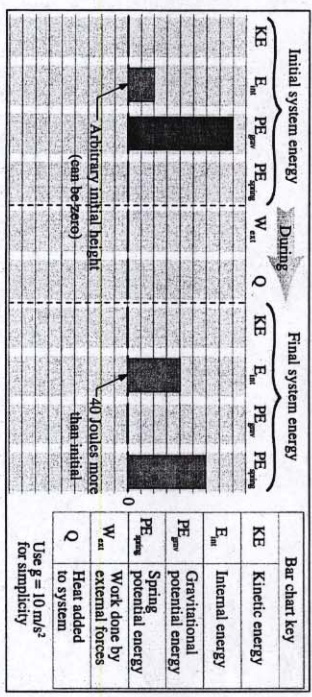
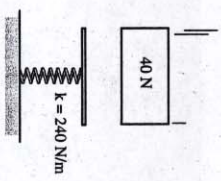
Explain.

Answer: The bar for the initial kinetic energy is 5 units high, so the final kinetic energy bar must be 20% less or 4 units high. The initial and final internal energies of the ball are the same (the ball stays at a constant temperature). The gravitational potential energy is also the same, since the ball is at the same height. The air is not part of the system, and the force of the air on the ball is in the opposite direction to the displacement. The kinetic energy lost by the ball is equal to the work done on the ball by the air.

NT6F-BCT63: BOX DROPPED ONTO A SPRING—ENERGY BAR CHART FOR BOX, EARTH, & SPRING

A 40-N box is dropped from a vertical height of 3 meters above the top of an uncompressed spring that has a spring constant of 240 N/m. The box-spring system comes to rest momentarily when the spring is compressed a vertical distance of 1 meter below its initial position. (Note: The system will start oscillating after reaching this point.) Ignore air resistance.

Complete the energy bar chart below for the box-earth-spring system for this process where the zero point for the gravitational potential energy is the location of the box when the spring has been compressed 1 m (lowest point).



Explain.

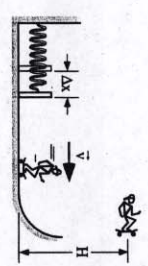
Answer: The initial gravitational potential energy of the block and the earth is $mg = (40 \text{ N})(3 \text{ m}) = 120 \text{ J}$. Since this is represented by a bar 8 units high, the scale of the bar chart is 20 J per unit height. The initial spring potential energy is zero since the spring is initially unstretched and uncompressed, and the final spring potential energy is $(0.5)(240 \text{ N/m})(1 \text{ m})^2 = 120 \text{ J}$. There is no initial kinetic energy and no final kinetic energy. There is not as much gain in spring potential energy as there is loss of gravitational potential energy, suggesting that the block hitting the spring caused the internal energy of the block and spring to increase. (The temperature of the block and the spring increase as a result of the block hitting the spring.)

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NT6F-CT65: SKATEBOARDER LAUNCHED BY A SPRING—SPEED & HEIGHT

A circus performer on a skateboard is launched by a spring initially compressed a distance Δx as shown at right. His speed on the horizontal portion of the ramp is v , and he rises to a height H after he leaves the ramp. He then conducts a second launch with the spring initially compressed a distance $2\Delta x$.

a) For the second launch, will the speed of the skateboarder on the horizontal portion of the ramp be greater than $2v$, less than $2v$, or equal to $2v$?



Explain.

Answer: Equal to. If we choose the instant of release as our initial situation, and an instant when the performer is in motion along the horizontal surface as our final situation, and zero height as the height at the bottom of the ramp, then the initial energy is spring potential only and the final energy is total energy at the instant shown.

b) Will the height reached by the skateboarder for the second launch be greater than $2H$, less than $2H$, or equal to $2H$?

Explain.

Answer: Greater than. With the same initial situation and zero height reference as part (a), our final situation here will be when the performer reaches maximum height. For the final situation, all of the energy is gravitational potential energy. So the height is proportional to the square of the initial spring compression, and will quadruple.

