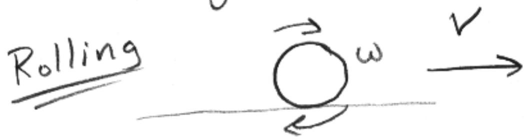
Kinetic Energy of Rotation

- Rotating objects have rotational kinetic energy

- They can have both translational and rotational kinetic energies

$K = \frac{1}{2}mv^2$ $K_R = $?



Use linear to angular analogies.

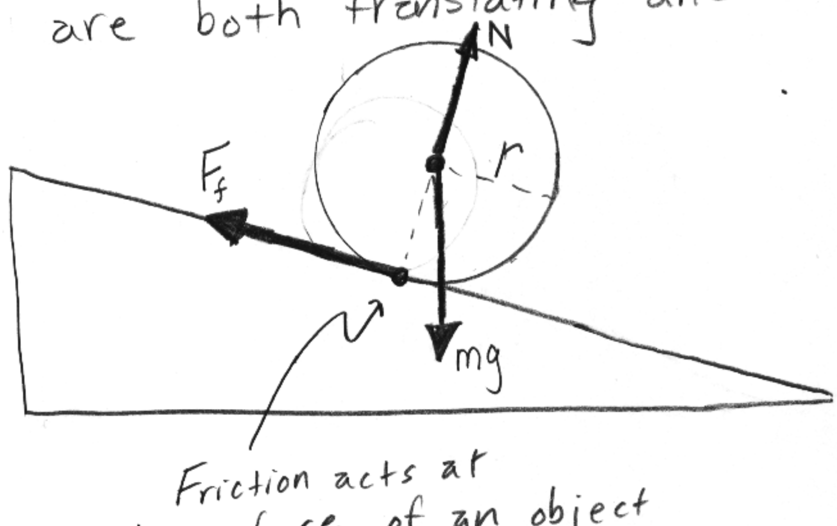
$\bar{v} \longrightarrow \omega$

$m \longrightarrow I$

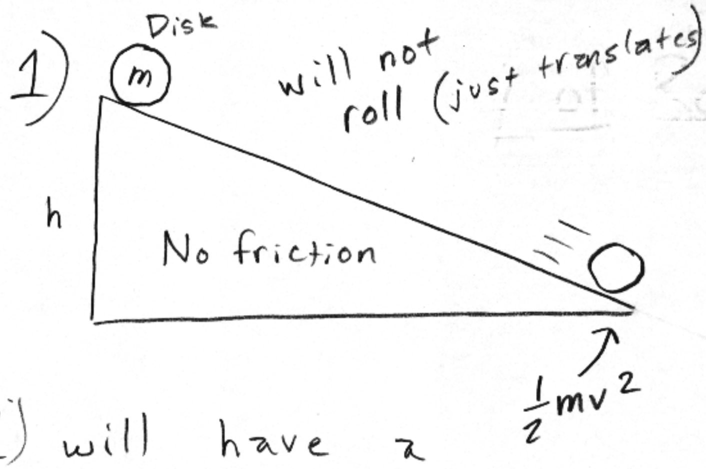
$$\boxed{K_R = \frac{1}{2} I \omega^2}$$

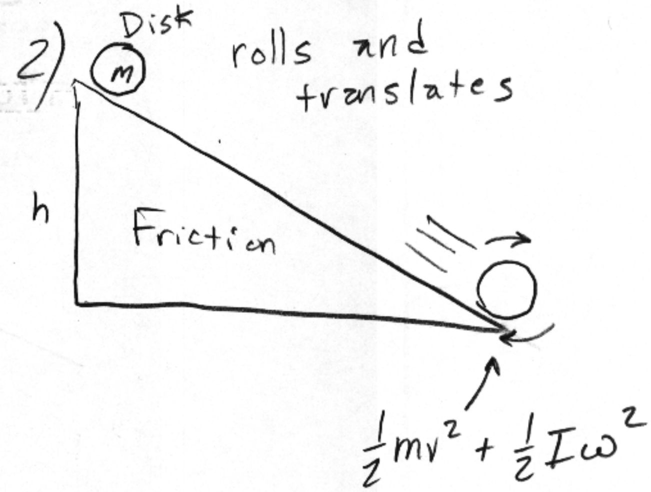
Rolling Motion

- Objects are both translating and rotating.



So, what causes torque?





(1) will have a greater (v)

2) will have a smaller (v) because some of the potential energy became rotation.

COE

1

$U_0 = K$

$mgh = \frac{1}{2}mv^2$

$$\boxed{v = \sqrt{2gh}}$$

2

$\omega = \frac{v}{r}$

$I = \frac{1}{2}mr^2$

$U_0 = K + K_R$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

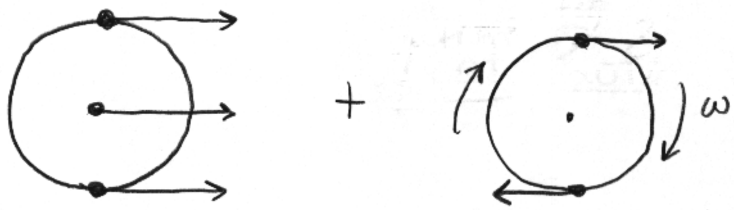
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{4}m\cancel{r^2}\frac{v^2}{\cancel{r^2}}$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{4}\cancel{m}v^2$

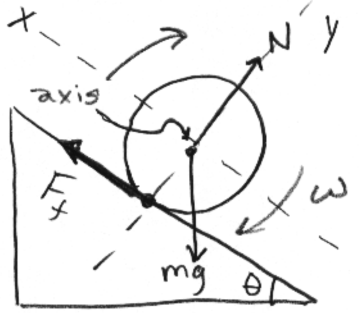
$gh = \frac{3}{4}v^2$

$$\boxed{v = \sqrt{\frac{4}{3}gh}}$$



Translational Rotational

I prefer to take the torque about the center of the object.



Force of friction causes torque about the axis.

Force of gravity translates the object.

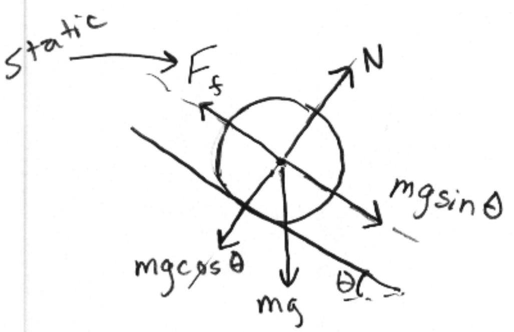
Translation	Rotation
$\Sigma F = ma$	$\Sigma \tau = I\alpha$

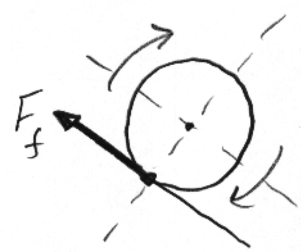
$F_f = \mu N$

$N = mg\cos\theta$

$I_{disk} = \frac{1}{2}mr^2$

$\alpha = \frac{a}{r}$





$mg\sin\theta - F_f = ma$

$\cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta = \cancel{m}a$

$g\sin\theta - \mu g\cos\theta = a$

$F_f \cdot r = I\alpha$

$\mu \cancel{m} g\cos\theta \cdot \cancel{r} = \left(\frac{1}{2}\cancel{m}r^{\cancel{2}}\right)\left(\frac{a}{\cancel{r}}\right)$

$\mu g\cos\theta = \frac{1}{2}a$

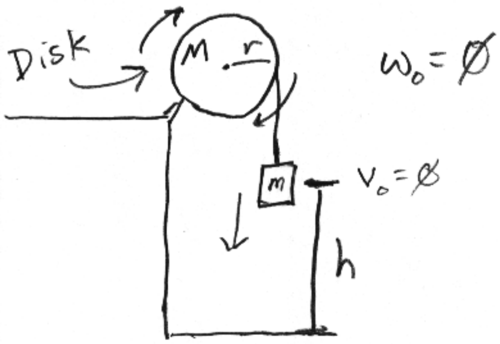
$g\sin\theta - \frac{1}{2}a = a$

$g\sin\theta = a + \frac{1}{2}a$

$g\sin\theta = \frac{3}{2}a$

$\boxed{a = \frac{2}{3}g\sin\theta}$

Pulley with Energy



What is the speed of (m) at bottom.

COE

$$U_0 = K + K_R$$

block Pulley

$$I = \frac{1}{2}Mr^2$$

$$\omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}M\cancel{r^2}\frac{v^2}{\cancel{r^2}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$$

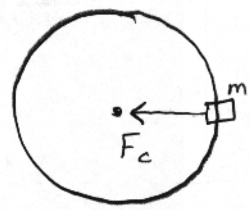
$$mgh = v^2\left(\frac{1}{2}m + \frac{1}{4}M\right)$$

$$\boxed{v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{4}M}}}$$

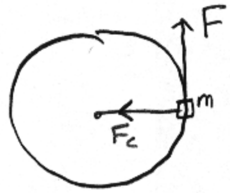
Rotational Work and Power

UCM

No work done



$W = F \cdot d$



$$W = F \cdot r \, \Delta\theta = \tau \, \Delta\theta$$

$$\boxed{W = \tau \, \Delta\theta}$$

$$W = \Delta K$$

$$W = \Delta K_R = \tfrac{1}{2} I \omega^2 - \tfrac{1}{2} I \omega_0^2$$

Power

$$P = \frac{W}{t} = \frac{F \cdot d}{t} = F \cdot v$$

$$\boxed{P = \tau \cdot \omega}$$