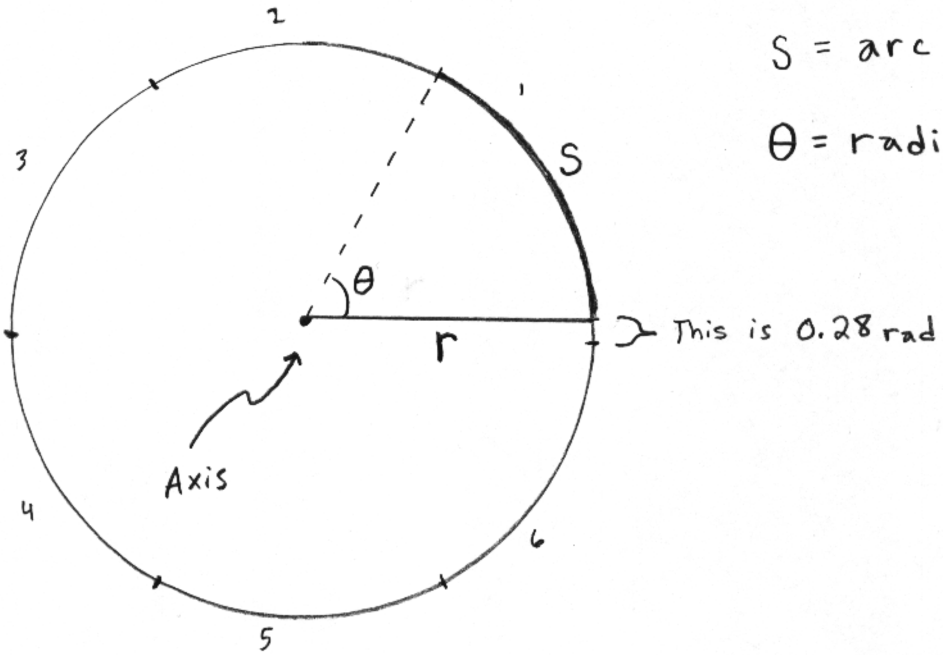
AP1 Rotation Notes Part 1

Angular Quantities vs. linear

r = radius (m)

S = arc length (m)

θ = radian (unitless)



To find a radian imagine taking the radius and projecting it along the outer edge of the circle.

The amount of angular displacement that correlates to this is equal to <u>1 radian</u>.

Count how many radiuses you can project around the entire circle.

- You find it to be just over 6
- in fact it is 0.28 over, thats $\boxed{6.28 \text{ radians}}$ -or- $\boxed{2\pi \text{ radians}}$
- This is also equal to $360°$ or 1 revolution

So, where is the circumference from

This equation relates linear and angular quantities

$$\boxed{S = \theta \cdot r}$$

radius (m)

arc length (m)

Angular displacement (radians)

For 1 rev

$$S = 2\pi \text{rad} \cdot r \quad \text{-or-} \quad 2\pi r$$

$$C = 2\pi r$$

is an arc length

Is the radian really a unit?

$$\theta = \frac{S\,(\cancel{m})}{r\,(\cancel{m})} = \text{unitless}$$

- We use (rad)

· Converting Quantities

$$2\pi \text{rad} = 360°$$

- Radians into degrees

$$\frac{1\text{rad} \cdot 360°}{2\pi \text{rad}} = 57.3°$$ ← 1 radian

$$\frac{180° \cdot 2\pi \text{rad}}{360°} = \boxed{\pi \text{rad} \text{ -or- } 3.14 \text{rad}}$$

- Revolutions to radians

$$1\text{rev} = 2\pi \text{rad}$$

$$\frac{4\,\cancel{\text{rev}} \cdot 2\pi \text{rad}}{1\,\cancel{\text{rev}}} = \boxed{8\pi \text{rad} \text{ -or- } 25.13 \text{rad}}$$

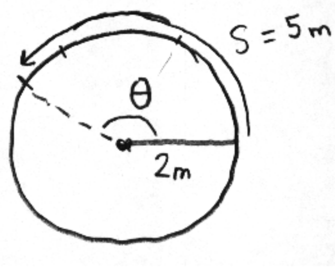
- linear displacement to Angular displacement

$S = X$ same idea $\qquad S = \theta \cdot r \qquad \theta = \frac{S}{r}$

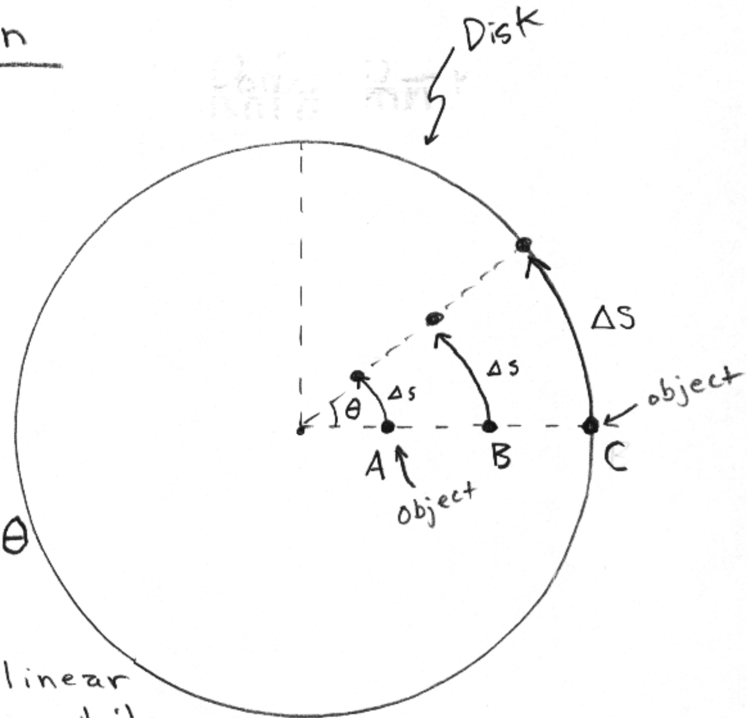
$S = 5m$

$r = 2m$

$$\theta = \frac{5\cancel{m}}{2\cancel{m}} = \boxed{2.5 \text{rad}}$$



Angular Motion



The objects are revolving at the same rate

C has greatest ΔS

A, B, and C have same $\Delta\theta$

Notice that the linear displacements differ while angular displacement is the same.

$$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{\Delta S\,(m)}{\Delta t\,(s)}$$ velocity is displacement over time $\left(m/s\right)$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$ angular velocity is angular displacement over time

Angular velocity called omega

$$\omega = \frac{\Delta\theta\,(rad)}{\Delta t\,(s)} = rad/s \text{ units}$$

A, B, C have the same (ω) but very different (v)

C covers a greater arc length in the same time.

For any circle you can convert to and from angular and linear velocity since $S = \theta \cdot r$

$$\frac{\Delta S}{\Delta t} = \bar{V} \qquad \frac{\Delta\theta}{\Delta t} = \bar{\omega}$$

$$\boxed{\bar{V} = \bar{\omega} \cdot r} \text{ -and- } \boxed{\bar{\omega} = \frac{\bar{V}}{r}}$$

Non-Uniform Circular Motion

What happens when ω is changing (slowing or increasing spin)

Well, this indicates <u>angular acceleration</u>.

<u>Alpha</u> $\boxed{\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}}$ $\frac{rad/s}{s} = rad/s^2$ units

Angular acceleration

If the rate of rotation changes so does linear acceleration.

$\boxed{\bar{a} = \bar{\alpha} \cdot r}$ -or- $\boxed{\bar{\alpha} = \frac{a}{r}}$