ROTATION AND ANGULAR MOTION

r $\quad \omega_0 = \emptyset$ $\quad \alpha$ $\quad T$

1. A wheel of .5 m radius starts from rest and accelerates at 5 revs/sec^2 over a period of 3 seconds. $\alpha = 10\pi \text{ rad/s}^2$

   a) What is the angular speed (in rads/sec) of the wheel at the end of the 3 second period?
   $\omega = \omega_0 + \alpha t = \boxed{30\pi \text{ rad/s}}$

   b) How fast is the center of the wheel moving at this time ?
   $V = r \cdot \omega/2 = \boxed{7.5 \text{ m/s}}$

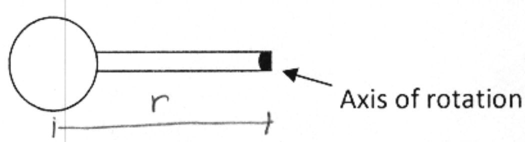
   c) What is the angular displacement (in radians) of the wheel after 3 seconds ?
   $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \boxed{45\pi \text{ rad}}$

   d) How far has the wheel rolled over the 3 second period ?
   $\Delta x = S = \theta \cdot r = \boxed{70.7 \text{ m}}$

   e) If the wheel has a moment of inertia of 5 kg m^2, what is its rotational KE after 3 seconds ?
   $K = \frac{1}{2} I \omega^2 = \boxed{22{,}210 \text{ J}}$

2. A 5 kg ball is attached to one end of a 3 m long rod. The rod has a mass of 2 kg. The rod is free to swing from a pivot on the end of the rod that does not have the mass on it as shown below. The size of the ball is negligible compared to the length of the rod.

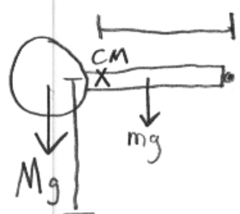
   M $\quad r$ $\quad m$



The rod-Ball combination is released from the horizontal position and falls until it reaches a vertical position

a. What is the moment of inertia of the rod-ball system about the pivot ?
$I_{tot} = I_b + I_r = Mr^2 + \frac{1}{3}mr^2 = \boxed{51 \text{ Kg}\cdot\text{m}^2}$

b. How far away from the axis is the center of mass of the rod-ball system ?
$X_{cm} = \frac{Mr + m\frac{1}{2}r}{M+m} = \boxed{2.57 \text{ m}}$



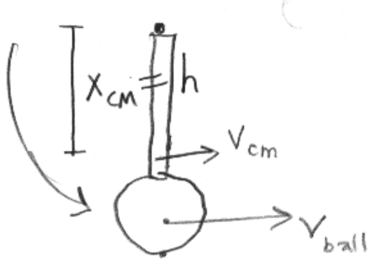
c. What is the net torque acting on the rod-Ball system when it is in its horizontal position ?
$\Sigma \tau = \tau_1 + \tau_2 = Mgr + \frac{1}{2}mgr = \boxed{180 \text{ N}\cdot\text{m}}$

d. What is the initial angular acceleration of the rod ball system when it is released from its horizontal position ?
$\Sigma \tau = I\alpha \quad \alpha = \frac{\Sigma \tau}{I} = \frac{180}{51} = \boxed{3.53 \text{ rad/s}^2}$

e. What is the angular velocity of the rod ball system as it reaches the vertical position ?
COE $(M+m)gX_{cm} = \frac{1}{2}I\omega^2 \quad \omega = \sqrt{\frac{2(M+m)gX_{cm}}{I}} = \boxed{2.66 \text{ rad/s}}$

f. What is the linear speed of the center of mass of the rod-ball system as it moves through its vertical position ?
$V_{cm} = \omega \cdot X_{cm} = \boxed{6.84 \text{ m/s}}$

g. What is the linear speed of the ball at the end of the rod as it moves through the vertical position ?
$V = \omega \cdot r = \boxed{7.98 \text{ m/s}}$



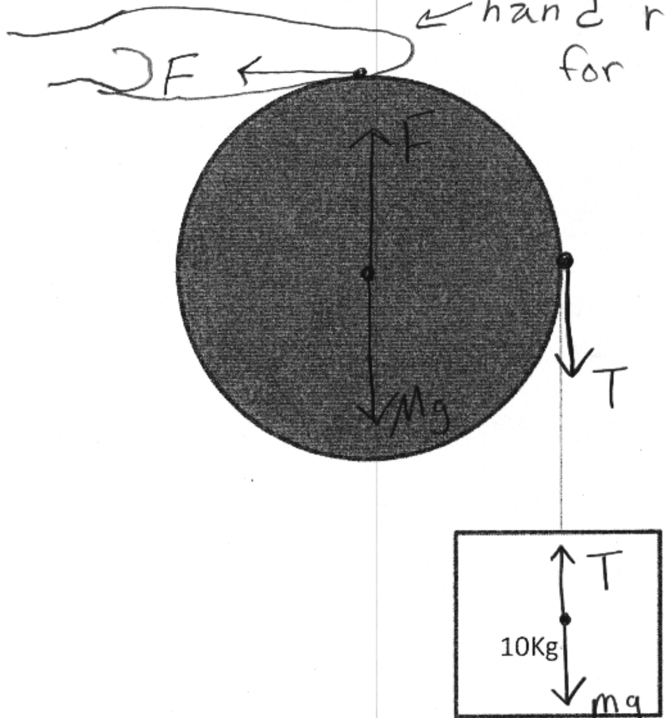
height is X_{cm}

[redacted]

A mass is connected across a pulley of radius 1.5m as shown. The pulley has a moment of inertial of $1.75 Kgm^2$.

← hand removes for part(b)

Draw FBD



a. Assuming the system above is stationary, how much torque is the mass applying to the pulley?

$$\tau = F \cdot r = T \cdot r = \boxed{mgr}$$

b. When the system is allowed to move the torque on the pulley is decreased to a value less than your answer to part a. Explain why?

$$mg - T = ma \qquad T = mg - ma \qquad \boxed{\tau = T \cdot r = (mg-ma)r}$$

c. Write Newton's Second Law for rotation for the pulley in this example.

$$\Sigma \tau = I\alpha$$

$$\underbrace{(mg-ma)}_{\text{"T"}}r = I\alpha$$

d. Determine the angular acceleration of the pulley and the linear acceleration of the mass.

"Pulley"

$$\Sigma \tau = I\alpha$$

$$T \cdot \cancel{r} = I\frac{a}{\cancel{r}}$$

$$T = Ia \quad \text{sub}$$

"Mass"

$$\Sigma F = ma$$

$$mg - T = ma$$

$$mg - Ia = ma$$

$$mg = a(I+m)$$

$$a = \frac{mg}{I+m}$$

$$\boxed{a = 8.51 m/s^2}$$

$$\boxed{\alpha = \frac{a}{r} = 5.67 rad/s^2}$$

e. How much rope will be required if the mass needs to fall for a time of 5 seconds?

$t = 5s$

$$\Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$$

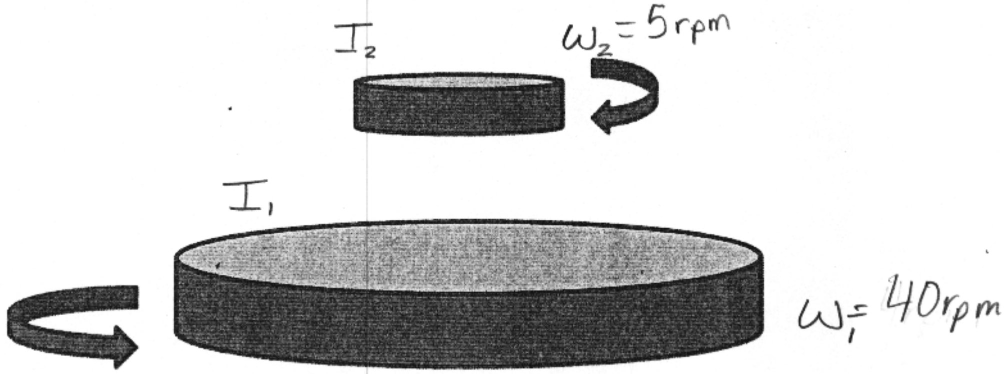
$$\Delta x = \frac{1}{2}at^2 = \boxed{106.4m}$$

[redacted]

I_1

A disk with moment of inertia 10Kgm^2 is rotating at a velocity of 40rpm counterclockwise. A second disk of moment of inertia 4Kgm^2 rotating at 5rmp clockwise is placed on top of the first disk.

I_2



a. Why can we say that angular momentum will be conserved in this situation?

It is always conserved when the $\Sigma\tau = \emptyset$

Newton's 3rd Law says the torque 1 applies on 2 is equal but opposite to 2 on 1. $\Sigma\tau$ of system $= \emptyset$

b. Determine the angular velocity of the two disk system after the small disk is placed on top of the large disk.

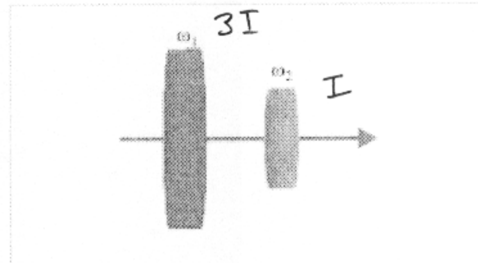
$$\omega_1 = \frac{40\text{rev}}{\text{min}} \cdot \frac{2\pi \text{rad}}{1\text{rev}} \cdot \frac{1\text{min}}{60\text{s}} = 4.18\text{rad/s}$$

$$L_o = L_f$$

$$\omega_2 = \frac{5\text{rev}}{1\text{min}} \cdot \frac{2\pi \text{rad}}{1\text{rev}} \cdot \frac{1\text{min}}{60\text{sec}} = 0.52 \text{ rad/s}$$

$$I_1\omega_1 - I_2\omega_2 = (I_1 + I_2)\omega_f$$

$$\omega_f = \frac{I_1\omega_1 - I_2\omega_2}{(I_1 + I_2)} = \boxed{2.84\text{rad/s}}$$



3. Two disc of different sizes rotate around the same axis at different speeds, so that disc 1 has an angular velocity of ω_1, and disc 2 has an angular velocity of ω_2. The moment of inertia of disc 1 is 3I , and the moment of inertia of disc 2 is I. The discs are force together until friction between the discs cause them to rotate at the same speed. COAM

a. What is the total angular momentum of the two discs before they are pressed together ?

$L = 3I\omega_1 + I\omega_2 = 4I(\omega_1 + \omega_2)$

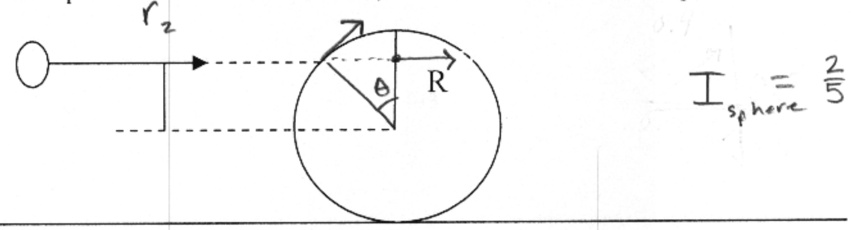
b. What is the total angular momentum of the two discs after they are combined ?

same

c. What is the final angular speed of the combined 2 disc system ?

$$\omega = \frac{4I(\omega_1+\omega_2)}{4I} = \omega_1 + \omega_2$$

4. A .2 kg blob of clay is thrown with a velocity of 20 m/s at a 5 kg sphere initially at rest. The clay blob hits and sticks to the sphere at a point .3 m above its center, where the radius of the sphere is .5 = r_1 meters.



$I_{sphere} = \frac{2}{5}Mr^2$

a. What is the angular momentum of the clay blob about the center of the sphere before it strikes the sphere ? $L = mvr_2 = 0.2kg(20)(0.3m) = \boxed{1.2 \ kg \cdot m^2/s}$

b. What is the total angular momentum of the clay blob- sphere combination immediately after the collision ? same COAM

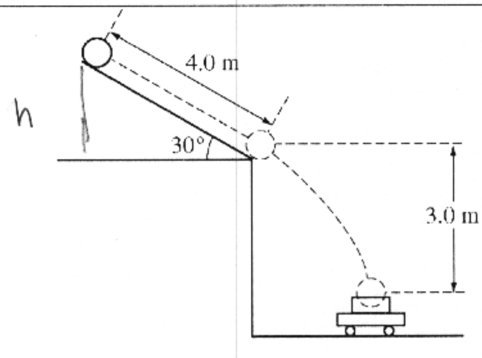
c. What is the final angular velocity of the clay blob- sphere combination immediately after the collision ?

$$L_0 = L$$

$$mvr_2 = I_{tot}\,\omega$$

$$mvr_2 = \left(\frac{2}{5}Mr_1^2 + mr_1^2\right)\omega$$

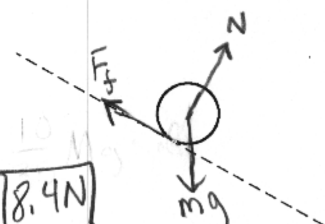
$$\omega = \left[\frac{1.2 \ kg \cdot m^2/s}{\left(\frac{2}{5}(5)(0.5)^2 + (0.2)(0.5)^2\right)}\right] = \boxed{2.18 \ rad/s}$$



Note: Figure not drawn to scale.

APC 2010 Mech. 2.
A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



$F_f = \frac{2}{5}M\left(\frac{5}{7}g\sin\theta\right)$

$F_f = \frac{10}{35}Mg\sin\theta = \boxed{8.4N}$

b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$\Sigma\tau = I\alpha$

$F_f \cdot R = \frac{2}{5}MR^2\frac{a}{R}$

$F_f = \frac{2}{5}Ma$

$\Sigma F = Ma$

$Mg\sin\theta - F_f = Ma$

$Mg\sin\theta - \frac{2}{5}Ma = Ma$

$\frac{7}{5}a = g\sin\theta$

$a = \frac{5}{7}g\sin\theta$

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

$h = 4\sin\theta = 2m$

Kinematics $\Delta X = 4m$

$v^2 = v_0^2 + 2a\Delta X$

$v = \sqrt{2(\frac{5}{7}g\sin\theta)(4)} = \boxed{5.34 m/s}$

COE

$Mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2\frac{v^2}{R^2}$

$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2$

$v^2 = \frac{10}{7}gh$

$v = \sqrt{\frac{10}{7}gh}$

$\boxed{v = 5.34 m/s}$

d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

COM

ONLY need X-velocity

$V_x = V_0\cos\theta$

$V_x = 4.62 m/s$

$\overset{P_0}{M_b V_x} = \overset{P}{(M_c + M_b)V_f}$

$V_f = \frac{6(4.62)}{6+12} = \boxed{1.54 m/s}$