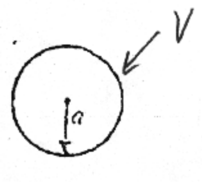
Rec 1



$$\Delta V_{\infty \to a} = -\int_{\infty}^{a} \frac{kQ}{r^2}\, dr$$

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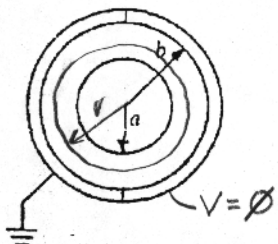
1988E1. The isolated conducting solid sphere of radius a shown above is charged to a potential V.

a. Determine the charge on the sphere.

$$V = \frac{kQ}{a} \qquad \boxed{Q = \frac{Va}{k}}$$



Two conducting hemispherical shells of inner radius b are then brought up and, without contacting the solid sphere are connected to form a spherical shell surrounding and concentric with the solid sphere as shown below The outer shell is then grounded.

b. By means of Gauss's law, determine the electric field in the space between the solid sphere and the shell at a distance r from the center.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{en}}{\varepsilon_0} \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Va}{k4\pi\varepsilon_0 r^2} = \boxed{\frac{Va}{r^2}}$$

$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

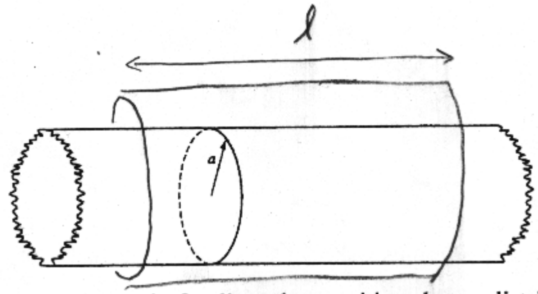
c. Determine the potential of the solid sphere relative to ground.

$$\Delta V_{(b\to a)} = -\int \vec{E}\, d\vec{r} \qquad V_{(a)} = -Va\int_b^a \frac{1}{r^2}\, dr \qquad \boxed{V_{(a)} = Va\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$V_{(a)} - \cancel{V_{(b)}} = -\int_b^a \frac{Va}{r^2}\, dr \qquad V_{(a)} = -Va\left[-\frac{1}{r}\right]_b^a$$

d. Determine the capacitance of the system in terms of the given quantities and fundamental constants.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Va\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{\cancel{Va}}{k\cancel{Va}\left[\frac{1}{a} - \frac{1}{b}\right]} = \boxed{\frac{1}{k\left[\frac{1}{a} - \frac{1}{b}\right]}}$$



3

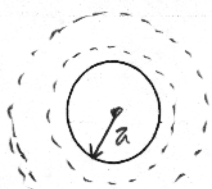
1995E1. A very long nonconducting rod of radius a has positive charge distributed throughout its volume. The charge distribution is cylindrically symmetric, and the total charge per unit length of the rod is λ .

a. Use Gauss's law to derive an expression for the magnitude of the electric field E outside the rod.

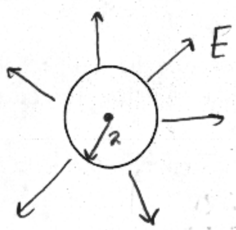
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{en}}{\varepsilon_0} \qquad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} \qquad \lambda = \frac{Q}{\ell}$$

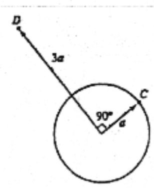
$$E 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0}$$

b. The diagrams below represent cross sections of the rod. On these diagrams, sketch the following.
i. Several equipotential lines in the region r > a



ii. Several electric field lines in the region r > a





c. In the diagram above, point C is a distance a from the center of the rod (i.e., on the rod's surface), and point D is a distance 3a from the center on a radius that is 90° from point C. Determine the following.

i. The potential difference Vc - VD between points C and D

$$\Delta V_{C\to D} = -\int_a^{3a} E\,dr \qquad \Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^{3a}\frac{1}{r}\,dr$$

$$\Delta V = -\int_a^{3a}\frac{\lambda}{2\pi\varepsilon_0 r}\,dr \qquad \Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln 3a - \ln a\right]$$

ii. The work required by an external agent to move a charge + Q from rest at point D to rest at point C

$$W_{done} = \Delta U_e = \Delta V q$$

$$\boxed{\Delta U_e = \frac{\lambda}{2\pi\varepsilon_0}(\ln 3a - \ln a)Q}$$

Inside the rod (r < a), the charge density ρ is a function of radial distance r from the axis of the rod and is given by $\rho = \rho_0(r/a)^{1/2}$, where ρ_0 is a constant.

d. Determine the magnitude of the electric field E as a function of r for r < a. Express your answer in terms of ρ_0, a, and fundamental constants.

$$\oint \vec{E}\, d\vec{A} = \frac{Q_{en}}{\varepsilon_0}$$

$$\rho = \frac{dq}{dV}$$

$$\int_0^r dq = \int_0^r \rho\, dV$$

$$Q_{en} = \int_0^r \rho_0 \left(\frac{r}{a}\right)^{\frac{1}{2}} A\, dr$$

$$Q_{en} = \int_0^r \rho_0 \left(\frac{r}{a}\right)^{\frac{1}{2}} 2\pi r l\, dr$$

$$Q_{en} = 2\pi l \frac{\rho_0}{\sqrt{a}} \int_0^r (r)^{\frac{3}{2}}\, dr$$

$$Q_{en} = 2\pi l \frac{\rho_0}{\sqrt{a}} \left[\frac{2}{5} r^{\frac{5}{2}}\right]_0^r$$

$$Q_{en} = \frac{4}{5}\pi l \frac{\rho_0}{\sqrt{a}} r^{\frac{5}{2}}$$

$$E\, 2\pi r l = \frac{\frac{4}{5}\pi l \frac{\rho_0}{\sqrt{a}} r^{\frac{5}{2}}}{\varepsilon_0}$$

$$E = \frac{4\rho_0 r^{\frac{5}{2}}}{10\sqrt{a}\,\varepsilon_0 r}$$

$$\boxed{E = \frac{4\rho_0 r^{\frac{3}{2}}}{10\sqrt{a}\,\varepsilon_0}}$$

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
[redacted]SCORING GUIDELINES

[redacted]

4

15 points total — **Distribution of points**

(a)

i) 2 points

For using the equation relating the potential difference to the magnitude of the work done on the electron — 1 point

$$W = |q||\Delta V| = \left(1.6\times 10^{-19}\ \text{C}\right)\left[\left((2)(20\ \text{m})^2 + (7)(20\ \text{m}) - (15)\right) - (0+0-15)\right]$$

For a correct answer with units — 1 point

$$W = 1.50\times 10^{-16}\ \text{J} = 940\ \text{eV}$$

ii) 2 points

For using the equation relating the magnitude of the work done on the electron to the speed of the electron — 1 point

$$W = \Delta K = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}mv^2$$

For correct substitutions — 1 point

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{(2)\left(1.50\times 10^{-16}\ \text{J}\right)}{\left(9.11\times 10^{-31}\ \text{kg}\right)}}$$

$$v = 1.82\times 10^{7}\ \text{m/s}$$

(b) 2 points

For taking the derivative of the electric potential to calculate the electric field — 1 point

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(2x^2 + 7x - 15\right)$$

For a correct answer — 1 point

$$E_x = -4x - 7$$

Question [redacted] (continued)

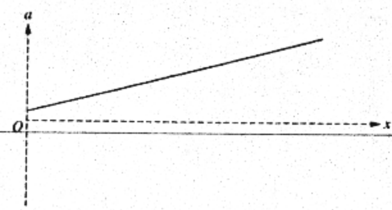
4

	Distribution of points

4

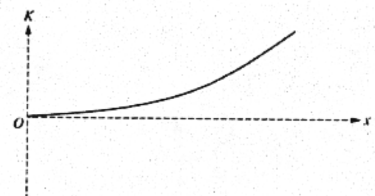
(c)

i) 2 points



For a straight line for $x > 0$ with non-zero slope — 1 point
For a line with positive slope and positive y intercept — 1 point

ii) 2 points



For a graph starting at the origin and extending into the first quadrant — 1 point
For a concave up curve with $K \geq 0$ — 1 point

(d) 2 points

For selecting only "$x = -2$ m" — 1 point
For a correct justification — 1 point
Example: Electrons accelerate opposite the direction of the electric field. Therefore the electron will accelerate in the negative direction only if the electric field is directed in the positive direction. The electric field is positive only for $x < -1.75$ m.
Note: If incorrect selection is made, justification cannot earn credit.

Question [redacted] (continued)

4

	Distribution of points

4

(e) 3 points

Starting with the integration of the net electric field to determine the potential difference
$V_{20} - V_0 = -\int E \cdot dx$

For recognizing that $E = E_1 + E_2$ — 1 point
$V_{20} - V_0 = -\int E_{net}' \cdot dx = -\int (E_1 + E_2) \cdot dx$

For the substitution of the correct expressions for the electric fields — 1 point
$\Delta V = -\int (E_1 + E_2)\,dx = -\int [(-4x - 7) + 7]\,dx = -\int -4x\,dx = \int_{x=0}^{x=20} 4x\,dx$

For a substitution with the correct limits and signs — 1 point
$\Delta V = \left[2x^2\right]_{x=0}^{x=20} = (2)(20)^2 - (2)(0)^2$
$\Delta V = 800$ V

Alternate solution — *Alternate points*
For correctly recognizing that the potential difference is equal to the sum of potential differences due to each electric field. — *1 point*
$\Delta V = \Delta V_1 + \Delta V_2$
For determining the potential difference of the original field from part (a) — *1 point*
$\Delta V_1 = V(20) - V(0) = 925 - (-15) = 940$ V
For correctly determining the potential difference due to the field generated by the charged object — *1 point*
$\Delta V_2 = -\int_0^{20} E\,dx = -E\Delta x = -\left(7\frac{\text{V}}{\text{m}}\right)(20\text{ m}) = -140$ V
$\Delta V = 940 + (-140) = 800$ V

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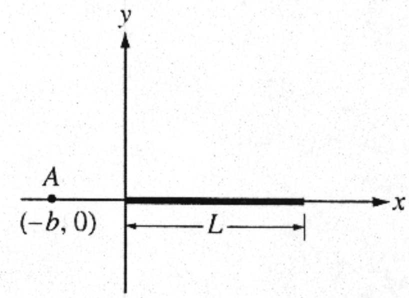
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

A rod of length L lies along the x-axis with its left end at the origin, as shown in the figure above. The rod has a nonuniform linear charge density $\lambda = \alpha x$, where α is a positive constant.

(a) Determine the units of the constant α.

$$\lambda = \alpha x \qquad \alpha = \left(\frac{C}{m^2}\right)$$

$$\alpha = \frac{\lambda}{x} = \frac{Q}{Lx}$$

(b) Derive an expression for the total charge on the rod. Express your answer in terms of α, L, and physical constants, as appropriate.

$$\lambda = \frac{Q}{L} = \frac{dq}{dl} \qquad \int_0^L dq = \int_0^L \lambda\, dx$$

$$dl = dx \qquad Q = \int_0^L \alpha x\, dx$$

$$Q = \frac{1}{2}\alpha x^2 \Big|_0^L$$

$$\boxed{Q = \frac{1}{2}\alpha L^2}$$

GO ON TO THE NEXT PAGE.

The rod is replaced with one of the same length but with a uniform positive linear charge density λ_0.

(c) Show that the electric field at point A, which is a distance b to the left of the rod, has a magnitude of $E = \dfrac{\lambda_0 L}{4\pi\varepsilon_0 b(b+L)}$.

$$dq = \lambda\, dr$$

$$E = \int \frac{k\, dq}{r^2}$$

$$E = \int_b^{b+L} \frac{k \lambda\, dr}{r^2}$$

$$E = k\lambda \int_b^{b+L} \frac{1}{r^2}\, dr$$

$$E = k\lambda \left[-\frac{1}{b} + \frac{1}{(b+L)} \right]$$

$$E = k\lambda \left[\frac{1}{b+L} - \frac{1}{b} \right]$$

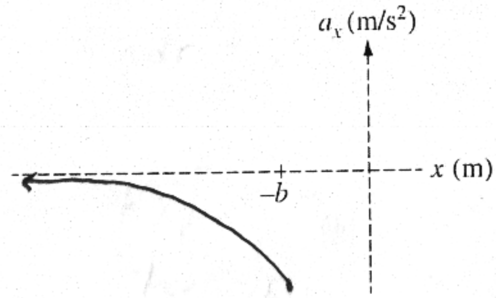
$$E = k\lambda \left[\frac{b}{b(b+L)} - \frac{b+L}{b(b+L)} \right]$$

$$E = k\lambda_0 \frac{L}{b(b+L)}$$

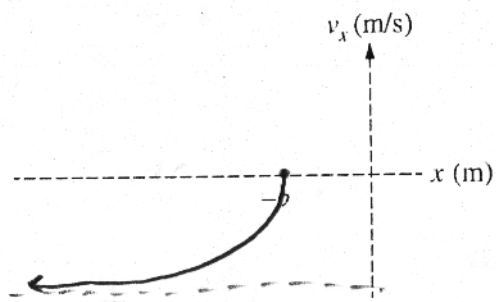
$$\boxed{E = \frac{\lambda_0 L}{4\pi\varepsilon_0 b(b+L)}}$$

(d) A proton is placed at point A, at position $(-b, 0)$, near the positively charged rod and released from rest.

i. Sketch a graph of the horizontal component of the acceleration of the proton a_x as a function of its position x for the region in which the proton travels after it is released. Sketch any asymptote, but do not indicate its value.



ii. Sketch a graph of the horizontal component of the velocity of the proton v_x as a function of its position x for the region in which the proton travels after it is released. Sketch any asymptote but do not indicate its value.



Question 1 continues on next page.