

# Energy in a Capacitor (RC) circuits

- Can be thought of a work done to charge the capacitor.

$$dW = Vdq = \frac{q}{C} dq$$

$$W = F \cdot d$$

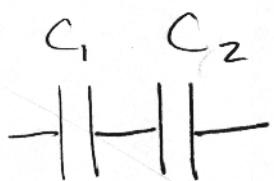
$$W = \int_0^Q \frac{q}{C} \cdot dq = \frac{Q^2}{2C}$$

$$U_E = Vq$$

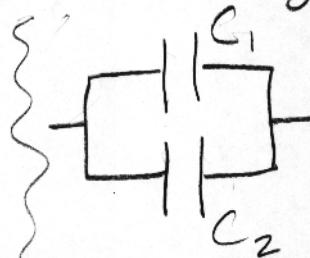
$$V = \frac{Q}{C}$$

$$\boxed{U_c = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2}$$

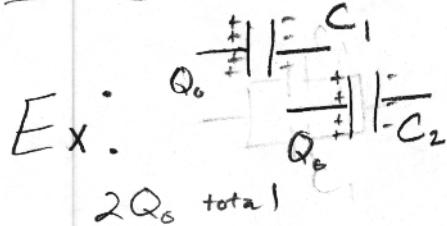
## Combining Capacitors & Energy



$$\frac{1}{C_s} = \sum \frac{1}{C_i}$$

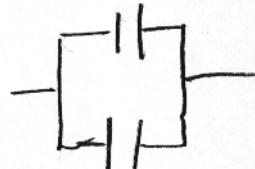


$$C_p = \sum C_i$$



$$C_1 = C_2 = C_F$$

$$Q_1 = Q_2 = Q_0$$



$C_1$  is charged then connected to  $C_2$  which is also charged.  
IF they are placed in parallel what is...

$$U_c = \frac{1}{2} CV^2$$

a) the new potential difference (conserve charge)

$$V = \frac{Q}{C}$$

$$V_0 = \frac{Q_0}{C_1} \quad V_0 = \frac{Q_0}{C_2}$$

$$C_1 V_0 + C_2 V_0 = (C_1 + C_2) V$$

$$V = \frac{2CV_0}{2C} = V_0$$

b) New charge

Same, just combined

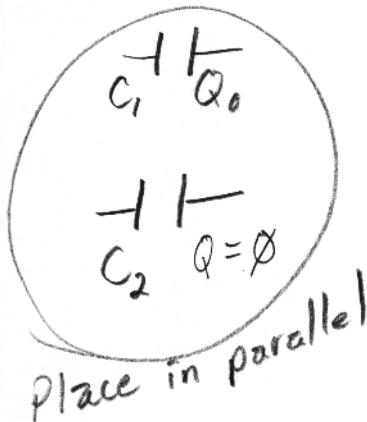
$$[2Q_0]$$

c) New potential energy

$$U_0 = \frac{1}{2} CV_0^2 \Rightarrow CV_0^2$$
$$U_f = \frac{1}{2}(CV)^2 = \frac{1}{2} 2C V_0^2 = \boxed{CV_0^2}$$
$$U_0 = \frac{1}{2} CV_0^2$$

why not go up? Because, it was not connected to a battery (Emf source). Work is required to combine.

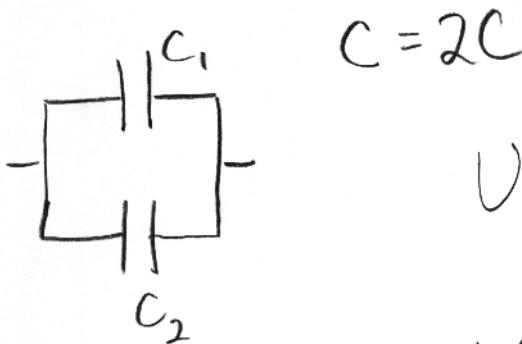
Ex 2.



$$C_1 = C_2 = C$$

$$Q_{\text{tot}} = Q_0 \quad \text{conservation of charge}$$

$$V_0 = \frac{Q_0}{C_1} \quad C_1 V_0 = C_1 + C_2$$



$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q_0^2}{2C} = \frac{1}{4} \frac{Q_0^2}{C}$$

$$U_0 = \frac{1}{2} \frac{Q_0^2}{C}$$

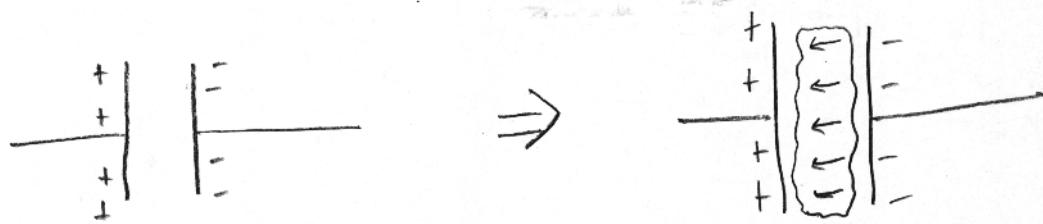
$$U = \frac{1}{2} U_0$$

Energy loss!

However, when connected to battery, the potential will not drop!

$$\text{So... } U = \frac{1}{2} CV^2$$

# Adding Dielectric & Energy



E-field goes down!

$$V = E \cdot d$$

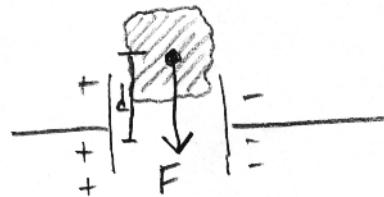
$$C = \frac{Q}{V} \quad \begin{matrix} \uparrow \\ \text{(on plate stays same)} \\ \downarrow \end{matrix}$$

$$U_0 = \frac{1}{2} CV^2$$

$$\downarrow U = \frac{1}{2} \cancel{CV^2} = \frac{1}{2} QV$$

Energy goes down when dielectric is first added.

Why?



It takes work from the capacitor to pull the dielectric in.

It performs work and loses energy.

However, if the capacitor is attached to an EMF source the potential will always match the EMF source. This means that the energy will increase.

$$U = \frac{1}{2} CV^2$$

$$C = \frac{\epsilon_0 A}{d}$$

