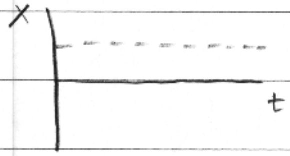
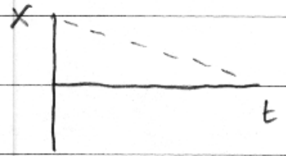
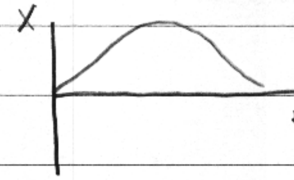
Intro to Calculus APC Physics



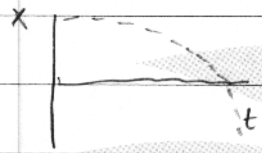
$\frac{\Delta x}{\Delta t} = \emptyset$ zero slope



$\frac{\Delta x}{\Delta t} = (-)$ slope

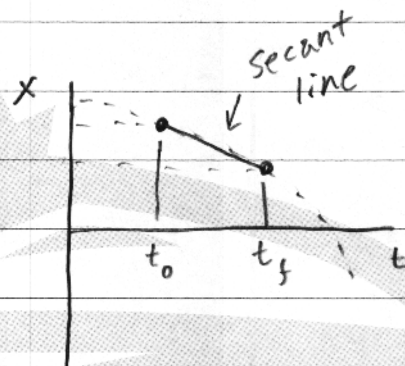


$\frac{\Delta x}{\Delta t} =$ very complex slope

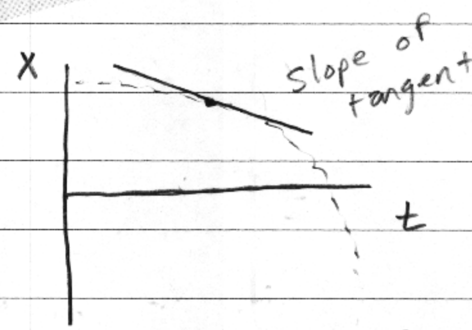


$\frac{\Delta x}{\Delta t}$ = changing slope

Finding slope



1) Average value (Avg. velocity) $\frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0} = \bar{V}$



2) At a given point (more exact rate) draw tangent line to that point. This gives an estimate of instantaneous velocity.

3) limit

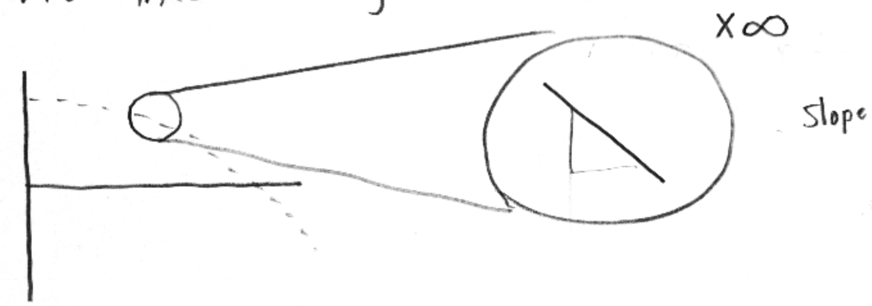
With a tagent it is impossible to get a true value due to the limitations of my scale. However if we shrink the change in time down we can get more accurate.

$$\bar{V} = \frac{\Delta x}{\Delta t} \qquad \lim_{\Delta t \to \emptyset} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = V_{(t)}$$

derivative notation

Instantaneous velocity at any time (t)

Its like zooming in.



$\boxed{\text{Show limit}}$

take the motion equation of an accelerated object.

$$\Delta X = \underset{\substack{\uparrow \\ \text{constant}}}{V_0}\Delta t + \frac{1}{2}\underset{\substack{\uparrow \\ \text{constant}}}{a}\Delta t^2$$

★ note difference between t and Δt

Lets take a limit

$$X_0 = V_0 t + \frac{1}{2}at^2$$

for final position time
$t = (t + \Delta t)$

$$X_f = V_0(t+\Delta t) + \frac{1}{2}a(t+\Delta t)^2$$

$$\lim_{\Delta t \to 0} \frac{X_f - X_0}{t_f - t_0} = \lim_{\Delta t \to 0} \frac{V_0(t+\Delta t) + \frac{1}{2}a(t+\Delta t)^2 - (V_0 t + \frac{1}{2}at^2)}{t + \Delta t - t}$$

to find $V_{(t)}$

$$\lim_{\Delta t \to 0} \frac{V_0 t + V_0 \Delta t + \frac{1}{2}a(t^2 + 2t\Delta t + \Delta t^2) - (V_0 t + \frac{1}{2}at^2)}{t + \Delta t - t}$$

$$\lim_{\Delta t \to 0} \frac{\cancel{V_0 t} + V_0\Delta t + \cancel{\frac{1}{2}at^2} + at\Delta t + \frac{1}{2}a\Delta t^2 - \cancel{V_0 t} - \cancel{\frac{1}{2}at^2}}{\cancel{t} + \Delta t - \cancel{t}}$$

$$\lim_{\Delta t \to 0} \frac{V_0\cancel{\Delta t} + at\cancel{\Delta t} + \frac{1}{2}a\Delta t^{\cancel{2}}}{\cancel{\Delta t}}$$

$$\lim_{\Delta t \to 0} V_0 + at + \cancel{\frac{1}{2}a\Delta t}$$

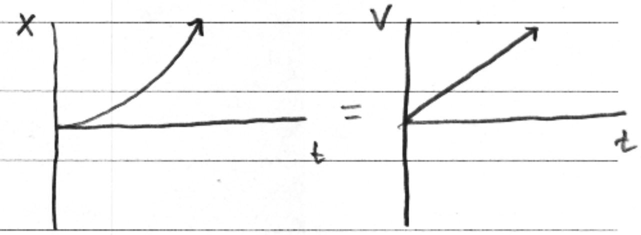
so...

$$\boxed{V_{(t)} = V_0 + at}$$

We will use power rule for derivatives this shortens taking the full limit.

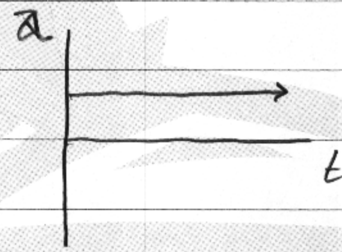
$$\frac{d}{dx}(x^n) = nx^{n-1}$$ Power Rule is used in most polynomials

Ex: $x_{(t)} = t^2$



derivative $\frac{dx}{dt} = 2t = v_{(t)}$

$v_{(t)} = 2t$



derivative $\frac{dv}{dt} = 2 = a_{(t)}$

---

look where kinematics come from w/ calculus

$x_{(t)} = x_0 + v_0 t + \frac{1}{2}at^2$ derivative

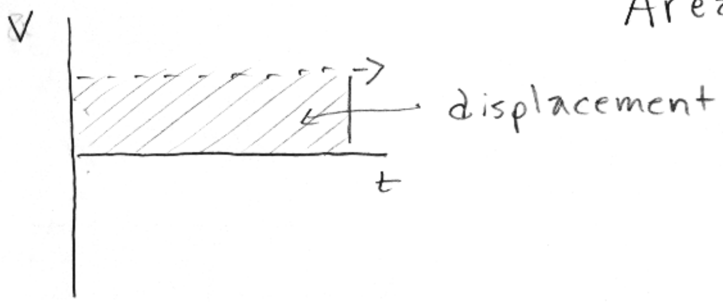
constants go to zero

$\frac{dx}{dt} = v_{(t)} = v_0 + at$

$\frac{dv}{dt} = a$ ← uniform acceleration!

Antiderivatives = Integration

• Area under slope

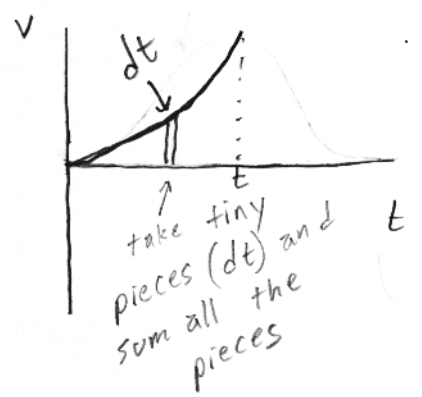


$l \cdot w = v \cdot t = \Delta x$

What if... Not so easy

$\sum$ Areas of rectangle = displacement (Not very accurate)

• Integration (Antiderivative)



Symbol for integral

$$\int v\,dt = x$$

Since the intergal is the inverse of taking the derivative.

$$\frac{dx}{dt} = v$$

Lets apply the function

$V_{(t)} = 2t^2$

to the above graph to find the displacement (Area under slope) from 0 to t.

$$x = \int v\,dt$$

$$x = \int_0^t 2t^2\,dt$$

$$x = \left[\frac{2}{3}t^3\right]_0^t$$

$$x = \left[\frac{2}{3}t^3 - \emptyset\right]$$

$$X_{(t)} = \frac{2}{3}t^3$$

Position for any given time