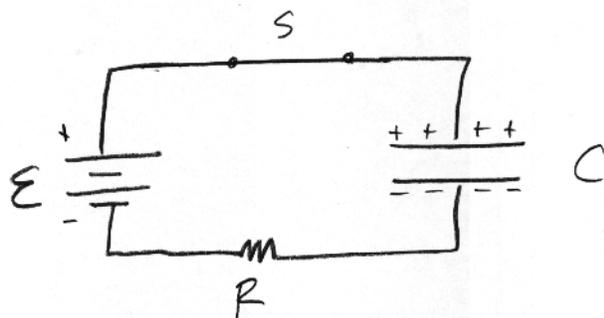
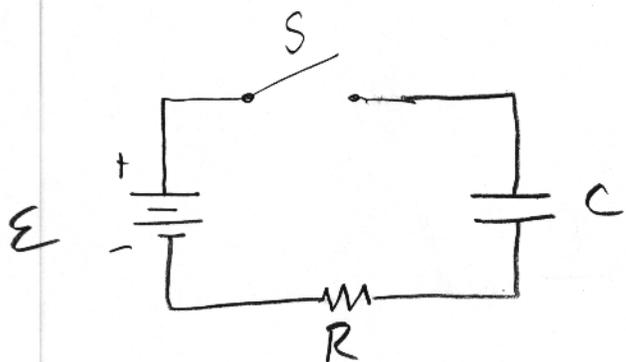


RC - circuits



Charges until it matches the source EMF. At this point current stops flowing on that path.

• Before RC circuits, we treat currents as a constant. Not any more.

$$I = \frac{dQ}{dt} \quad \text{Calculus! } \ddot{Q}$$

Let's apply loop rule to the RC capacitor above.

$$\varepsilon - V_{(t)} - I_{(t)}R = 0$$

Notice that the capacitor will act as a voltage drop while charging since current flows from (+) to (-) across it.

$I_{(t)}$ = current at time (t) during charge

$V_{(t)}$ = potential on capacitor at time (t) during charge.

$\varepsilon = \text{constant}$ $I = \frac{dQ}{dt}$ $V = \frac{Q}{C}$

$$\frac{d}{dt} [\varepsilon = I_{(t)}R + V_{(t)}]$$

$$0 = \frac{dI}{dt}R + \frac{dV}{dt}$$

$$\Rightarrow 0 = \frac{dI}{dt}R + \frac{dQ}{dtC}$$

$$\frac{dI}{dt} R = - \frac{dQ}{dt} C \quad I = \frac{dQ}{dt}$$

$$\frac{dI}{dt} R = - \frac{I}{C}$$

$$\int_{I_0}^I \frac{dI}{I} = - \int_0^t \frac{dt}{RC}$$

$$(\ln I - \ln I_0) = - \frac{t}{RC}$$

$$\ln \left| \frac{I}{I_0} \right| = - \frac{t}{RC}$$

$$e^{\ln \left| \frac{I}{I_0} \right|} = e^{-\frac{t}{RC}}$$

$$\frac{I}{I_0} = e^{-\frac{t}{RC}}$$

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

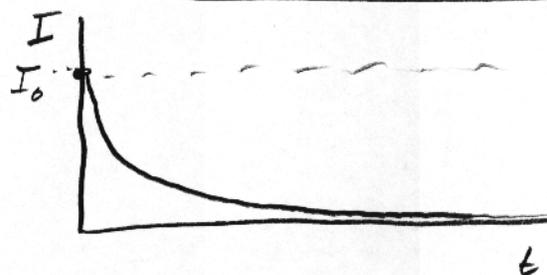
$$I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$$

differential Equation time
Need to integrate (I)
and (t)

$$\int \frac{dx}{x} = \ln x$$

$$I_0 = \frac{\mathcal{E}}{R}$$

Current of charging
RC circuit



Charge of the Capacitor in respect to time (When charging)

Well if $I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$

$Q = \int dQ$

$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

$dQ = \left(\frac{\mathcal{E}}{R} e^{-t/RC} \right) dt$

$\int_0^Q dQ = \int_0^t \frac{\mathcal{E}}{R} e^{-t/RC} dt$

$\int e^{ax} dx = \frac{1}{a} e^{ax}$

$Q(t) = \frac{\mathcal{E}}{R} \left[\frac{e^{-t/RC}}{-1/RC} \right]_0^t$

$Q(t) = \frac{\mathcal{E}}{R} \left[-RC e^{-t/RC} \right]_0^t$

$Q(t) = \frac{\mathcal{E}}{R} \left(-RC e^{-t/RC} + RC e^{-0/RC} \right)$

$x^0 = 1$

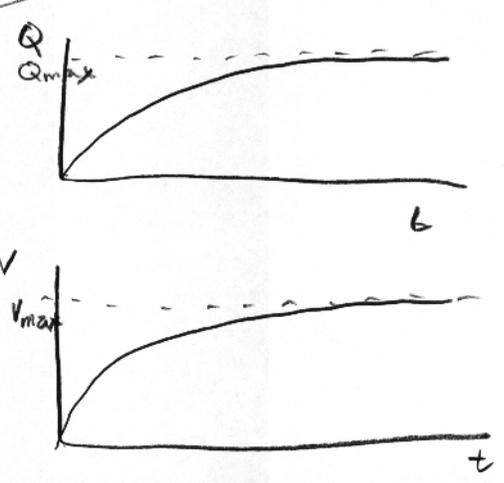
$Q(t) = \frac{\mathcal{E}}{R} RC \left(-e^{-t/RC} + 1 \right)$

$Q(t) = \mathcal{E}C \left(1 - e^{-t/RC} \right)$

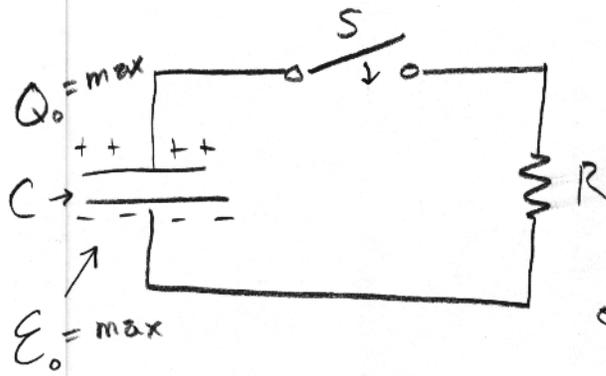
$Q = VC$

$Q(t) = Q_{max} \left(1 - e^{-t/RC} \right)$

$V = \frac{Q}{C}$
 $V(t) = \frac{Q}{C} \left(1 - e^{-t/RC} \right)$



Discharging RC - Circuits



Capacitor can act as a battery when it discharges it will produce a changing current,

At time $t=0$ the switch is closed.

at $t=0$ $I_0 = \max$

The capacitor will provide an immediate EMF of $\mathcal{E}_0 = \frac{Q_0}{C}$

This means the current starts at a max and decays as the EMF of the capacitor drains, due to its loss of charge. (Discharge)

$$Q_0 = \mathcal{E}_0 C \quad \mathcal{E}_0 = I(t) R$$

$$Q(t) = I(t) RC$$

$$Q(t) = -\frac{dQ}{dt} RC$$

differential Equation
time

$$\frac{Q}{C} = -\frac{dQ}{dt} R$$

$$\int_{Q_0}^Q \frac{dQ}{Q} = \int_0^t -\frac{dt}{RC}$$

$$\ln Q - \ln Q_0 = -\frac{t}{RC}$$

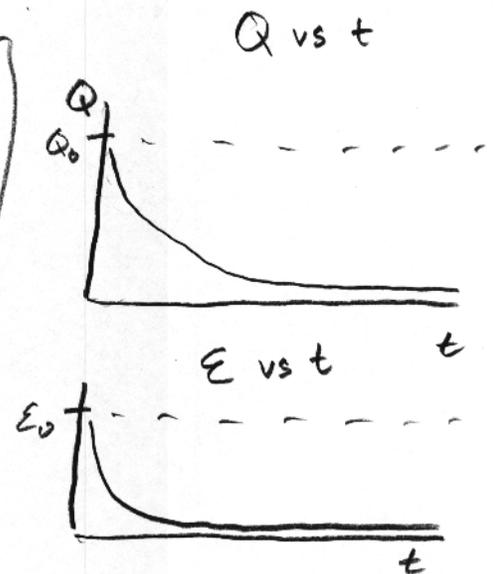
$$\ln \left| \frac{Q}{Q_0} \right| = -\frac{t}{RC}$$

$$e^{\ln \left| \frac{Q}{Q_0} \right|} = e^{-\frac{t}{RC}}$$

$$\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$$

$$Q(t) = Q_0 e^{-\frac{t}{RC}}$$

$$Q(t) = \mathcal{E}_0 C e^{-\frac{t}{RC}}$$



Change in current when a capacitor (discharges).

$$Q(t) = \epsilon_0 C e^{-t/RC}$$

$$I = -\frac{dQ}{dt}$$

take derivative!

$$\frac{d}{dt} [Q(t) = \epsilon_0 C e^{-t/RC}]$$

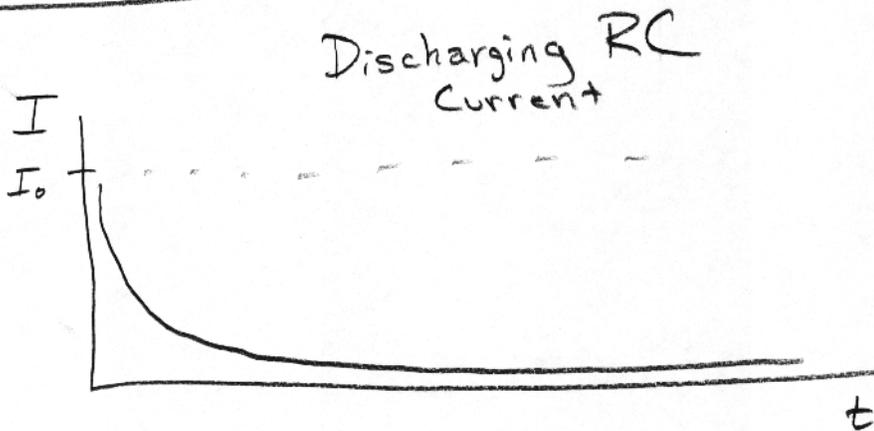
$$\frac{d}{dx} e^{ax} = a e^{ax}$$

$$-\frac{dQ}{dt} = -\frac{1}{R} \epsilon_0 C e^{-t/RC}$$

$$I(t) = +\frac{\epsilon_0 C}{R} e^{-t/RC}$$

$$I(t) = \frac{\epsilon_0 C}{R} e^{-t/RC}$$

$$I_0 = \frac{\epsilon_0 C}{R} = \text{max!}$$



Time Constant in RC circuits.

The time it takes for (I) current to decrease to $\frac{1}{e}$ of its initial value.

$$\frac{1}{e} = 0.37 = 37\% \text{ of } I_0$$

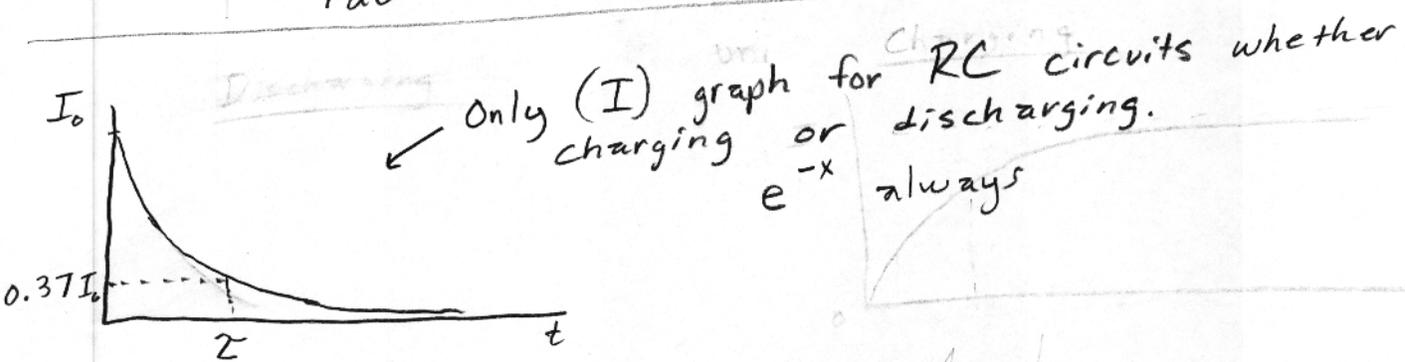
So how do we find this?

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \leftarrow \text{it's right here}$$

time constant (τ) = RC

\uparrow
tau

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$



$$I(t) =$$

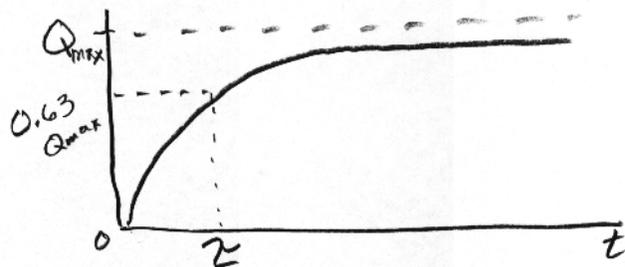
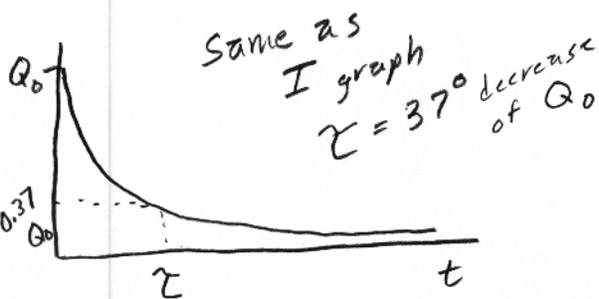
Charge vs. time Graphs will be both e^{-x} and $1 - e^{-x}$

$$Q(t) = C\mathcal{E} e^{-t/RC}$$

(discharging)

$$Q(t) = C\mathcal{E} (1 - e^{-t/RC})$$

(charging)



$1 - \frac{1}{e} = 0.63 = 63\%$ increase from zero to Q_{max} .