Argumentation

Blake and Carlos are trying to predict whether the coin will slip if the coin is "too close" to or "too far" from the axis of rotation. The students reason as follows:

Blake: "I think that the coin will slip if it is too close to the axis. It is like if a car takes a turn too tightly, the car can slide out of control. There's not enough force if the radius is too small.

Carlos: "I think that the coin will slip if it is too far from the axis. It's like a merry-go-round; if I ride a merry-go-round near the center, then I don't feel much force pulling me to the outside, but if I ride near the outside, there is more force pulling me away from the axis.

PART C: For each student's statement, state whether the inequality written in Part B provides support for that statement. If so, explain how. If not, explain why not. Ignore whether the student's statement is correct or incorrect for this part.

Blake's Statement	***Carlos's Statement***
Blake is supporting "too close" by indicating that radius has an inverse relationship with centripetal force $\left(\Sigma F_c \propto \frac{1}{r}\right)$.	Carlos is supporting "too far" by saying he has experienced a greater (sensation) of being pulled outward from having a larger radius.

TRUTH: $\Sigma F_c = \frac{mv^2}{r}$ $\quad v = \frac{2\pi r}{T}$ in this case.....

$$\Sigma F_c = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{m4\pi^2 r^{\cancel{2}}}{\cancel{r}T^2} = \frac{m4\pi^2 r}{T^2} \qquad \Sigma \overset{\uparrow}{F_c} \propto \overset{\uparrow}{r}$$

PART D: State whether the coin will slip when it is "too close" to or "too far" from the axis.

_____ too close ✓ too far

PART E: Angela and Dominique are arguing over how the mass of the coin affects whether it will slip or not. Angela believes that a lighter coin is less likely to slip because a lighter coin requires less force. Dominique believes that a heavier coin is less likely to slip because a heavier coin can have a greater amount of friction. Using your equations along with other physical principles, explain how the coin's mass affects its likelihood of slipping.

It does not affect the situation.

$$\Sigma F_c = m\frac{v^2}{r} \qquad \mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$$

$$F_{f_s} = m\frac{v^2}{r} \qquad \mu_s g = \frac{v^2}{r}$$

(mass cancels out)

$$\mu_s N = \frac{mv^2}{r}$$

more mass = more centripetal force

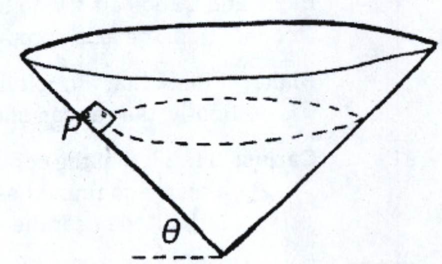
more mass = more friction

NAME ____________________ DATE ____________

6

Scenario

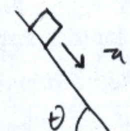
Consider a cone made of a material for which friction may be neglected. The sides of the cone make an angle θ with the horizontal plane. A small block is placed at point P. In Case 1, the block is released from rest and slides down the side of the cone toward the point at the bottom. In Case 2, the block is released with initial motion so that the block travels with constant speed along the dotted circular path.



Data Analysis

PART A: In Case 1, the block is released from rest. Is the block accelerating?

✓ Yes ________ No

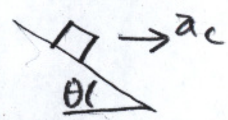


Explain, and if yes, determine the direction of the acceleration.

The block will accelerate down the incline plane. $\Sigma F = ma$ $(a = g\sin\theta)$
$\cancel{m}g\sin\theta = \cancel{m}a$

In Case 2, the block is released so that it travels with a constant speed along the dotted circular path. Is the block accelerating?

✓ Yes ________ No

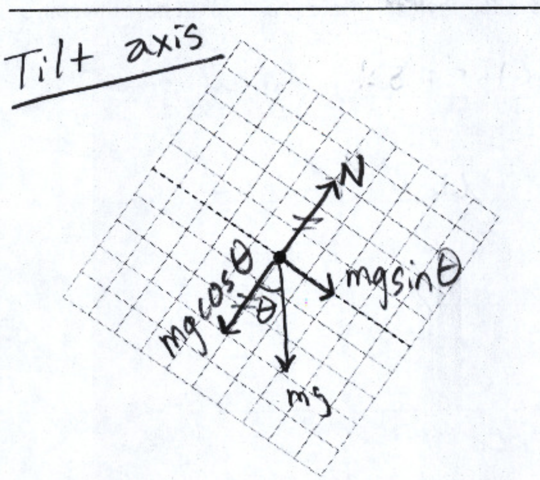
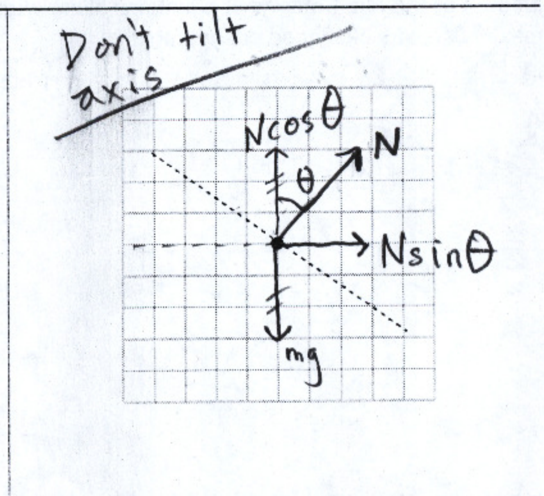


Explain, and if yes, determine the direction of the acceleration.

The block is now accelerating directly toward the center of the cone. This acceleration is centripetal and is changing the direction of the tangent velocity. $\left(a_c = \frac{v^2}{r}\right)$

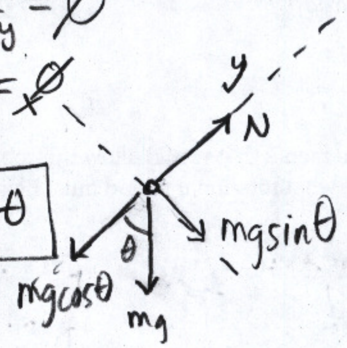
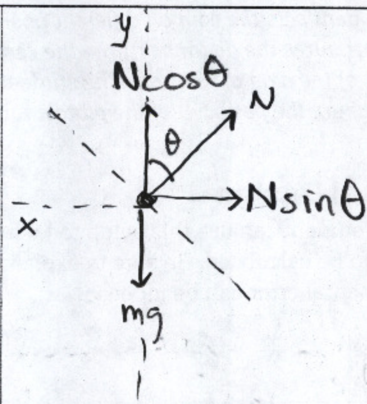
Using Representations

PART B: In both diagrams below, the weight F_g of the block is drawn. Draw the normal force exerted in each case on the corresponding diagram. Use the grids provided to make each normal force have the proper length. (In each case, breaking one of the forces into components will help you find the direction of the acceleration.)

Case 1	*Case 2*
	

Quantitative Analysis

PART C: Derive an expression for the magnitude of the normal force exerted on the object in each case in terms of F_g, θ, and physical constants as necessary.

Case 1	*Case 2*
$\Sigma F_y = \emptyset$ $N - mg\cos\theta = \emptyset$ $\boxed{N = mg\cos\theta}$ 	 $\Sigma F_y = \emptyset$ $N\cos\theta - mg = \emptyset$ $N\cos\theta = mg$ $\boxed{N = \frac{mg}{\cos\theta}}$

PART D: Use the diagrams in Part B to explain why the normal force is greater in Case 2. Then use your equations in Part C to explain why the normal force is greater in Case 2.

The Normal force does not contribute to the acceleration in along the incline in case 1. In addition the normal force is equal to a vector component of gravity in case 1, where as the gravity is equal to a vector component of normal force in case 2.

In this example
$\cos\theta < 1$ so...

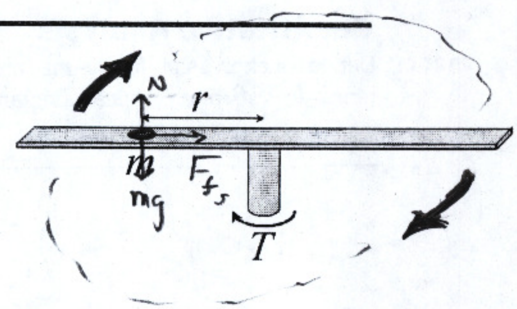
$$\underbrace{N = mg\cos\theta}_{\text{case 1}} < \underbrace{\frac{mg}{\cos\theta} = N}_{\text{case 2}}$$

NAME ______________________ DATE ______________

7

Scenario

A student is attempting to determine the coefficient of static friction μ_s between a coin and a steel plate. The student attaches the center of the plate to a freely rotating axis. For each trial, the student sets the coin on different positions on the steel plate and measures the distance r from the center of the coin to the center of the axis of rotation. The student also has a stopwatch to measure the period T of the plate's rotation.



Experimental Design

PART A: Explain how the student can use this setup to take measurements that would allow the coefficient of static friction to be calculated. Be sure to explain clearly what rotational period must be measured and how experimental error can be reduced.

The student needs to find the period and radius when the coin just slides. This can be done by placing the coin and finding the constant rotation speed just before the coin slips. The period can be found by timing 10 revolutions and dividing by 10.

Quantitative Analysis

PART B: Starting with Newton's laws and basic equations for circular motion, derive an equation that relates μ, r, T, and fundamental constants.

$\Sigma F_c = m a_c$ $F_{f_s} = m\frac{v^2}{r}$	The centripetal force is provided by static friction.
$F_{f_{s_{Max}}} = \mu_s N$ $F_{f_{s_{Max}}} = \mu_s mg$	The static friction needs to be maximized to solve for μ_s. The normal force is equal to the weight.
$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$ $v = \frac{2\pi r}{T}$	The mass cancels. Relate speed to period
$\mu_s g = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$	$\mu_s g = \frac{4\pi^2 r^{\cancel{2}}}{T^2 \cancel{r}}$ sub speed into the equation.

$\mu_s = \frac{4\pi^2 r}{T^2 g}$	solve for μ_s.

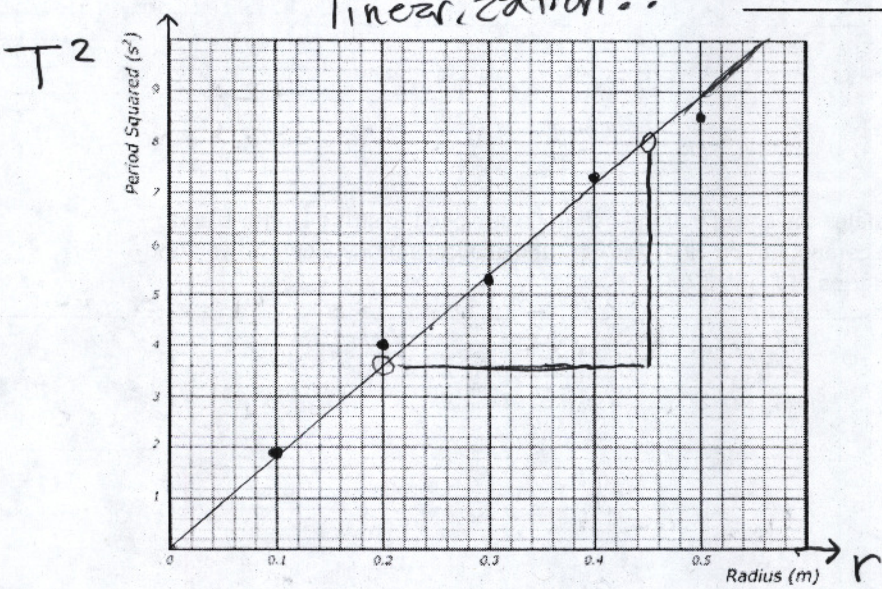
The student collects the data shown in the table above.

PART C: Plot the data on the T^2 vs. r graph shown below. Draw a best-fit line to the data and calculate the slope of the best-fit line.

r (m)	T (s)	T^2 (s^2)
0.1	1.4	1.96
0.2	2.0	4.00
0.3	2.3	5.29
0.4	2.7	7.29
0.5	2.9	8.41

$$(T^2) = \frac{4\pi^2}{\mu_s g}(r)$$

linearization!!



$$\text{Slope} = \frac{4\pi^2}{\mu_s g}$$

$$\mu_s = \frac{4\pi^2}{g(\text{slope})}$$

PART D: Use your equation from Part B along with the slope of your best-fit line from Part C to calculate the value of μ.

(must be w/ the line)

$$\text{slope} = \frac{T^2 - T_0^2}{r - r_0} = \frac{8. - 3.6}{0.45 - 0.2} = 17.6$$

$$\text{slope} = \frac{4\pi^2}{g\mu_s} \qquad \mu_s = \frac{4\pi^2}{g(\text{slope})} = \frac{4\pi^2}{g(17.6)} = \boxed{0.23}$$

NAME ______________________ **DATE** ____________

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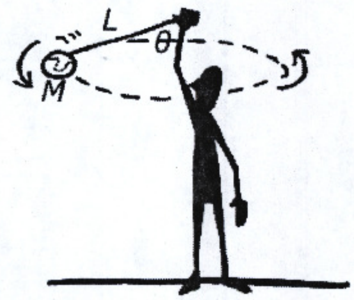
Scenario

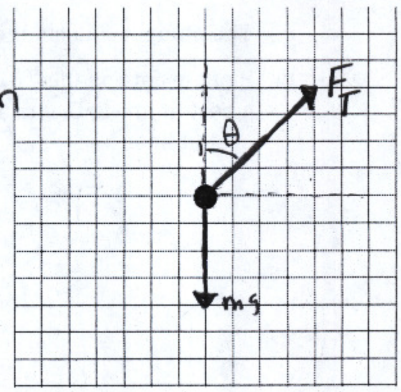
Consider a ball of mass M connected to a string of length L. A student holding the free end of the string whirls the ball in a horizontal circle with constant speed. The angle between the string and the vertical is θ. The student attempts to whirl the ball faster and faster in order to make the string become horizontal. No matter how fast the student whirls the ball, the string is never exactly horizontal.



Using Representations

PART A: i. The dot below represents the ball at the instant it appears in the diagram. Draw a free-body diagram showing and labeling the forces (not components) exerted on the ball. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.

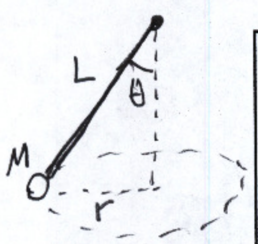
ii. By discussing specific features of your force diagram, explain why the rope cannot become completely horizontal no matter how fast the ball is whirled.

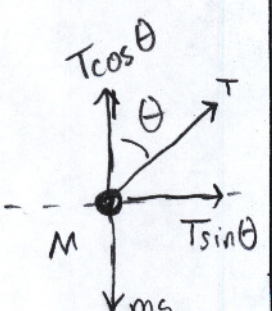


The vertical component of Tension must cancel the force of gravity.

Create an Equation

PART B: i. Derive an equation that relates the speed v of the ball in its circle to the string length L and angle θ. [Hint: What force component provides the centripetal acceleration? How can you find the radius of the circle in terms of L and θ?]





$\Sigma F_c = ma_c$ $T\sin\theta = m\frac{v^2}{r}$	The centripetal force is provided by the horzontal component of the tensian.
$\Sigma F_y = \varnothing$ $T\cos\theta - mg = \varnothing$	The vertical component of tension equals the force of gravity.
$T\cos\theta = mg$ $T = \frac{mg}{\cos\theta}$	$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \frac{\cancel{m}v^2}{r}$ The tension can be substituted into the centripetal force
$g\tan\theta = \frac{v^2}{r}$ $r = L\sin\theta$	$\boxed{v = \sqrt{g\tan\theta L\sin\theta}}$ The radius of the circular path is $L\sin\theta$

UNIT 3 ***Circular Motion and Gravitation*** | 3.L Inertia in Space

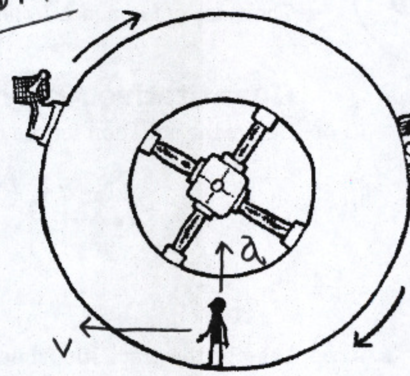
NAME ______________________ **DATE** __________

9

Scenario

Not in orbit

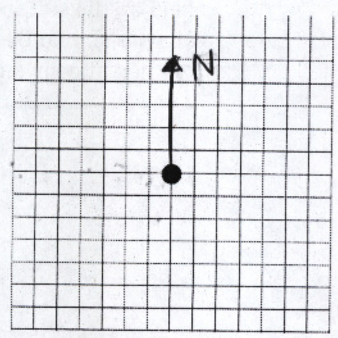
A doughnut-shaped space station is built far away from the gravitational fields of Earth and other massive bodies. For the comfort and safety of the astronauts, the space station is rotated to create an artificial internal gravity. The rotation speed is such that the apparent acceleration due to gravity at the outer surface is 9.8 $\mathrm{m/s^2}$. *The space station rotates clockwise.*



Using Representations

PART A: On the image at right, sketch and label vectors that represent the astronaut's velocity and acceleration.

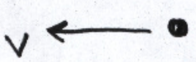
PART B: The dot at right represents the astronaut standing in the space station. Draw a free-body diagram showing and labeling the forces (not components) exerted on the astronaut at the instant shown. Draw the relative lengths of all vectors to reflect the magnitudes of all the forces.

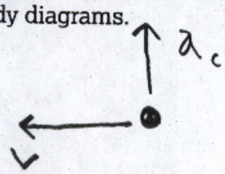


FBD

PART C: The astronaut drops a ball. On the following diagrams, sketch the velocity and acceleration vectors for the ball as seen by an observer outside the space station in an inertial frame of reference. These are NOT free-body diagrams.

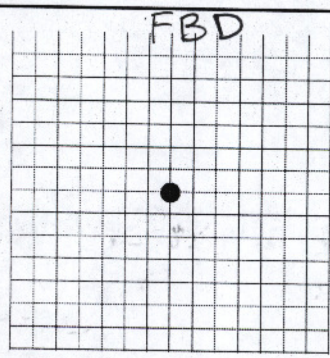
while falling

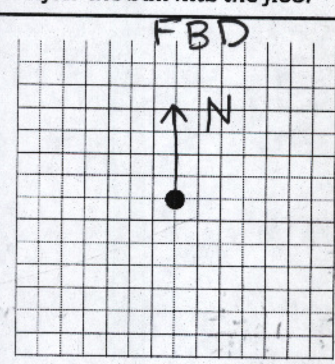




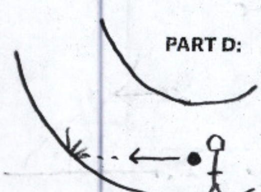
After the ball is released and before it hits the floor	***After the ball hits the floor***
FBD	FBD

NO Forces





on floor



PART D: From the point of view of a person watching from outside the space station, what does the path of the ball look like?

The path of the ball will be tangent to the circular path

PART E: From the point of view of the astronaut inside the space station, what does the path of the ball look like?

The path of the ball will look like if you dropped a ball on Earth.

NAME ______________________ **DATE** ____________

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Scenario

The mass of Mars is 1/10 times that of Earth; the diameter of Mars is 1/2 that of Earth.

Quantitative Analysis

PART A: Derive the equation for gravitational, g, due to a planet.

Force on planet surface

$$F_g = G\frac{Mm}{r^2} \qquad \cancel{m}g = G\frac{M\cancel{m}}{r^2} \qquad \boxed{g = G\frac{M}{r^2}}$$

"gravity field"

PART B: Let g be the gravitational field strength on Earth's surface. Derive an expression for the gravitational field on the surface of Mars without plugging in a value for the mass or radius of Mars. Your answer should be a number multiplied by g. For each line of the derivation, explain what was done mathematically (i.e., annotate your derivation).

$g = G\frac{M}{r^2}$	Start w/ the equation for gravity field
$g \propto 1 \cdot \frac{(\frac{1}{10})}{(\frac{1}{2})^2}$	Create a proportional relationship w/ values that are the same as (1)
$g \propto \frac{(\frac{1}{10})}{(\frac{1}{4})} \propto \frac{4}{10}$	This shows that "g_{mars}" is $\frac{4}{10}$ of "g_{Earth}"
$g_E = 9.81\,m/s^2$ $g_{Mars} = g_E\left(\frac{4}{10}\right)$	Multiply this proportion by g_{Earth}
$g_{Mars} = 3.9\,m/s^2 \approx 4\,m/s^2$	

Argumentation

$$\Delta y = \cancel{v_{0y}t} - \frac{1}{2}gt^2 \qquad t = \sqrt{\frac{2\Delta y}{-g}}$$

PART C: A rock is dropped 2.0 meters above the surface of Mars. Does this rock take a longer or a shorter time to fall than a rock dropped 2.0 m above the surface of Earth? Justify your answer without using equations.

$$\uparrow t = \sqrt{\frac{2h}{g_\downarrow}}$$

Longer the acceleration is less on mars so it will gain less velocity over the same distance, thus taking longer.

$$\overset{\uparrow}{t}_{mars} = \sqrt{\frac{2hG}{M_m R_m^2 \downarrow}} > \underset{\downarrow}{t}_{Earth} = \sqrt{\frac{2hG}{M_E R_E^2 \uparrow}}$$

PART D: On the internet, a student finds the following equation for the time an object will take to fall to the ground from a height h, depending on the mass and radius of the planet the object is on: $t = \sqrt{\frac{2hG}{MR^2}}$

Regardless of whether this equation is correct, does it agree with your qualitative reasoning in Part C? In other words, does this equation for t have the expected dependence as reasoned in Part C?

✓ Yes ______ No

Briefly explain your reasoning without deriving an equation for t.

This equation shows greater time for the fall on mars b/c the product of mar's mass and $(\text{radius})^2$ is less than the product of Earth's mass and $(\text{radius})^2$. This is on the bottom of the fraction so that will cause time to be greater.

PART E: Another student deriving an equation for the time it takes for an object to fall from height h makes a mistake and comes up with: $t = \sqrt{\frac{R^2}{2GMh}}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

This equation show height (h) on the bottom of the time equation. That would imply less height means longer time!

$$t = \sqrt{\frac{2h}{g}}$$

Correct derivation

$$t = \sqrt{\frac{2h}{g}} \qquad g = G\frac{M}{R^2}$$

$$t = \sqrt{\frac{2h}{\left(\frac{GM}{R^2}\right)}} = \sqrt{\frac{2hR^2}{GM}}$$

NAME ____________________ DATE __________

11

Scenario

A student is given the following set of orbital data for some of Jupiter's moons and is asked to use the data to determine the mass M_J of Jupiter. Assume that the orbits of these moons are circular.

***Orbital Period* T** (*seconds*)	***Orbital Radius* R** (*meters*)	T^2 (*s^2*)	R^3 (*m^3*)
2.08×10^7	1.12×10^{10}	4.33×10^{14}	1.4×10^{30}
2.49×10^7	1.26×10^{10}	6.2×10^{14}	2.00×10^{30}
4.05×10^7	1.71×10^{10}	16.4×10^{14}	5.00×10^{30}
5.03×10^7	2.02×10^{10}	25.3×10^{14}	8.24×10^{30}

Create an Equation

PART A: Write an algebraic expression for the gravitational force between Jupiter and one of its moons.

$$F_g = G\frac{M_j m}{R^2}$$

PART B: Use your expression from Part A and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$\Sigma F_c = m a_c$	circular orbit is UCM so you have centripetal force
$F_g = m\frac{v^2}{R}$	The centripetal force is caused by gravity
$G\frac{M_j \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$	sub F_g
$G\frac{M_j}{R} = \left(\frac{2\pi R}{T}\right)^2$	Circular speed can be found as $v = \frac{2\pi R}{T}$
$G\frac{M_j}{R} = \frac{4\pi^2 R^2}{T^2} \rightarrow T^2 = \frac{4\pi^2 R^3}{GM_j}$	Algebra shows $T^2 \propto R^3$
$T = \sqrt{\frac{4\pi^2 R^3}{GM_j}}$	solved

Data Analysis

PART C: Which quantities should be graphed to yield a straight line whose slope could be used to determine the mass of Jupiter?

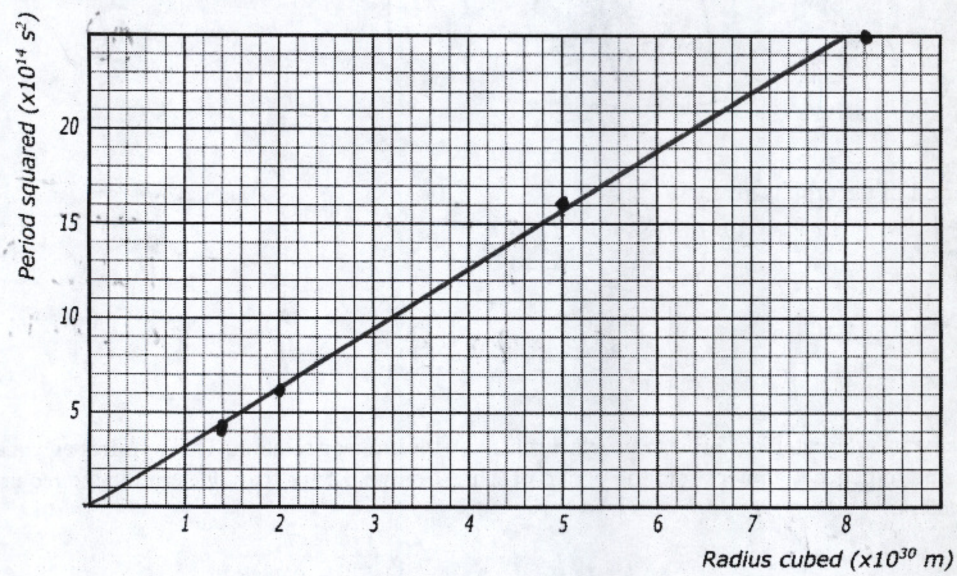
T^2 and R^3 those values are the variables

Linearization!!

PART D: Complete the table by calculating the two quantities to be graphed. Label the top of each column, including units.

PART E: Plot the graph on the axes below. Label the axis with the variables used and appropriate numbers to indicate the scale.

T^2



R^3

$$(T^2) = \frac{4\pi^2}{GM_j}(R^3)$$

$$\uparrow$$

$$\text{Slope} = \frac{4\pi^2}{GM_j}$$

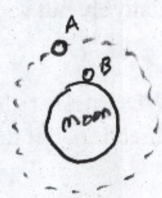
$$M_j = \frac{4\pi^2}{G(\text{slope})}$$

PART F: Two identical probes are sent to study one of Jupiter's moons. Probe A is in geosynchronous orbit around the moon while probe B rests on the surface of the moon and rotates with the moon.

Rank the magnitudes of the following gravitational forces from greatest to least. If two or more quantities are the same, say so clearly.

a. The force of the moon on probe A
b. The force of the moon on probe B — 3rd law (b & d)
c. The force of probe A on the moon — 3rd law (a & c)
d. The force of probe B on the moon
e. The force of probe A on probe B — 3rd law (e & f)
f. The force of probe B on probe A

$$F_g = G\frac{m_1 m_2}{r^2}$$



Greatest b=d, a=c, e=f ______ ______ Least

Justify your ranking.

You do this!

- Explain why the forces are equal (3rd law pairs)

- Explain how radius impacts F_g

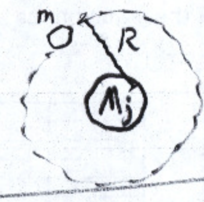
- Explain how the product of the masses impact F_g

12

2000M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by



$$\Sigma F_c = m a_c$$
$$F_g = m\frac{v^2}{R}$$
$$G\frac{M_j \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$$
$$\boxed{v = \sqrt{\frac{GM_j}{R}}}$$

ii. The period of the orbit is given by

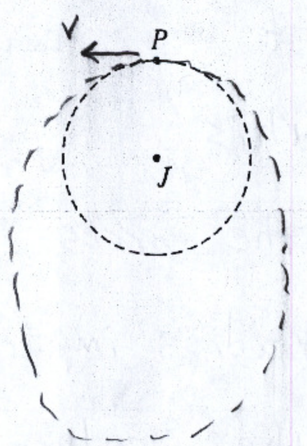
$$v = \frac{2\pi R}{T}$$
$$G\frac{M_j \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$$
$$G\frac{M_j}{R} = \left(\frac{2\pi R}{T}\right)^2$$
$$G\frac{M_j}{R} = \frac{4\pi^2 R^2}{T^2}$$
$$\boxed{T = \sqrt{\frac{4\pi^2 R^3}{GM_j}}}$$

b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

$$T^2 = \frac{4\pi^2 R^3}{GM_j}$$
$$4\pi^2 R^3 = GM_j T^2$$
$$R = \sqrt[3]{\frac{GM_j T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(1.9\times10^{27})(3.55\times10^4)^2}{4\pi^2}} = \boxed{2.0 \times 10^{13}\ \text{m}}$$

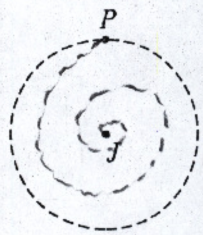
c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



Ellipse

ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>slower</u> than the correct speed for a circular orbit of that radius.



Fall too fast