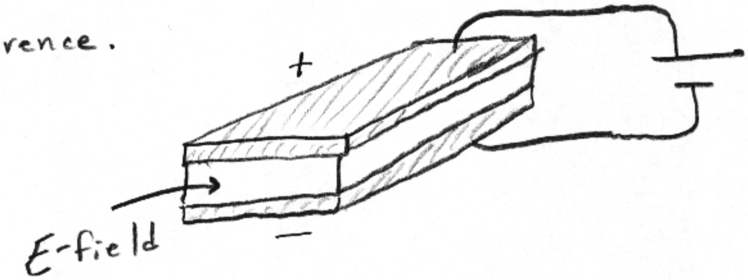
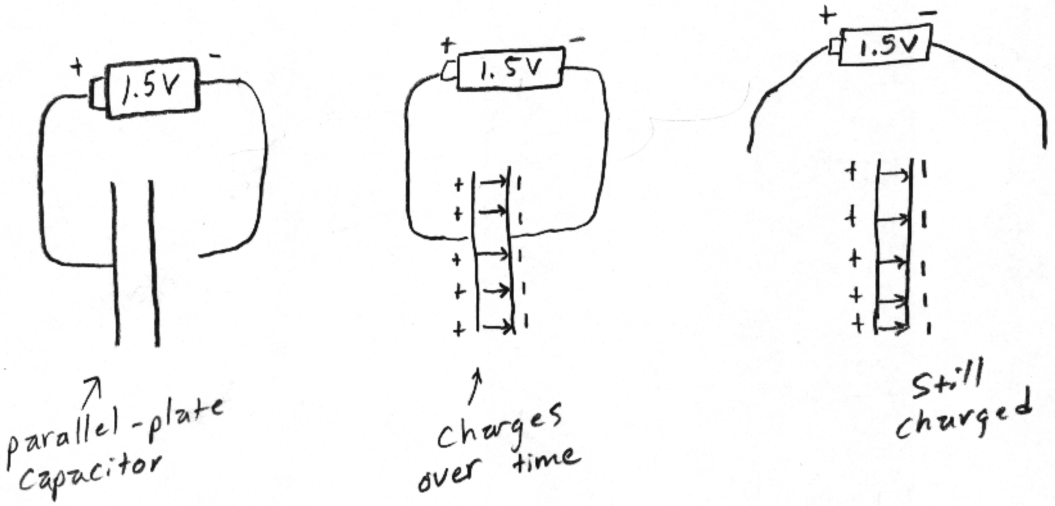
Capacitance and Dielectrics

- Storing charges
  - 2 conductors of any shape placed near and insulated from each other.
  - This insulative region can be air or some material called a dielectric.
  - The E-field inbetween the plates forms a potential difference.





The potential difference represents the potential energy in charges which have the ability to do work

For conducting sheets.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$E \cdot A = \frac{Q_{in}}{\varepsilon_0} \rightarrow E \cdot \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Delta V = -\int E \cdot dr \qquad \text{-or-} \qquad \frac{\Delta V}{d} = E = \frac{\sigma}{\varepsilon_0}$$

$$V_b - V_a = -\int_a^b \frac{\sigma}{\varepsilon_0} \cdot dr \qquad \frac{\Delta V}{d} = \frac{Q}{\varepsilon_0 A}$$

$$V_b - V_a = -\frac{\sigma}{\varepsilon_0} \Big| r \Big|_a^b \qquad \underline{\Delta V = \frac{Q \cdot d}{\varepsilon_0 A}}$$

$$V_b - V_a = -\frac{\sigma}{\varepsilon_0}(b-a)$$

$$V_b - V_a = \frac{\sigma}{\varepsilon_0}(a-b)$$

$$\Delta V = \frac{\sigma}{\varepsilon_0} d = E \cdot d = \underline{\frac{Q\,d}{\varepsilon_0 A}}$$

amount of charge

$$\Delta V = \frac{Q \cdot d}{\varepsilon_0 A}$$

geometry of plates

potential

So, where is capacitance hiding?

well... $Q \propto \Delta V$ true...

C = constant of proportionality

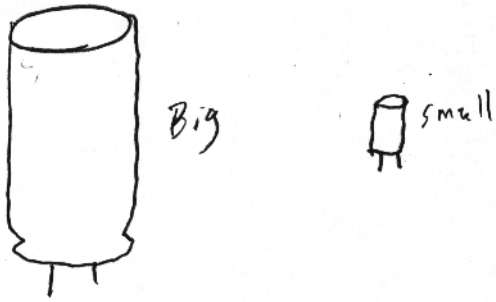
$$Q = CV$$

$$C = \frac{Q}{V} = \text{capacitance} \qquad \frac{\text{Coulombs}}{\text{Volts}} = \text{Farad } (F)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}\,\varepsilon_0 A}{\cancel{Q} \cdot d} = \boxed{\frac{\varepsilon_0 A}{d}}$$

You can change capacitance only by changing geometry of the plates!

$$C \propto \frac{A}{d} \longrightarrow C = \varepsilon_0 \frac{A}{d}$$



How much area is needed for a 1 F parallel-plate capacitor seperated by 1mm of distance?

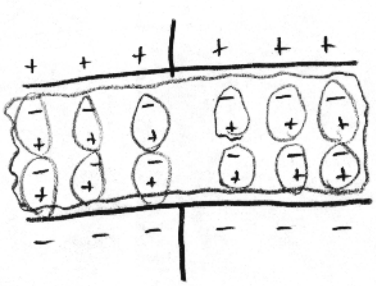
So, how is that practical?

Dielectrics

- Dielectrics add mass, more mass more atoms, more atoms, more charge...

$$C = k\,\varepsilon_0 \frac{A}{d}$$

k = Dielectric constant "kappa"



dielectric

Polarizes material forms E-fied in opposite direction insile.

This decreases E-field and potential diff. (ΔV) when not connected to a voltage source.

Q remains constant

C increases by (k)

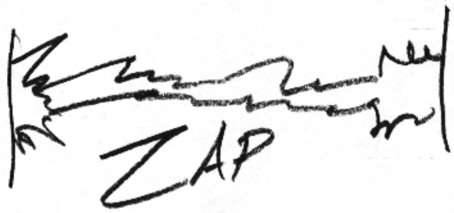
$$C^{\uparrow} = \frac{Q}{\Delta V_{\downarrow}}$$

- If still connected to a source voltage (ΔV) stays constant and Q increases.

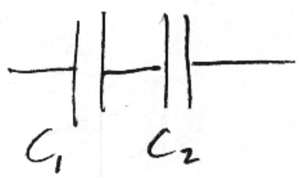
$$\overset{\uparrow}{Q} = \overset{\uparrow}{C}\,\Delta V$$

Dielectrics, including air, limit the (ΔV) that can form between plates.

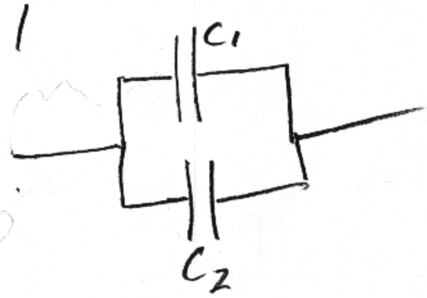
if ΔV gets too high, dielectric breakdown occurs.



- Using capacitors in combinations
  - Series



  - Parallel



Series

- Charge by induction
- share same charge (Q)
- Potential is divided $\quad V_{tot} = V_1 + V_2 + V_3 \ldots$

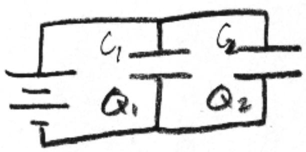
$$V = \frac{Q}{C} \qquad \frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} \ldots$$

$$\boxed{\frac{1}{C_s} = \sum \frac{1}{C_i}}$$

Parallel

- Voltage is the same on each (All touch touch the source)
- Charge is split

$$Q_{Tot} = Q_1 + Q_2 + Q_3, \ldots$$



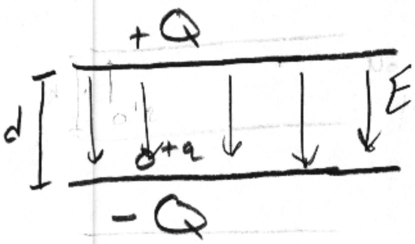
$$Q = CV$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 \ldots$$

$$V_T = V_1 = V_2 = V_3$$

$$\boxed{C_p = \Sigma C_i}$$

Stored Energy in Capacitors



requires work to move

$$V = E \cdot d$$

$$C = \frac{Q}{V}$$

$$dW = E \cdot d \, dq$$

$$dW = V \, dq$$

$$dW = \frac{Q}{C} dq$$

$$\int_{\emptyset}^{Q} dW = \int_{\emptyset}^{Q} \frac{Q}{C} dq$$

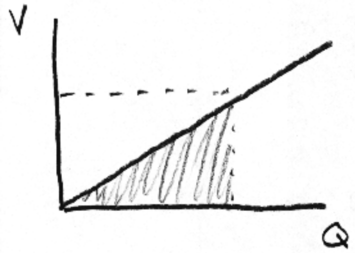
$$U_E = \frac{1}{C} \int_{\emptyset}^{Q} Q \, dq$$

$$U_E = \frac{1}{C} \left[\frac{1}{2} Q^2 \right]_0^Q$$

$$\boxed{U_E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2}$$

$$C = \frac{Q}{V} \qquad C = \frac{Q}{V}$$

• Stored Energy is really "potential"



Area = Energy

$$U_c = \frac{1}{2} bh = \frac{1}{2} QV = \text{Joules}$$

$V - \left(\frac{J}{C}\right)$ $Q - (C)$ $\frac{J}{\cancel{C}}\cancel{C} = J$ ✓

Sweet!