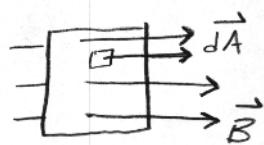


Flux

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

$$\Phi_B = BA$$



Faraday's Law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

← must be changing
for Emt.

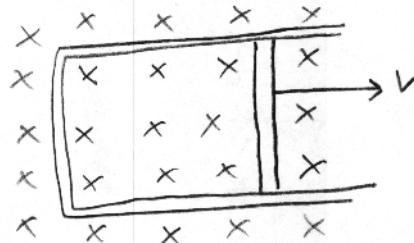
$$\mathcal{E} = -N \frac{d\Phi}{dt}$$

of
turns in
loop

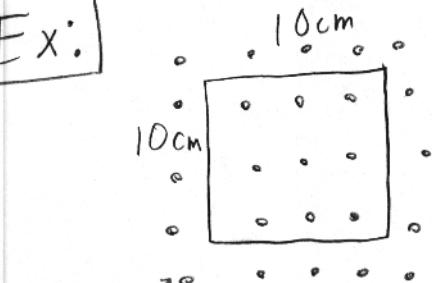
Changing Flux

- $\Phi = B \cdot A$
 - change B
 - change A
 - change θ (changes dot product)

Conservation of B-field



Ex:



$$B = 5t^2$$

Uniform
but changing
strength

uniform
pull it
out

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

$$\Phi = BA$$

$$\Phi = 5t^2(0.01m)$$

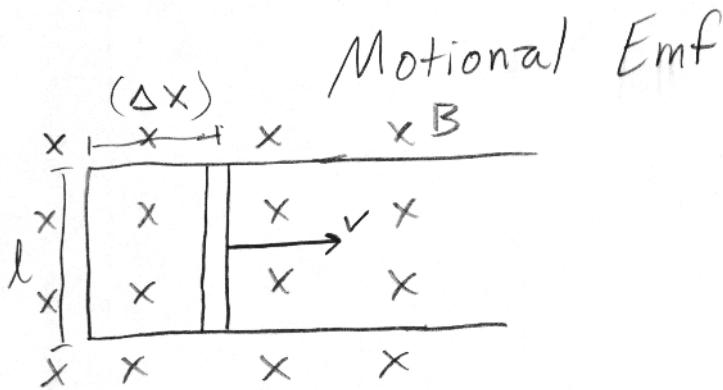
$$\Phi = .05t^2$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -\frac{d}{dt} 0.05t^2$$

$$\mathcal{E} = -0.1t$$

Goes back to
definition of flux
it is not stronger
in one part of
area than
the other.



$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -\frac{Bldx}{dt}$$

$\mathcal{E} = Blv$

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

$$\Phi = Bldx$$

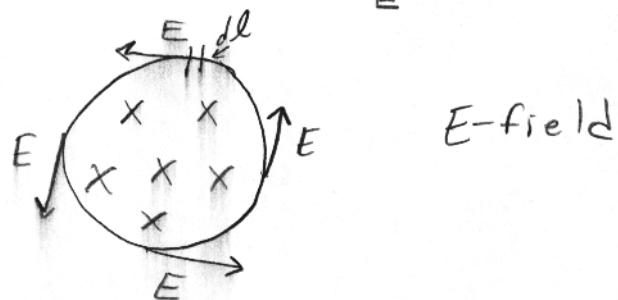
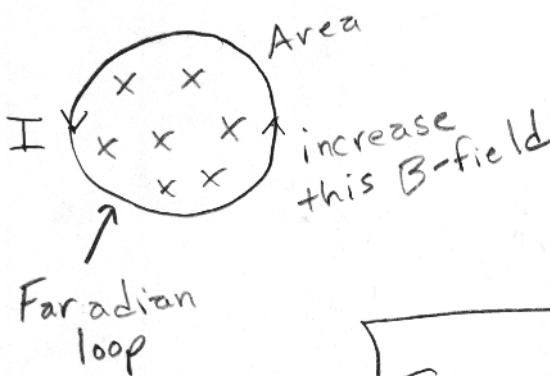
$$\Phi = Bldx$$

To find current

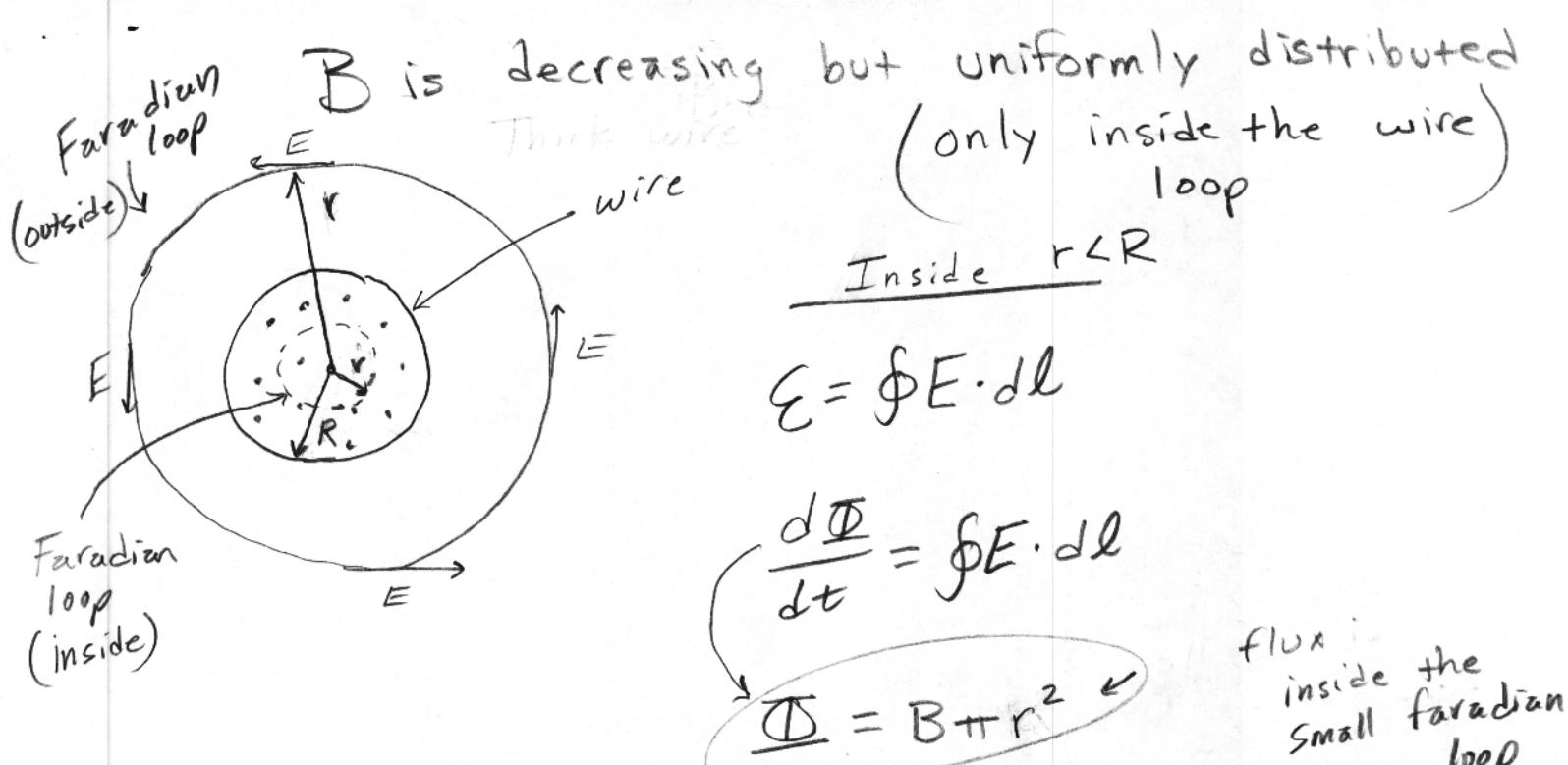
$$I = \frac{\mathcal{E}}{R}$$

Faraday & E-fields

$$\frac{dl}{E}$$



$$\mathcal{E} = -\frac{d\Phi}{dt} = \oint \vec{E} \cdot d\vec{l}$$



$$\frac{d\Phi}{dt} = \oint E \cdot dl$$

$$\Phi = B\pi r^2$$

flux inside the small faradian loop

Outside $r > R$

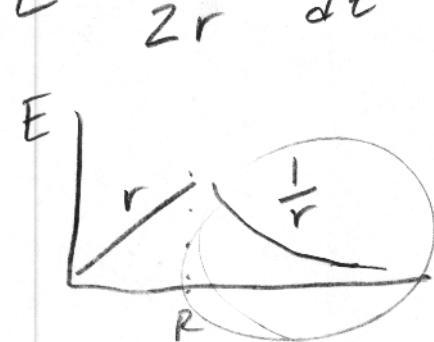
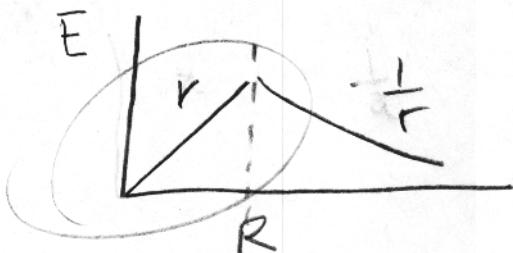
$$\oint E \cdot dl = \frac{d\Phi}{dt}$$

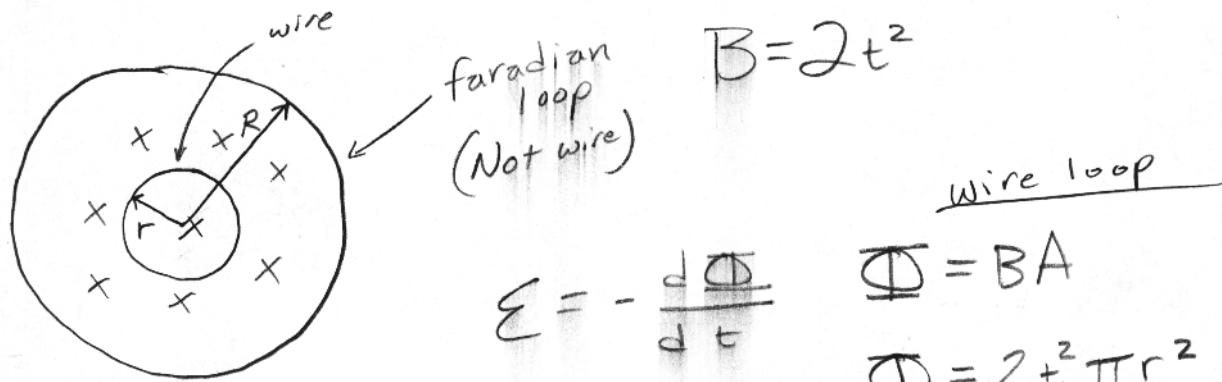
$$\Phi = B\pi R^2$$

$$\frac{dB\pi r^2}{dt} = E \oint dl$$

$$\frac{dB\pi r^2}{dt} = E 2\pi r$$

$$E = \frac{r}{2} \cdot \frac{dB}{dt}$$





$$\mathcal{E} = -\frac{d\Phi}{dt} \quad \Phi = BA$$

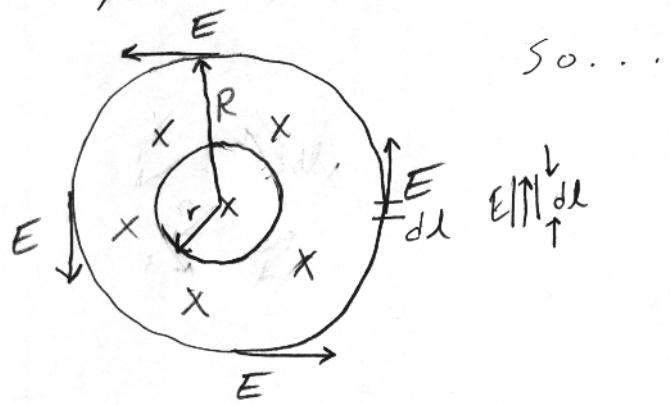
$$\Phi = 2t^2 \pi r^2$$

$$\mathcal{E} = \frac{d}{dt} (2t^2 \pi r^2)$$

$$\mathcal{E} = 2\pi r^2 \frac{dt^2}{dt}$$

$$\mathcal{E} = 4\pi r^2 t$$

Now look at Faradian loop



So...

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l}$$

Faradian loop

$$4\pi r^2 t = \oint \mathbf{E} \cdot d\mathbf{l}$$

$$4\pi r^2 t = E 2\pi R$$

$$E_{(t)} = \frac{2r^2 t}{R}$$

Inductance

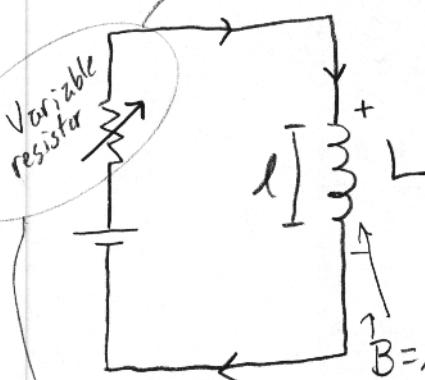
Analogy

$$F = m \frac{dv}{dt}$$

Inertia
resist change
in motion

$$\mathcal{E}_{\text{back}} = -L \frac{dI}{dt}$$

electrical "inertia"
resist change in current



If current is constant the inductor does nothing.
(Not changing resistor)

$$B = \mu_0 n I$$

$$n = \frac{N}{L}$$

→ If resistance is decreased

$$\downarrow R = \frac{V}{I} \uparrow$$

$$B \uparrow$$

$$\uparrow \text{ } \textcircled{D}$$

\mathcal{E} -back emf occurs

I increasing



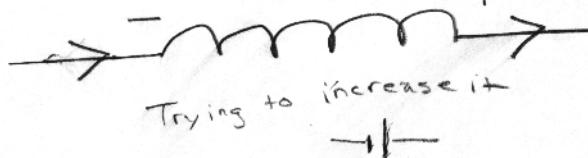
$$\mathcal{E}_{\text{back}} \leftarrow +$$

I = constant

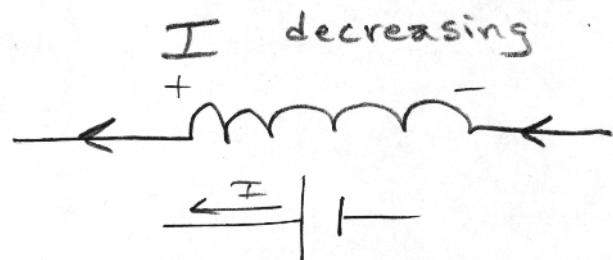
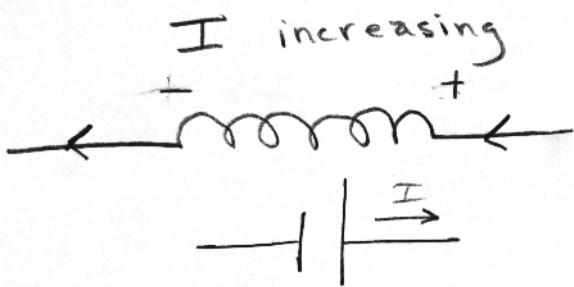


$$\mathcal{E} = \emptyset$$

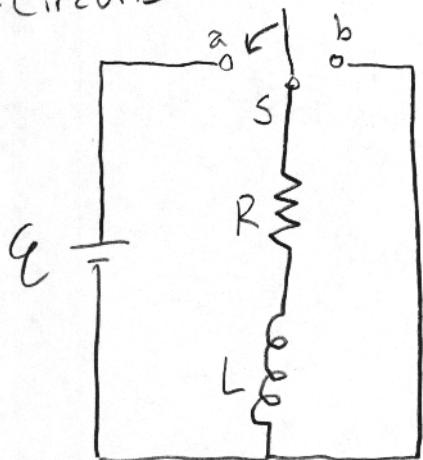
I decreasing



I want " I " to stay constant
- Inductor



RL-Circuits



Use Kirchhoff's loop rule
When switched to (a)
 $t = 0 \quad I = 0$

$$E - IR - L \frac{dI}{dt} = 0$$

divide both sides by R

$$\frac{E - IR}{R} = \frac{L}{R} \frac{dI}{dt}$$

U substitution

$$U = \frac{E}{R} - I$$

$$\frac{dU}{dI} = -1$$

$$dU = -dI$$

or

$$dI = -dU$$

also, find U_0 now

$$U_0 = \frac{E}{R} - I_0 \quad I_0 = 0$$

$$U_0 = \frac{E}{R}$$

$$\frac{E}{R} - I = \frac{L}{R} \frac{dI}{dt}$$

$$U = -\frac{L}{R} \frac{dI}{dt}$$

$$\int_{U_0}^U \frac{dU}{U} = \int_0^t \frac{R}{L} dt$$

$$\ln \left| \frac{U}{U_0} \right| = -\frac{Rt}{L}$$

$$e^{\ln \left| \frac{U}{U_0} \right|} = e^{-\frac{Rt}{L}}$$

$$\frac{U}{U_0} = e^{-\frac{Rt}{L}}$$

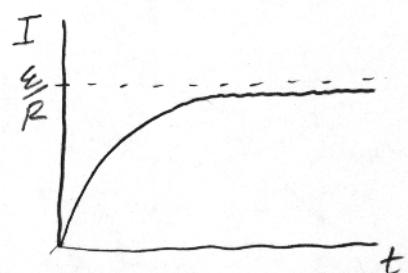
$$\frac{E}{R} - I = \frac{E}{R} \left(e^{-\frac{Rt}{L}} \right)$$

$$I = \frac{E}{R} - \left(\frac{E}{R} e^{-\frac{Rt}{L}} \right)$$

$$I_{(t)} = \frac{E}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$\gamma = \frac{R}{L} = \text{time constant}$$

$$I_{(t)} = \frac{E}{R} \left(1 - e^{-\frac{t}{\gamma}} \right)$$



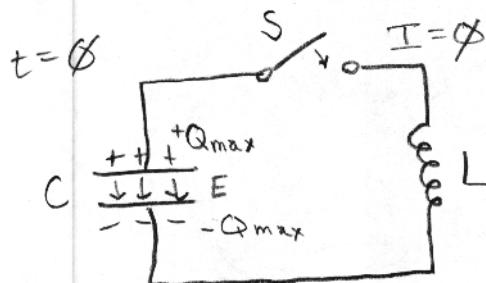
Energy in an Inductor

$$U_L = \frac{1}{2} L I^2$$

- Some energy is dissipated as heat by resistance,
- The rest is stored in the magnetic field of the inductor

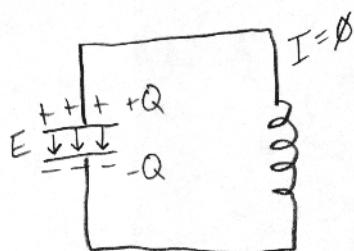
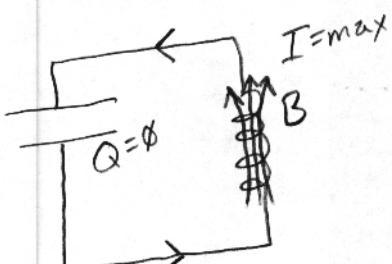
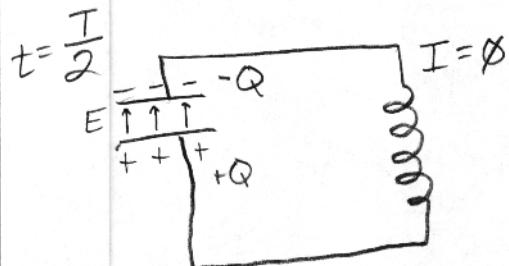
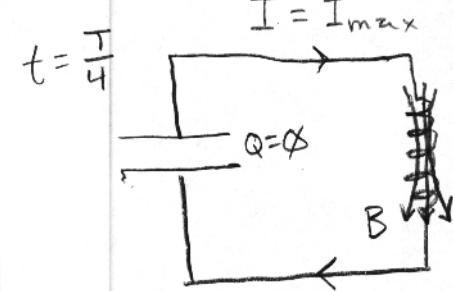
LC-Circuits

Assume no current when (S) open

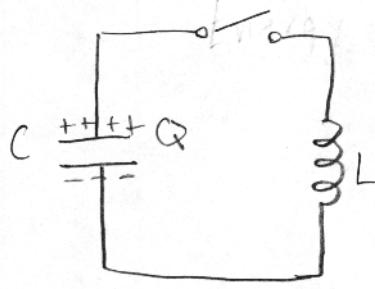


$t = 0$ (S) closed

- 1) The inductor prevents the capacitor from discharging too quickly.
- 2) As current gradually rises the Energy from the capacitor is stored in (L)'s B-field
- 3) The current in the (L) is now at a max and it delivers charge back to the capacitor charging the opposite side.
- 4) The current decreases to zero as the capacitor charges back up.
- 5) Capacitor starts to discharge again, this time in the opposite direction from before.
- 6) This repeats (oscillating). 1 cycle is a return to the capacitor original current direction.



← This was 1 cycle.



Apply Kirchhoff's Rule, well it's
tricky.

Look at it in Energy terms.

$$U_{\text{tot}} = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

Assume No Energy is lost

$$\frac{dU}{dt} = \emptyset$$

so...

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{Q^2}{2C} + \frac{1}{2}LI^2 \right)$$

$$\frac{dU}{dt} = \frac{Q}{C} \cdot \frac{dQ}{dt} + LI \frac{dI}{dt} = \emptyset$$

$$\frac{Q}{C} \cdot \frac{dQ}{dt} + L \frac{dQ}{dt} \cdot \frac{dI}{dt} = \emptyset$$

$$\frac{dQ}{dt} \left(\frac{Q}{C} + L \frac{dI}{dt} \right) = \emptyset$$

$$\frac{Q}{C} = -L \frac{dI}{dt}$$

$$\frac{Q}{C} = -L \frac{d^2Q}{dt^2}$$

$$-\frac{1}{LC}Q = \frac{d^2Q}{dt^2}$$

Second
order
differential
Similar to
oscillations

Analogy Time!

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x = -\omega^2 x$$

k = spring constant
 $\omega = \sqrt{k/m}$

Position $\rightarrow x = A \cos(\omega t + \phi)$

Charge $\rightarrow Q_{(t)} = Q_{\max} \cos(\omega t + \phi)$

Angular frequency

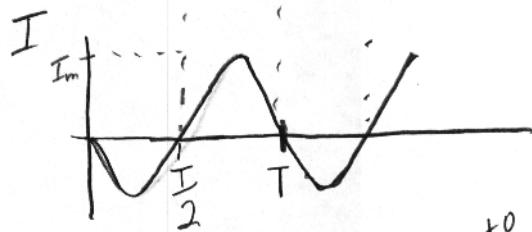
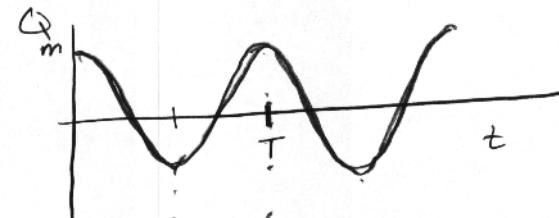
$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

Current $\rightarrow I_{(t)} = \frac{dQ}{dt} = -\omega Q_{\max} \sin(\omega t + \phi)$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$I_{\max} = \omega Q_{\max}$$

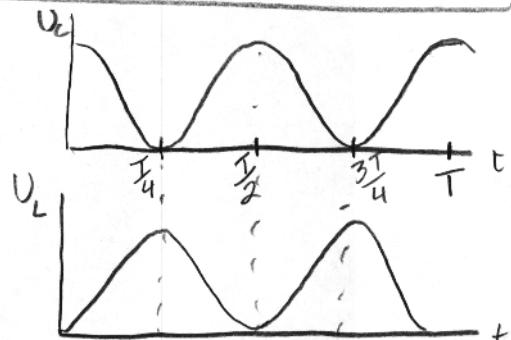


Refer to diagrams!

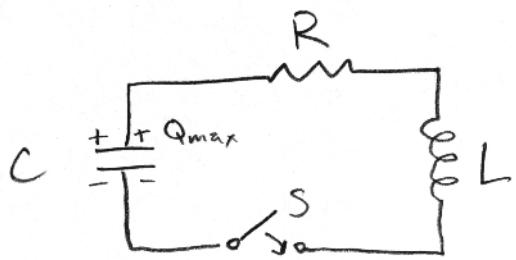
Energy Oscillation LC-circuit

$$U = U_c + U_L = \frac{Q_{\max}^2}{2C} \cos^2 \omega t + \frac{L I_{\max}^2}{2} \sin^2 \omega t$$

Should make sense \rightarrow



RLC - Circuits



Total Energy is no longer constant!
Resistor dissipates energy as heat.

$$P = I^2 R$$

(heat dissipation)

$$\frac{dU}{dt} = -I^2 R$$

↑
 Shows
 U decreasing
 w/ time

Sub this into LC-Circuit

$$LI \frac{dI}{dt} + \frac{Q}{C} \frac{dQ}{dt} = -I^2 R$$

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = 0$$

This is like having a damping effect

