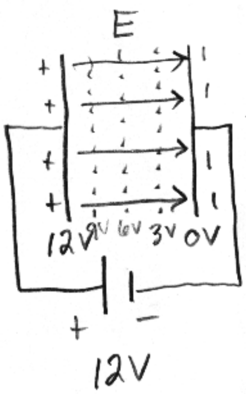
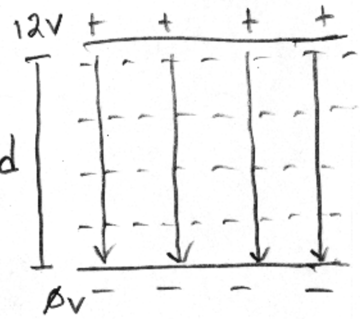
Equipotential Lines

- Always $\perp$ to E-field lines.

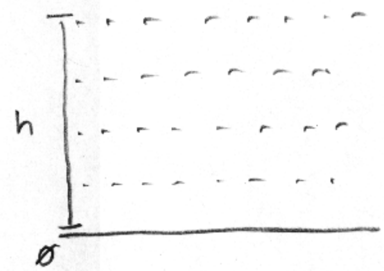
Parallel-Plates



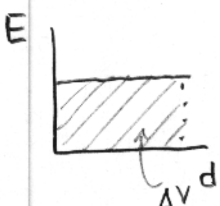
$E = \text{constant}$



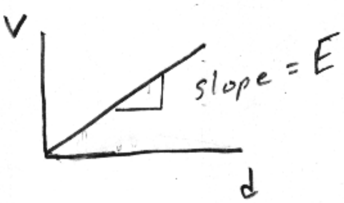
Analogy is elevation



Topography Maps ≈ Equipotential lines



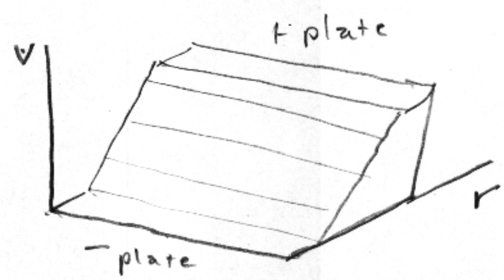
$E = \frac{\sigma}{\varepsilon_0}$



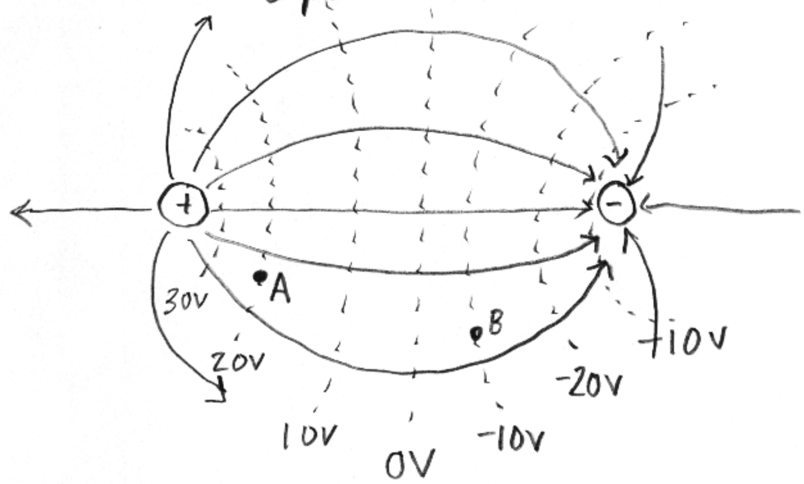
$V = E \cdot d$

$$\Delta V = -\int E \cdot dr$$

$$E = -\frac{dV}{dr}$$

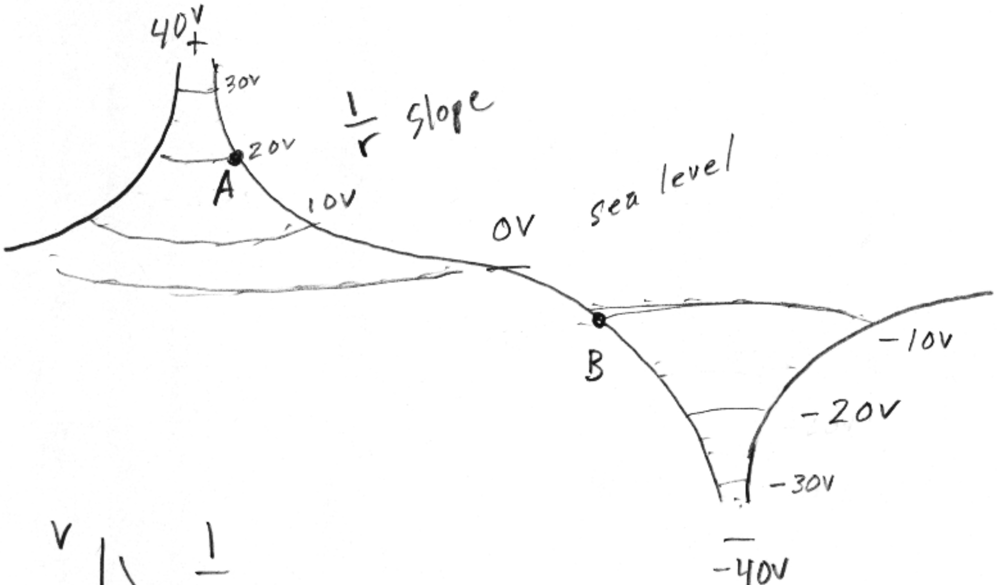


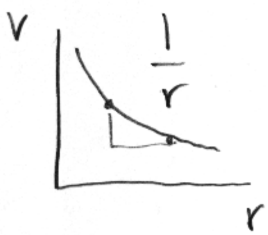
dipole from point charges.

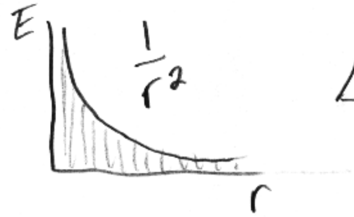


$$\Delta V = V_A - V_B = 20V - (-10V)$$

$$\Delta V = 30V$$







$$E = -\frac{dV}{dr}$$

$$\Delta V = -\int E \cdot dr$$

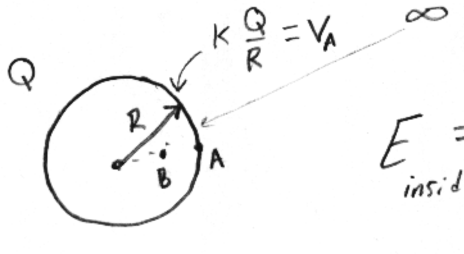
$$\Delta V = -\int_{\infty}^{r} \frac{kQ}{r^2} \cdot dr$$

$$E = -\frac{d}{dr} kQ \frac{1}{r} = \frac{kQ}{r^2}$$

$$\Delta V = kQ\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Delta V = \frac{kQ}{r}$$

Voltage inside an insulative sphere. (as a function of r)



$$E_{inside} = \frac{kQ}{R^3} r$$

$$\Delta V = -\int E \cdot dr$$

$$\Delta V = -\int_R^r \frac{kQ}{R^3} r \cdot dr$$

$$\Delta V = -\frac{kQ}{R^3} \int_R^r r \cdot dr$$

$$\Delta V = -\frac{kQ}{R^3} \left[\frac{r^2}{2} \right]_R^r$$

$$\Delta V = \frac{-kQ}{2R^3} \left[r^2 - R^2 \right]$$

$$V_B - V_A = \frac{kQ}{2R^3} \left[R^2 - r^2 \right]$$

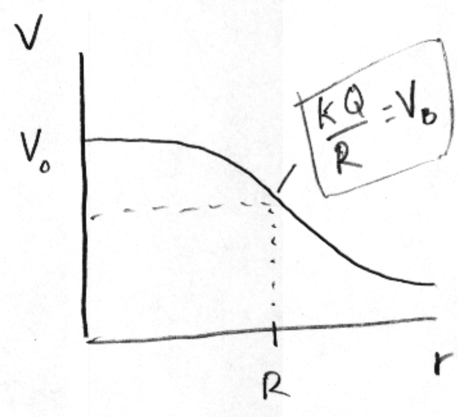
$$V_B - k\frac{Q}{R} = \frac{kQ}{2R^{\cancel{3}}} \cancel{R^2} - \frac{kQr^2}{2R^3}$$

$$V_B = \frac{kQ}{2R} + \frac{kQ}{R} - \frac{kQr^2}{2R^3}$$

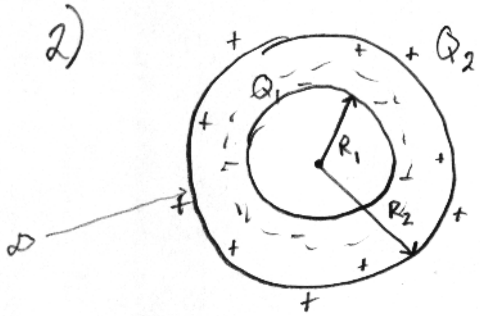
$$V_B = \frac{kQR^2}{2R^3} + \frac{2kQR^2}{2R^3} - \frac{kQr^2}{2R^3}$$

$$V_B = \frac{kQR^2 + 2kQR^2 - kQr^2}{2R^3} = \frac{3kQR^2 - kQr^2}{2R^3} = \frac{kQ}{2R}\left(\frac{3\cancel{R^2}}{\cancel{R^2}} - \frac{r^2}{R^2}\right)$$

$$\boxed{= \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$



2)



a) $\Delta V = -\int_{\infty}^{R_2} E \cdot dr$

$$V_2 - \emptyset = -kQ \int_{\infty}^{R_2} \frac{1}{r^2} \cdot dr = -kQ\left[-\frac{1}{r}\right]_{\infty}^{R_2}$$

$$V_2 = kQ\left[\frac{1}{R_2} - \frac{1}{\infty}\right]$$

$$\boxed{V_2 = \frac{kQ}{R_2}}$$ on surface of outer

$\boxed{900{,}000\,V}$

b) $\Delta V = -\int E \cdot dr$ must use relative to outer potential R_2 to R_1

$$E_{inside} = \frac{kQ_1}{r^2}$$

$$\Delta V = -\int_{R_2}^{R_1} \frac{kQ_1}{r^2} \cdot dr$$

$$V_1 - V_2 = -kQ_1 \int_{R_2}^{R_1} \frac{1}{r^2} \cdot dr$$

$$V_1 - V_2 = -kQ_1\left[-\frac{1}{r}\right]_{R_2}^{R_1}$$

$$\Delta V = kQ_1\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = -450{,}000V$$

c) $\Delta V = V_1 - V_2$

$$\Delta V + V_2 = V_1 = \boxed{450{,}000V}$$

d) $V =$ constant in conductor

$\boxed{450{,}000V}$

e) $\Delta V = 450{,}000V$

$W = \Delta V \cdot -e$

f.) $W = V_2 \cdot +e$

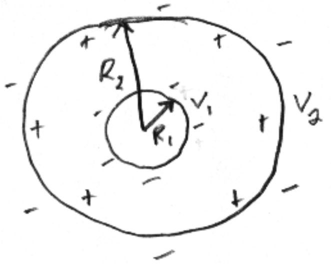
g) R_2'

$$V_2 = \frac{kQ}{R_2'}$$

$$\Delta V = kQ_1\left[\frac{1}{R_1} - \frac{1}{R_2'}\right]$$

h)

3.



gauss $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$\lambda = \frac{Q}{L}$

b) $\Delta V = -\int_{R_2}^{R_1} E \cdot dr$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{R_2}^{R_1} \frac{1}{r} \cdot dr$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln|r|\right]_{R_2}^{R_1}$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln R_1 - \ln R_2\right]$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_1}{R_2}\right)$ ← flip to rid (−)

$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$

$Q_{tot} = \emptyset$

a) since outside is $E = \emptyset$ we have no E-lines or potential!

b) $V_1 - V_2 = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right) = \frac{Q_1}{2\pi\varepsilon_0 L}\ln\left(\frac{R_2}{R_1}\right) = \boxed{-2500V}$

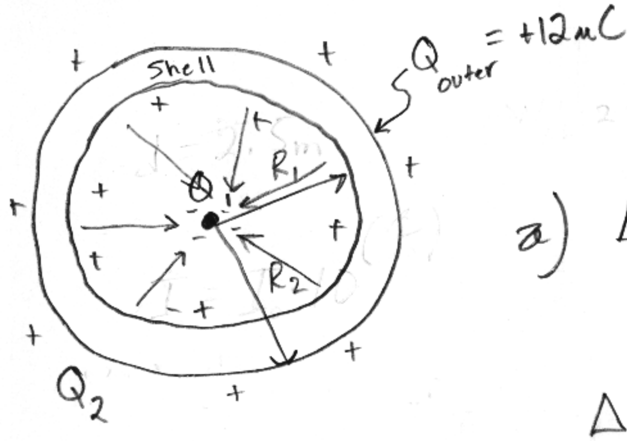
c) $\boxed{-2500V}$

d) $E = \emptyset$ $V = \text{constant}$ $\boxed{-2500V}$

e) $W = \Delta V e$

f) $\emptyset$ no potential

4.



a) $\Delta V = -\int_{\infty}^{R_2} \frac{kQ}{r^2} \cdot dr$

$\Delta V = -kQ \int_{\infty}^{R_2} \frac{1}{r^2} \cdot dr$

$\Delta V = kQ \left[\frac{1}{R_2} - \frac{1}{\infty} \right]$

$V_{R_2} = \frac{k Q_{outer}}{R_2} = \boxed{1{,}200{,}000\,V}$

b) $\Delta V = \varnothing$

V = constant inside

$\boxed{V_{R_1} = 1{,}200{,}000\,V}$

c)

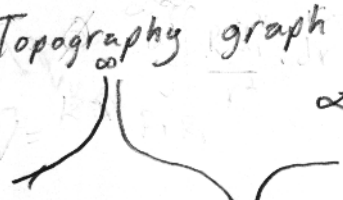
d) $E = k \frac{Q_1}{r^2}$ $r = 1.0\,cm$

$\Delta V = -\int_{R_1}^{r} E \cdot dr$

$\Delta V = kQ_1 \left[\frac{1}{r} - \frac{1}{R_1} \right] = -4.6 \times 10^6\,V$

$\Delta V = V_r - V_{R_1} = \boxed{-3.4 \times 10^6\,V}$

e) Topography graph



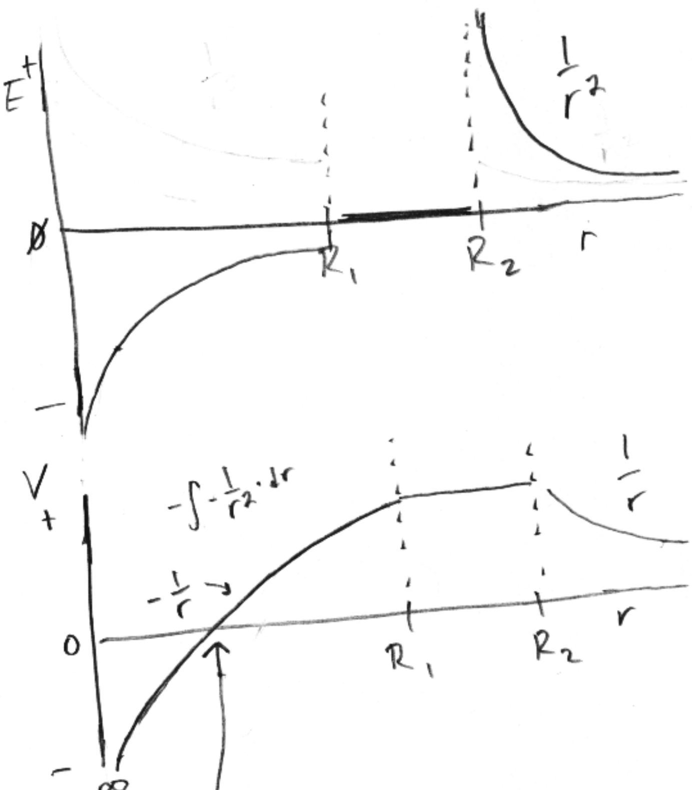
$\Delta V = -\int_{r}^{0} E \cdot dr$

∞

$\Delta V = kQ_1 \left[\frac{1}{r} \right]_r^0$

$\Delta V = kQ_1 \left[\frac{1}{0} - \frac{1}{r} \right] = \infty$

f)



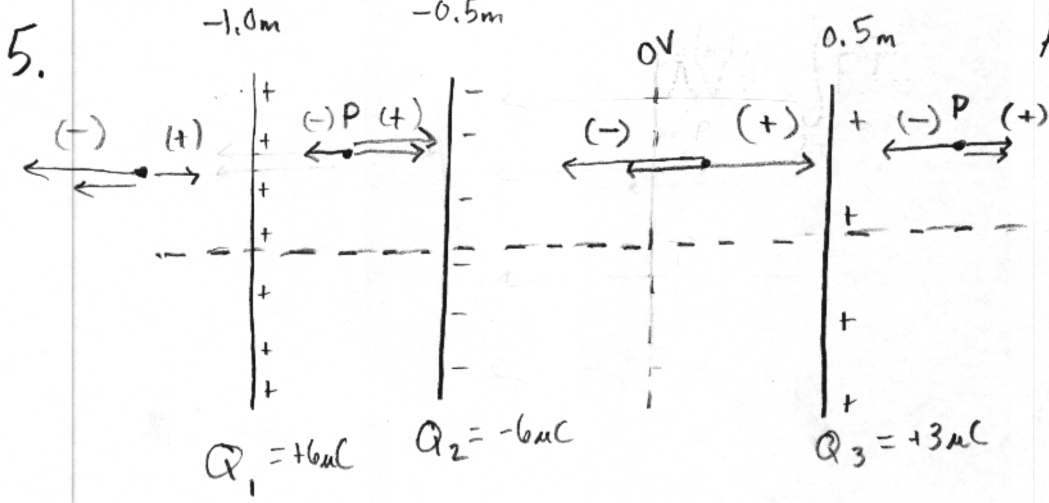
h)

$\Delta V = kQ_1 \left[\frac{1}{r} - \frac{1}{R_1} \right]$

$\cancel{V_r} - V_{R_1} = kQ_1 \left[\frac{1}{r} - \frac{1}{R_1} \right]$

$\frac{-V_{R_1}}{kQ_1} + \frac{1}{R_1} = \frac{1}{r}$ $r = \frac{1}{\frac{-V_{R_1}}{kQ_1} + \frac{1}{R_1}} =$ $\boxed{2.7\,cm}$

5.



$A = 3m^2$

a) $r > 0.5m$

$$E_{tot} = E_1 + E_2 + E_3$$

$$E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \underset{(1.13E^5)}{\frac{Q_1}{2\varepsilon_0 A}} + \underset{(-1.13E^5)}{\frac{Q_2}{2\varepsilon_0 A}} + \underset{(5.65E^4)}{\frac{Q_3}{2\varepsilon_0 A}} = \boxed{5.65\times10^4 N/C}$$

$0.5m > r < -0.5m$

$$(1.13E^5) + (-1.13E^5) - (5.65E^4)$$

$$E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = \boxed{-5.65\times10^4 N/C}$$

For direction

$-1.0m > r < -0.5m$

$$E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = \boxed{1.7\times10^5 N/C}$$

direction

$r < -1.0m$

$$E = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = \boxed{-5.65\times10^4 N/C}$$

direction

b) Q_3

$$\Delta V_{0\to 0.5m} = -\int_0^{0.5m} E \cdot dr \qquad E = -5.65\times10^4 N/C$$

$$\Delta V = -E\int_0^{0.5} dr$$

$$\Delta V = 5.65\times10^4 N/C \cdot [0.5m] = \boxed{2.83\times10^4 V}$$

$$V_3 = E \cdot d$$

Q_2

$$V_2 = E\cdot d = -2.83\times10^4 V \quad \left(\Delta V = -\int_0^{-0.5} E\cdot dr\right)$$

Q_3

$$E = 1.7\times10^5 N/C$$

$$\Delta V = -\int_{-0.5m}^{-1m} E\cdot dr$$

$$\Delta V = -E\left[-1m-(-0.5m)\right]$$

$$\Delta V = 8.5\times10^4 V$$

$$\Delta V = V_1 - V_2 \qquad V_1 = \Delta V + V_2 = \boxed{5.67\times10^4 V}$$

c) $\boxed{8.5\times10^4 V}$

d) $W = \Delta V \cdot d$

$$W = (V_3 - V_1)\cdot(1.5m) = \boxed{4.26\times10^4 J}$$