NAME DATE

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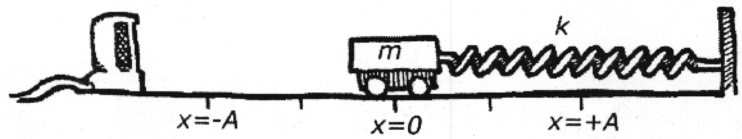
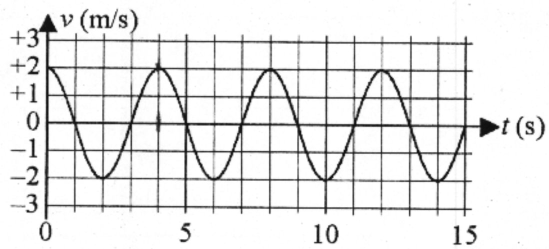
Scenario

A cart of mass m is connected to a spring of spring constant k and displaced to position $x = +A$. The cart is released and oscillates about the position $x = 0$. At time $t = 0$, the cart passes through the origin having rightward velocity. For the 15 seconds after this time, Angela and Blake use motion-sensing equipment to measure the cart's velocity (where right is positive). The graph below shows this velocity as a function of time.





$T = 4s$

$E = 10J$

Quantitative Analysis

PART A: The spring-cart system has 10 J of total energy. Calculate the values of m, k, and A. Explain your methods with words as you show calculations.

First, I solve for:

$E = K_{max} = U_{max}$

$E = \frac{1}{2} m v_{max}^2$

$m = \frac{2(E)}{v^2} = \frac{2(10J)}{(2)^2} = 5kg$

$T = 2\pi\sqrt{\frac{m}{k}}$

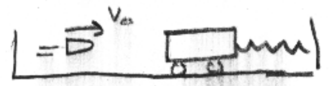
$k = \frac{m}{\left(\frac{T}{2\pi}\right)^2} = \frac{(5kg)}{\left(\frac{4}{2\pi}\right)^2} = 12.34 N/m$

$U_s = E$

$\frac{1}{2}kA^2 = E$

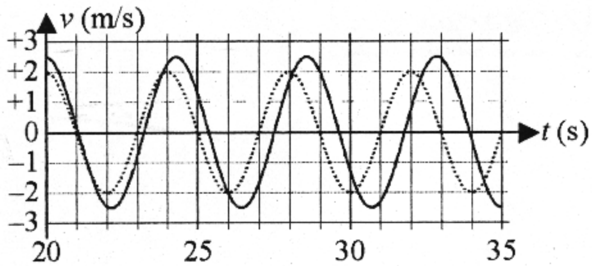
$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(10J)}{12.34}} = 1.27m$

6.I Changing Mass and Period of a Mass-Spring System

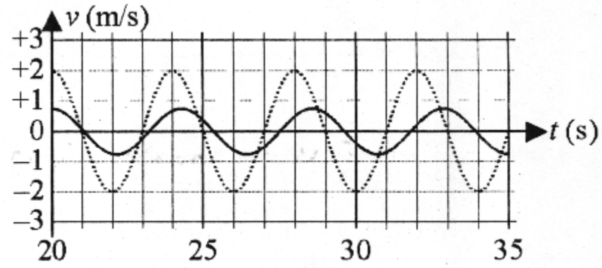


A dart is shot to the right into the cart at a moment in time when the cart's momentum has greater magnitude than the dart's momentum. In Case 1, the dart is shot into the cart at time $t = 12$ s. In Case 2, the experiment is repeated exactly as before, but the dart is shot into the cart at time $t = 14$ seconds. The dart embeds itself into the cart in both cases. The graphs below show the velocity of the cart as a function of time for both cases for the interval $20 \text{ s} < t < 35$ s. The dotted graph in each case is the graph of the cart's velocity vs. time had the dart not been shot into the cart.

Case 1



Case 2



Argumentation

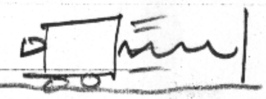
PART B: In a clear, coherent, paragraph-length response, explain how and why the maximum speed and period shown in each case is different from the maximum speed and period had the dart not been shot into the cart. Discuss specific physical principles as appropriate.

Checklist:

- ______ I answered the question directly.
- ______ I stated a law of physics that is always true.
- ______ I connected the law or laws of physics to the specific circumstances of the situation.
- ______ I compared the situation (stated what was the same in all cases).
- ______ I contrasted the situations (stated what was different in all cases).
- ______ I used physics vocabulary (period, mass, spring constant, force, velocity, displacement, equilibrium, momentum, energy).

Case 1

The amplitud does not much change because the spring is compressed when the collision occurs on the 3rd cycle This means the Energy of the system is still (10J) $E=\frac{1}{2}kA^2$ since the amplitude and constant did not change. The period does increase from the additional mass $T = 2\pi\sqrt{\frac{m}{k}}$.



Case 2

In this case the dart collides with the cart at equilibrium, this changes the kinetic Energy of the cart to a lower value. This results in less potential and thus total Energy of the system. The amplitude is reduced as $E = U_{max} = \frac{1}{2}kA^2$. The period is increased for the same reason as case 1 $\uparrow T = 2\pi\sqrt{\frac{m\uparrow}{k}}$.

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Scenario

Carlos and Dominique hang a spring with an unknown spring constant from a ceiling. The students connect objects with different amounts of mass m *to the spring and measure the oscillation period* T *of each object. Their data are shown in the table to the right. Note that the square of each period is also calculated.*

Mass *m* [kg]	**Period *T*** (sec)	T^2 (sec^2)
1.15	0.78	0.61
2.32	1.21	1.46
3.51	1.61	2.59
4.75	1.87	3.49
5.89	2.06	4.24

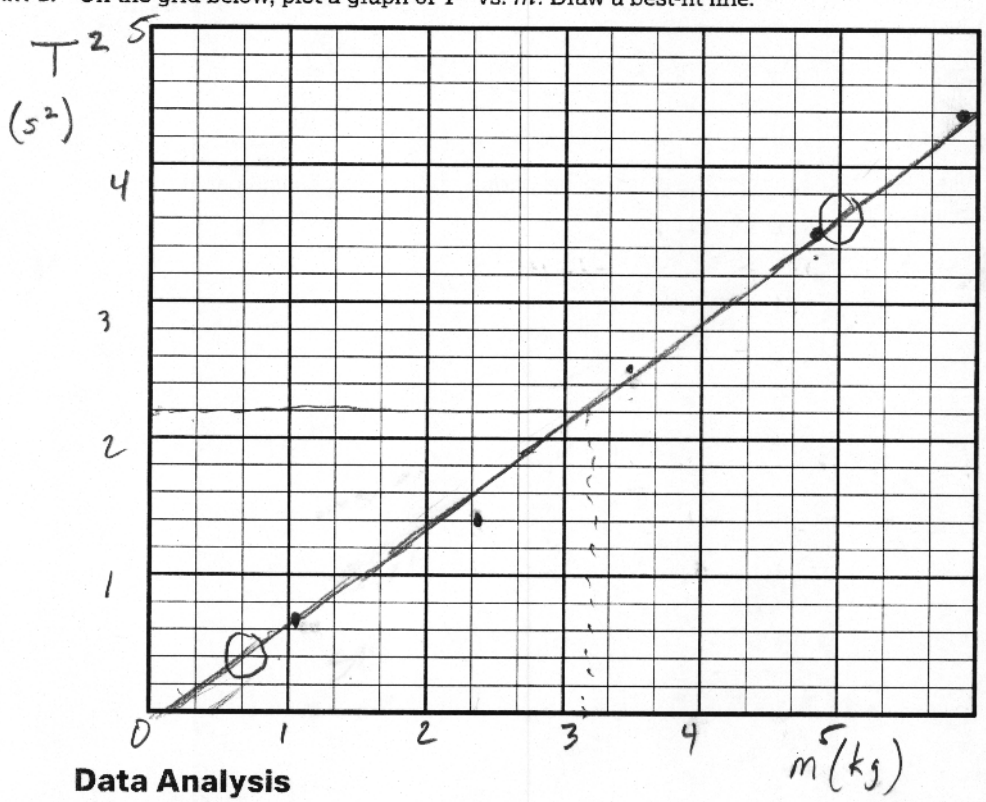
Argumentation

PART A: The students did not take care to ensure that the object oscillated with the same amplitude on each trial. Their teacher informs them that this oversight will not invalidate their experimental results. Briefly explain why this is the case.

Amplitude does not impact period. $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

Using Representations

PART B: On the grid below, plot a graph of T^2 vs. m. Draw a best-fit line.



Data Analysis

PART C: An unknown object hung on the spring oscillates with a period of 1.47 seconds. Using your best-fit line, calculate the mass of this object. Explain your method using words in addition to showing mathematical steps.

Find the value of mass at $(1.47 s)^2 = 2.16 s^2$

$\approx \boxed{3.15 \text{ kg}}$

PART D: Using your best-fit line (not merely a single data point or average), calculate the spring constant of the spring. Explain how you determined what the slope of the line represents algebraically.

$T = 2\pi\sqrt{\frac{m}{k}}$

$\text{Slope} = \frac{(3.6 - 0.2)}{(5 - 0.33)} = 0.73$

$T^2 = 4\pi^2 \frac{m}{k}$

$\text{Slope} = \frac{4\pi^2}{k}$

$\frac{\Delta T^2}{\Delta m} = \frac{4\pi^2}{k} = \text{Slope}$

$k = \frac{4\pi^2}{\text{slope}} = \frac{4\pi^2}{0.73} = \boxed{54 N/m}$

PART E: Suppose Carlos and Dominique also made a graph of T^2 vs. m. However, the graph they made has a best-fit line with a slightly steeper slope than the line you drew on the graph in Part B.

i. How would this affect their experimental value for the spring constant compared to yours in Part D? Explain.

smaller since the $\text{Slope}\uparrow = \frac{4\pi^2}{k\downarrow}$

ii. How would this affect their value for the mass with a period of 1.47 seconds compared to yours in Part C? Explain.

less $T^2 = 4\pi^2 \frac{m\downarrow}{k}$

$\downarrow$

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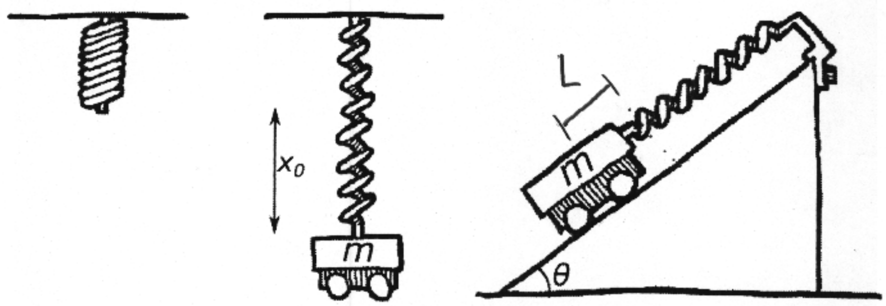
NAME ______________________ **DATE** ____________

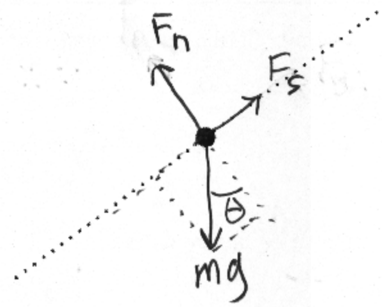
Scenario

A cart of mass m is attached to a vertical spring of spring constant k so that the spring stretches a distance x_0. When the cart is set into oscillatory motion on the vertical spring, the period of oscillation is T. The cart is then set on a smooth incline angled at θ above the horizontal and reattached to the spring.



Using Representations

PART A: The dot at right represents the cart at rest on the incline. Draw a free-body diagram showing and labeling the forces (not components) exerted on the block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot. (The dotted line represents the incline.)



Quantitative Analysis

PART B: Derive an expression for the stretch of the spring (L) while the cart is on the incline in terms of x_0, θ, and physical constants as necessary.

(L should be less than X_0)

When vertical

$\Sigma F = 0$

$F_s - mg = 0$

$F_s = mg$

$kx_0 = mg$

$k = \frac{mg}{x_0}$

on ramp

$\Sigma F_x = 0$

$mg\sin\theta - F_s = 0$

$F_s = mg\sin\theta$

$kL = mg\sin\theta$

$\frac{\cancel{mg}}{X_0} L = \cancel{mg}\sin\theta$

$\boxed{L = X_0 \sin\theta}$

$\Sigma F_y = 0$

$F_n - mg\cos\theta = 0$

$F_n = mg\cos\theta$

PART C: Does your equation make physical sense for $\theta = 0°$? Explain.

yes $L = X_0 \sin\theta = X_0 \sin(0) = \emptyset$.

It would be horizontal and not stretched.

Does your equation make physical sense for $\theta = 90°$? Explain.

yes $L = X_0 \sin\theta = X_0 \sin(90°) = X_0$

It would be hanging vertical.

PART D: How does the new period of oscillatory motion that the cart could undergo on the incline compare to the original period T when hanging vertically? Explain your reasoning.

Same m & k have not changed

$T = 2\pi\sqrt{\frac{m}{k}}$

PART E: The cart is pulled down the ramp so that it has been stretched a distance L past equilibrium. The cart is released and oscillates. A graph of the position of the cart as a function of time is sketched below. Sketch the following graphs for the cart as it oscillates on the incline: velocity vs. time and acceleration vs. time.