Name: KEY

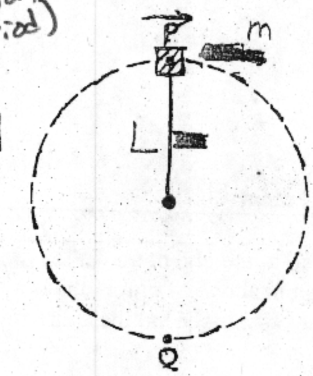
Part 2

Period: ____________

UCM Recitations [redacted]

Answer in terms of: L, T, m, g, H ↙ Force of tension (not period)

Symbolic



1. A [redacted] object rotates freely in a vertical circle at the end of a string of length [redacted] meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is [redacted]. Assume g [redacted] meters per second squared.

mass (m) L (T)

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P.

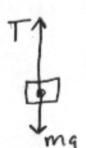
(b) Calculate the speed of the object at point P.

$\Sigma F_c = m a_c$ $r = L$

$T + mg = m\frac{v^2}{r}$

$$\boxed{v = \sqrt{\frac{L(T+mg)}{m}}}$$

(c) Calculate the tension in the string as the object passes through point Q.



$\Sigma F_c = m a_c$ $r = L$

$T_Q - mg = m\frac{v^2}{r}$

$T_Q = \frac{mv^2}{L} + mg$

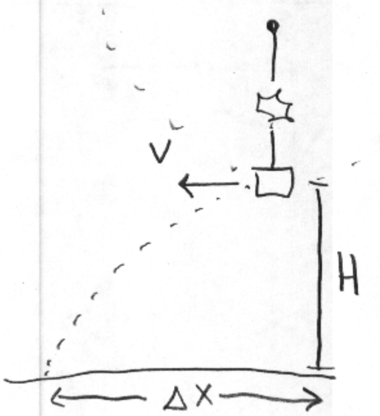
$T_Q = \frac{\cancel{m}\left(\sqrt{\frac{\cancel{L}(T+mg)}{\cancel{m}}}\right)^2}{\cancel{L}} + mg$

$$\boxed{T_Q = T + 2mg}$$

(d) If the string where to snap when the object is at point Q, how far horizontally would the mass travel? (Point Q is [redacted] above a horizontal surface)

H

Projectile



X-axis

$\Delta X = V_{ox} \cdot t$

$\Delta X = V\left(\sqrt{\frac{2H}{g}}\right)$

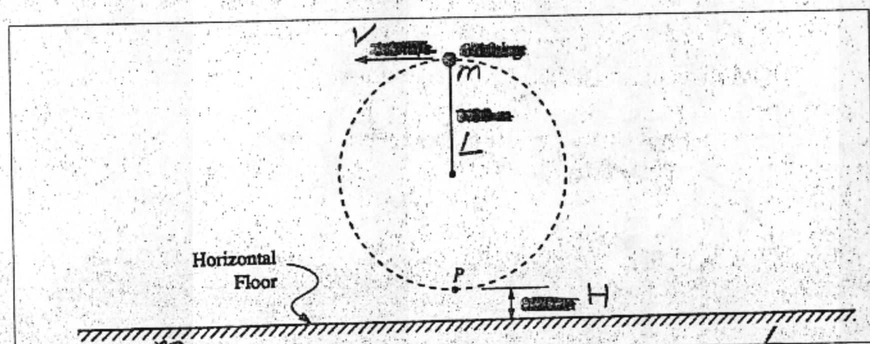
y-axis

$\Delta y = \cancel{v_i t} + \frac{1}{2}gt^2$

$t = \sqrt{\frac{2\Delta y}{-g}} = \sqrt{\frac{2H}{g}}$

$\Delta X = \sqrt{\frac{L(T+mg)}{m}} \cdot \sqrt{\frac{2H}{g}}$

$$\boxed{\Delta X = \sqrt{\frac{2HL(T+mg)}{mg}}}$$



symbolic
Answer in terms of: L, v, m, H, g

2. A [redacted] solid rubber ball is attached to the end of an [redacted] length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is [redacted] above the floor. The speed of the ball is [redacted] meters per second the entire revolution.

m L H v

$r = L$

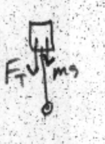
a. What is the frequency of the ball's revolution in hertz (1 rev/sec)?

$$v = \frac{2\pi r}{T} = \frac{2\pi L}{T} = 2\pi L f \qquad \boxed{f = \frac{v}{2\pi L}}$$

$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

b. Determine the tension in the thread at

i. the top of the circle;



$$\Sigma F_c = m a_c \qquad \boxed{F_T = \frac{mv^2}{L} - mg}$$

$$F_T + mg = m\frac{v^2}{r}$$

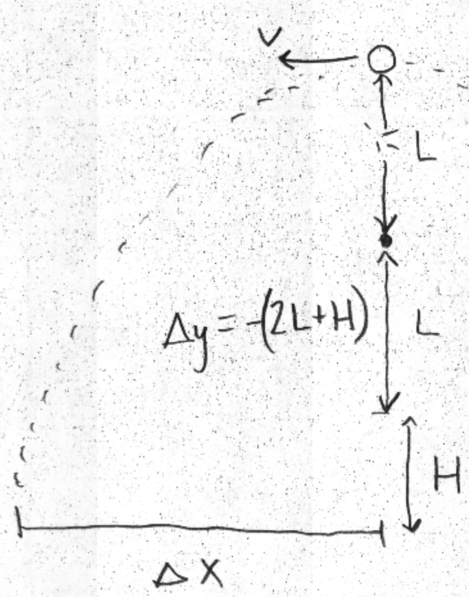
ii. the bottom of the circle.

$$\Sigma F_c = m a_c \qquad \boxed{F_T = \frac{mv^2}{L} + mg}$$

$$F_T - mg = m\frac{v^2}{r}$$

The ball again reaches the top of the circle when the thread breaks.

c. Determine the horizontal distance that the ball travels before hitting the floor.



x-axis

$$\Delta X = V_{0x} t$$

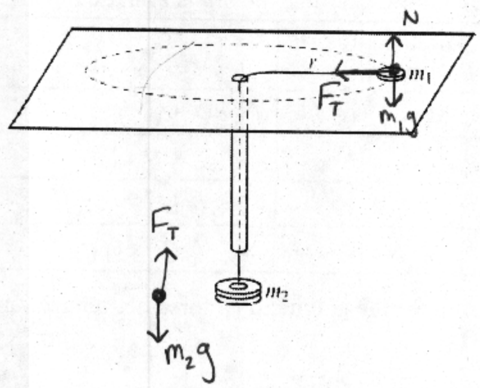
$$\boxed{\Delta X = v\sqrt{\frac{4L+2H}{g}}}$$

y-axis

$$\Delta y = \cancel{v_{0y} t} + \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2\Delta y}{-g}} = \sqrt{\frac{2(2L+H)}{g}}$$

$$t = \sqrt{\frac{4L+2H}{g}}$$



1. (15 points)

An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.

We usually use (T)

(a) Derive the equation $P = 2\pi\sqrt{\dfrac{m_1 r}{m_2 g}}$ that relates P and m_2.

$\Sigma F_c = m_1 a_c$

$F_T = m_1 \dfrac{v^2}{r}$

$m_2 g = m_1 \dfrac{v^2}{r}$

$\Sigma F = m_2 a$

$F_T - m_2 g = \emptyset$

$F_T = m_2 g$

$v = \dfrac{2\pi r}{P}$

$m_2 g = m_1 \dfrac{\left(\frac{2\pi r}{P}\right)^2}{r}$

$m_2 g = m_1 \dfrac{4\pi^2 r^{\cancel{2}}}{P^2 \cancel{r}}$

$P^2 = \dfrac{m_1 4\pi^2 r}{m_2 g}$

$P = \sqrt{\dfrac{m_1 4\pi^2 r}{m_2 g}}$

$\boxed{P = 2\pi\sqrt{\dfrac{m_1 r}{m_2 g}}}$

The procedure is repeated, and the period P is determined for four different values of m_2, where $m_1 = 0.012$ kg and $r = 0.80$ m. The data, which are presented below, can be used to compute an experimental value for g.

constants

variables

m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75

(b) What quantities should be graphed to yield a straight line with a slope that could be used to determine g?

$P = 2\pi\sqrt{\dfrac{m_1 r}{m_2 g}}$

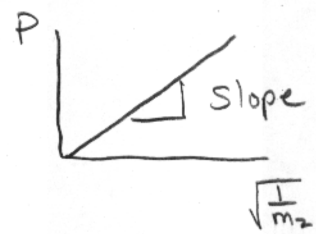
$P \propto \sqrt{\dfrac{1}{m_2}}$

proportional relationship.

$\boxed{P \text{ vs. } \sqrt{\dfrac{1}{m_2}}}$

– or –

$\boxed{P^2 \text{ vs. } \dfrac{1}{m_2}}$



$\text{Slope} = 2\pi\sqrt{\dfrac{m_1 r}{g}}$

$g = \dfrac{4\pi^2 m_1 r}{(\text{Slope})^2}$

2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	6.33×10^{24}
1.18×10^5	2.38×10^8	1.39×10^{10}	13.4×10^{24}
1.63×10^5	2.95×10^8	2.66×10^{10}	25.6×10^{24}
2.37×10^5	3.77×10^8	5.62×10^{10}	53.6×10^{24}

a. Write an algebraic expression for the gravitational force between Saturn (M_s) and one of its moons (m).

$$\boxed{F_g = G\frac{m_1 m_2}{r^2} = G\frac{M_s m}{R^2}}$$

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$$\Sigma F_c = ma_c$$

$$F_g = m\frac{v^2}{R}$$

$$G\frac{M_s \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$$

$$v^2 = G\frac{M_s}{R}$$

$$\left(\frac{2\pi R}{T}\right)^2 = G\frac{M_s}{R}$$

$$\frac{4\pi^2 R^2}{T^2} = G\frac{M_s}{R}$$

$$\boxed{T = \sqrt{\frac{4\pi^2 R^3}{GM_s}}}$$

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

$$T^2 \propto R^3$$

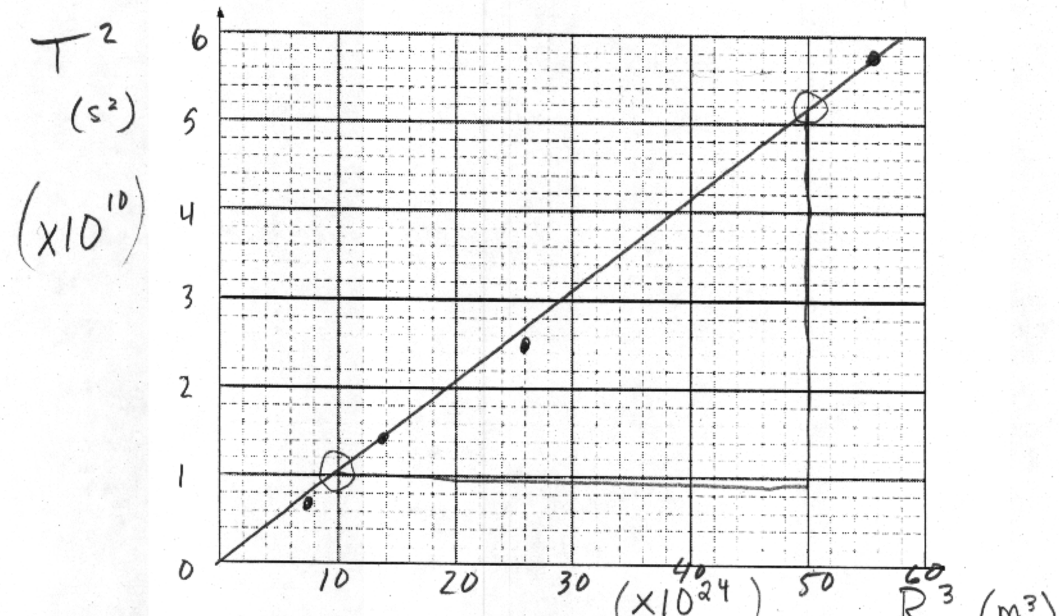
$$T^2 = \frac{4\pi^2 R^3}{GM_s}$$

$$\boxed{T^2 \text{ vs. } R^3}$$

linearization

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

$$Slope = \frac{(5.2\times10^{10}) - (1\times10^{10})}{(50\times10^{24}) - (10\times10^{24})} = \frac{4.2\times10^{10}}{4\times10^{25}} = 1.1\times10^{-15}$$

$$T^2 = \frac{4\pi^2 R^3}{GM_s}$$

$$Slope = \frac{4\pi^2}{GM_s}$$

$$M_s = \frac{4\pi^2}{G\cdot(slope)} = \frac{4\pi^2}{(6.67\times10^{-11})(1.1\times10^{-15})}$$

$$\boxed{M_s = 5.38\times10^{26}\,kg}$$