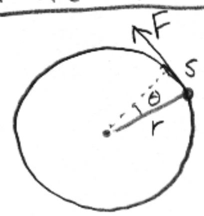
Work and Rotational Motion

Rotational Kinetic Energy $\boxed{K = \frac{1}{2} I \omega^2}$

How is this kinetic energy changed?



Imagine Work done by a force through at tiny distance in this case S (arc length)

$s = r\theta$ so... $ds = r\,d\theta$

$W = F \cdot d$ so... $dW = F \cdot ds$ -or- $dW = F \cdot r\,d\theta$

$F \cdot r = \tau$ so... $dW = \tau\, d\theta$

$\boxed{W = \int \tau\, d\theta}$

look familiar $W = \int F\, dx$

Rate at which work is done is Power

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

$\boxed{P = \tau\omega}$

$P = F \cdot v$

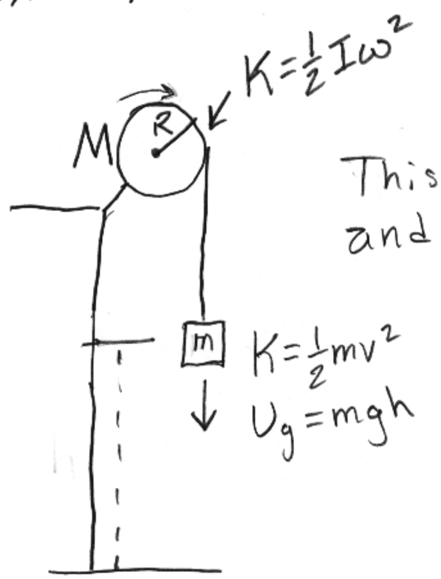
Work-Energy Theorem

$$\boxed{\Sigma W = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0^2}$$

Conservation of Energy

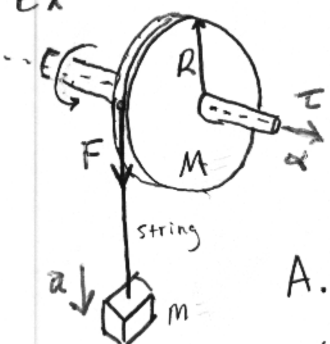
It is critical that when dealing with rotating objects (rolling) or pulley systems (with mass) that you recognize both translational and rotational kinetic energy.

Ex: Mass and Pulley System... this time the pulley has mass!



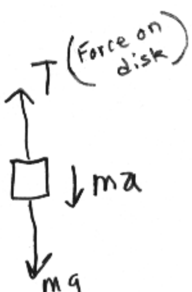
This system deals with translational, rotational and potential energies.

Ex



The wheel to the left is a solid disk of $M = 2.0\text{kg}$, $R = 30\text{cm}$, and $I = 0.09\text{kg}\cdot\text{m}^2$. The suspended block has a mass $m = 0.5\text{kg}$. The wheel starts at rest. (No slip)

A. Find angular acceleration (α) of the wheel.



(τ)orque in this scenario comes from the Tension (T) at a distance of (R) from the axis.

$\tau = I\alpha = TR$ $\qquad$ $I\alpha = TR$

$\Sigma F = ma$ $\qquad$ $a = \alpha \cdot r$

$mg - T = ma$

$T = mg - ma$

$T = mg - m(\alpha \cdot R)$ <2 unknowns (α & T)>

$I\alpha = (mg - m\alpha R)\cdot R$

$I\alpha = mgR - m\alpha R^2$

$I\alpha + m\alpha R^2 = mgR$

$\alpha(I + mR^2) = mgR$

$$\alpha = \frac{mgR}{I + mR^2} = \frac{0.5\text{kg}(10)(0.30)}{(0.09)+(0.5)(0.30)^2}$$

$$\boxed{\alpha = 11.1\,\text{rad}/\text{s}^2}$$

B. The linear acceleration of the block.

$$\alpha \cdot r = a$$

$$(11.1)(.3) = \boxed{3.33\,m/s^2}$$

C. The tension of the string.

$$T = mg - ma$$

$$T = (.5)(10) - (.5)(3.33) = \boxed{3.34\,N}$$

D. The angular velocity of the wheel @ $t = 1.0s$.

$$\omega = \cancel{\omega_0} + \alpha t$$

$$\omega = (11.1)(1) = \boxed{11.1\,rad/s}$$

E. The number of radians the wheel turns in 2 sec.

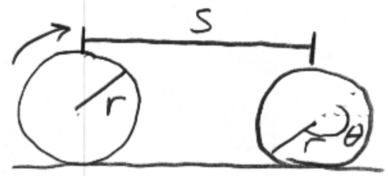
$$\Delta\theta = \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2$$

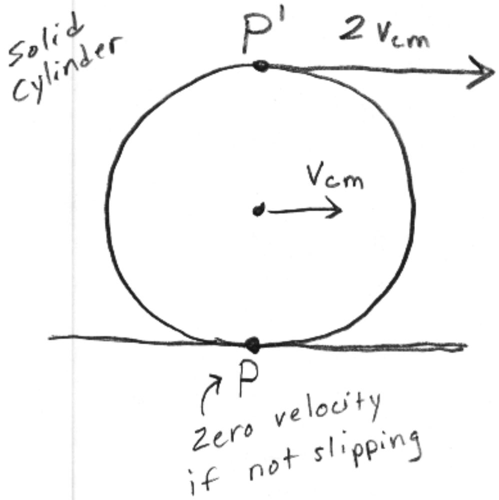
$$\Delta\theta = \frac{1}{2}(11.1)(2)^2 = \boxed{22.2\,rad} \text{ or } 3.5\,rev$$

Rolling Motion

- We will deal with pure rolling motion (No slip)

$$V_{cm} = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$





Total Kinetic Energy of rolling cylinder

$$K = \frac{1}{2} I_p \omega^2$$

I_p = is moment of inertia from point P the source of torque and axis

parallel-axis theorem will state

$$I_p = I_{cm} + MR^2$$

$$R^2\omega^2 = v^2$$

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} MR^2\omega^2$$

$$\boxed{K = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2}$$

We can use this in conservation of energy to calculate final velocity or height.

Friction provides torque for rolling objects!