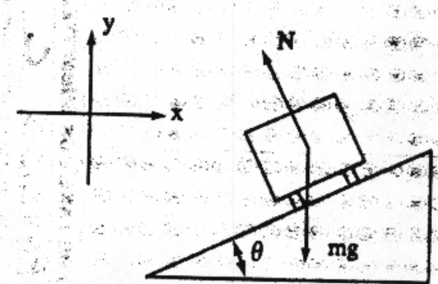
SOLUTIONS AND SCORING GUIDES
SECTION II
1988 AP Physics C Examination: Mechanics

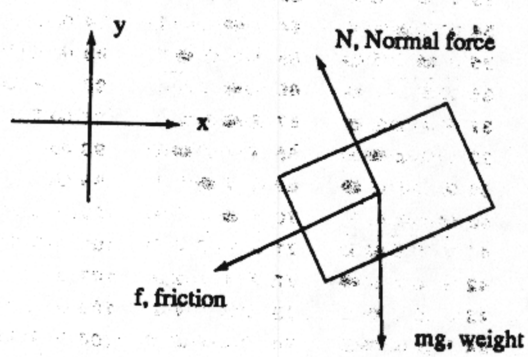
Solutions | **Distribution of points**

Mech 1. (a) 5 points



For Newton's 2nd law: $\sum \vec{F} = m\vec{a}$	1 point
$\sum F_y = N\cos\theta - mg = 0$	1 point
$\sum F_x = N\sin\theta = \frac{mv^2}{r}$	1 point
Eliminate N: $\tan\theta = \frac{v^2}{gr}$	1 point
$v = \sqrt{gr\tan 15°} = 16$ m/s	1 point

(b) 4 points



1 point awarded for each force with correct label	3 points
1 point awarded if no extraneous or incorrect forces are included. A pseudoforce is acceptable if it is correctly explained and used.	1 point

Solutions | **Distribution of points**

(c) 6 points

For proper resolution of Newton's 2nd law:

$$\left.\begin{aligned}\sum F_y &= N\cos\theta - f\sin\theta - mg = 0\\ \sum F_x &= N\sin\theta + f\cos\theta = \frac{mv^2}{r}\end{aligned}\right\}$$ 2 points

For solving for N, f:

$$\left.\begin{aligned}N &= \frac{mv^2}{r}\sin\theta + mg\cos\theta\\ f &= \frac{mv^2}{r}\cos\theta - mg\sin\theta\end{aligned}\right\}$$ 2 points

$$\mu \geqslant \frac{f}{N} = \frac{v^2\cos\theta - rg\sin\theta}{v^2\sin\theta + rg\cos\theta}$$ 1 point

$\mu_{min} = 0.32$ (correct calculation) 1 point

[redacted]

For linear force relation: $F = kx$ 1 point

$20\text{ N} = k_1(0.10\text{ m})$ (Correct use of values from graph) 2 points

$k_1 = 200\text{ N/m}$ 1 point

(b) 2 points

$$\left.\begin{aligned}&|\Delta K| = \Delta U = \frac{1}{2}k_1(x_{AB})^2 = \frac{1}{2}(200\text{ N/m})(0.10\text{ m})^2\\ &\text{OR } |\Delta K| = \text{area under graph} = \frac{1}{2}(20\text{ N})(0.10\text{ m})\\ &\text{OR } |\Delta K| = \int_0^{0.1\text{ m}} k_1 x\,dx = (200\text{ N/m})\frac{(0.10\text{ m})^2}{2}\\ &\text{OR } |\Delta K| = \bar{F}\Delta x = (10\text{ N})(0.1\text{ m})\end{aligned}\right\}$$ 1 point

$|\Delta K| = 1\text{ J}$ 1 point

(c) 3 points

$|\Delta K| = |\Delta U| =$ *area* under force graph from B to C
OR one of the equivalent methods in part (b) above 2 points

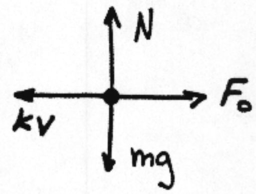
$|\Delta K| = 2\text{ J}$ 1 point

2

1993 **Physics C Solutions** — **Distribution of Points**

Mech 2.

(a) 3 points



1 point for F_0 correctly drawn and labeled	1 point
1 point for kv correctly drawn and labeled	1 point
1 point for N and mg correctly drawn and labeled	1 point

(b) 3 points

$F_{net} = ma$ — 1 point

But $F_{net} = F_0 - kv$, therefore: — 1 point

$F_0 - kv = ma$

Solving for a:

$a = (F_0 - kv)/m$ — 1 point

(c) 5 points

$a = \frac{dv}{dt}$ — 1 point

Using the equation from part b:

(1) $\frac{dv}{dt} = \frac{(F_0 - kv)}{m}$

Re-arranging and integrating:

(2) $\int \frac{dv}{F_0 - kv} = \int \frac{1}{m}\,dt$ — 1 point

Changing variables by letting $u = F_0 - kv$, $du = -k\,dv$:

$-\frac{1}{k} \int \frac{du}{u} = \int \frac{1}{m}\,dt$

$\ln(F_0 - kv) - \ln C = -\frac{k}{m}t$, where C is a constant

$v = \frac{1}{k}\left(F_0 - Ce^{-kt/m}\right)$

(the last three steps together) — 1 point

Mech 2. (continued)
(c) (continued)

To evaluate C, use initial conditions $t = 0$, $v = 0$ — 1 point

$C = F_0$

so $v = \frac{F_0}{k}\left(1 - e^{-kt/m}\right)$ — 1 point

Equation (2) can also be integated using limits 0 and v for the left-hand side and 0 and t for the right-hand side to obtain the same answer for full credit.

(Alternate Method to solve equation (1)) — (Alternate points)

Recognizing that the solution will be in exponential form, try:

$v = Ae^{Bt} + C$, where A, B, and C are constants — (1 point)

Substituting into equation (1)

$ABe^{Bt} = \frac{F_0}{m} - \frac{k}{m}(Ae^{Bt} + C)$

$ABe^{Bt} = \left(\frac{F_0}{m} - \frac{kC}{m}\right) - \frac{kA}{m}e^{Bt}$ — (1 point)

Equating coefficients to evaluate B and C,

$B = -\frac{k}{m}$, $C = \frac{F_0}{k}$

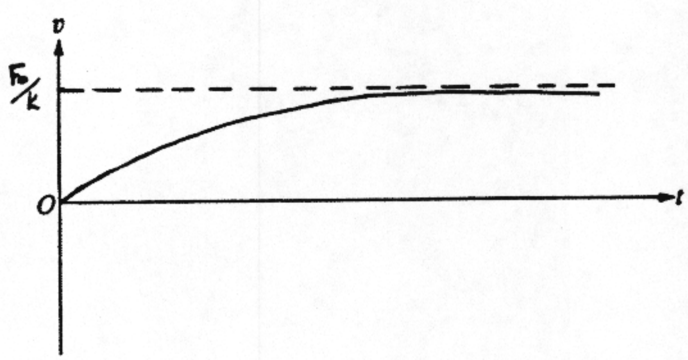
Therefore, $v = Ae^{-kt/m} + \frac{F_0}{k}$

To evaluate A, use initial conditions $t = 0$, $v = 0$ — (1 point)

$A = -\frac{F_0}{k}$

so $v = \frac{F_0}{k}\left(1 - e^{-kt/m}\right)$ — (1 point)

(d) 2 points



For correct maximum value F_0/k — 1 point

For correct shape of curve — 1 point

Mechanics 2 Scoring Guide (15 points)

Solutions | **Distribution of points**

3

(a) 3 points

For indicating, via an equation or a free-body diagram, that the net force is the sum of gravity and the normal force exerted by the platform — 1 point

Using Newton's second law:

$mg - N = ma_v$

Solving for the force exerted by the platform:

$N = m(g - a_v)$

For correctly substituting both accelerations — 1 point

$N = (300\ \text{kg})(10\ \text{m/s}^2 - 1.5\ \text{m/s}^2)$

For the correct answer — 1 point

$N = 2550\ \text{N}$ (or 2490 N if using $9.8\ \text{m/s}^2$)

(b) 2 points

For indicating that the frictional force is the only horizontal force exerted — 1 point

$f = ma_h$

$f = (300\ \text{kg})(2\ \text{m/s}^2)$

For the correct answer — 1 point

$f = 600\ \text{N}$

(c) 3 points

Expressing the frictional force in terms of the normal force:

$f = \mu N$

$\mu = f/N$

For correctly substituting the frictional force from part (b) — 1 point

For correctly substituting the normal force from part (a) — 1 point

$\mu = (600\ \text{N})/(2550\ \text{N})$

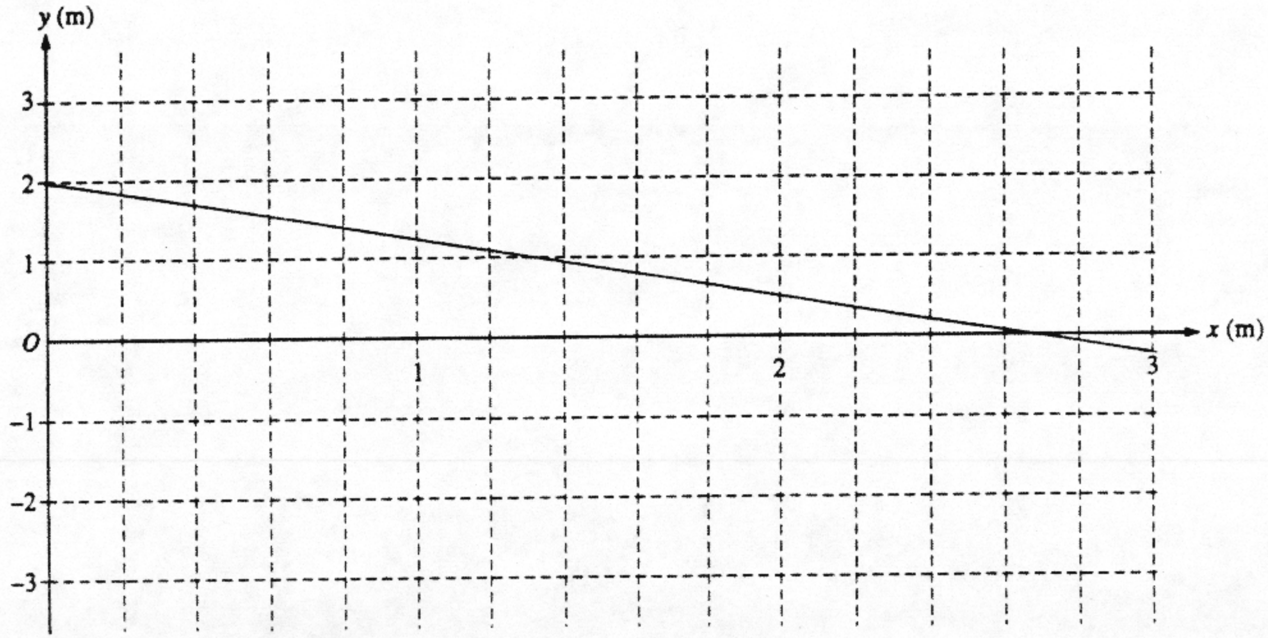
For the correct answer, with no units — 1 point

$\mu = 0.24$

Solutions	Distribution of points
(d) 4 points	
For writing the equation for the vertical motion, and indicating that $a_v = -1.5\ \text{m/s}^2$	1 point
$y = y_0 + \frac{1}{2}(-1.5)t^2$	
For indicating that $y_0 = 2$	1 point
For writing the equation for the horizontal motion, and indicating that $a_h = 2\ \text{m/s}^2$	1 point
$x = \frac{1}{2}(2)t^2$	
The relationship between x and y can be obtained by combining the equations for the horizontal and vertical motions, eliminating t^2.	
For a correct equation relating y and x	1 point
$y = 2 - 0.75x$	

Note: Alternate methods that derive this equation from a relationship between the components of acceleration or velocity could also receive full credit.

(e) 3 points



For a straight line with a negative slope, extending at least to $x = 1$ m	1 point
For a y-intercept at 2 m	1 point
For an x-intercept at $2\frac{2}{3}$ m or a slope of $-\frac{3}{4}$	1 point

Note: Credit was only awarded for showing the items above on the graph. Students were expected to show an understanding of the physics of the situation, and thus were not awarded full credit for graphing an incorrect equation from part (d).

Question 2

4

15 points total | **Distribution of points**

(a) 3 points

For some correct application of conservation of energy — 1 point

$U_i = K_f$

For substituting correct expressions for potential and kinetic energy — 1 point

$mgh_i = \frac{1}{2}mv_f^2$

$gh = \frac{1}{2}v_0^2$

For a correct answer — 1 point

$h = \frac{v_0^2}{2g}$

Alternate solution — *Alternate points*

For using correct kinematics and dynamics equations — *1 point*

$v_f^2 = v_i^2 + 2a\left(s_f - s_i\right)$

$F = ma$

For substituting correct variables — *1 point*

$mg\sin\theta = ma$, $\sin\theta = h/L$ *where L is the length of the ramp, so* $a = gh/L$

$v_0^2 = 0 + 2\left(\frac{gh}{L}\right)L$

For a correct answer — *1 point*

$h = \frac{v_0^2}{2g}$

(b)

i. 2 points

For selecting "Zero" — 1 point

For a correct explanation of why the vertical component of the net force is zero. The explanation must be linked to the fact that there is no acceleration. — 1 point

Example:

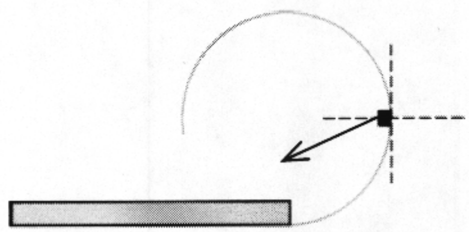
The block does not accelerate vertically, therefore the component of the net force in the vertical direction must be zero.

Note: No credit is earned if an incorrect choice is selected.

Distribution of points

(b) (continued)

ii. 3 points



Top View

For drawing a single arrow pointing into the third quadrant	2 points
For correct justification of both components	1 point

Example:

The horizontal component of the net force must provide a centripetal force and also slow down the block. So it must point inward and against the direction of motion.

Note: If components are drawn, partial credit can be earned for the following.

For drawing an arrow pointing left (toward the center of the circle) with correct justification	*1 point*
For drawing an arrow pointing toward the bottom of the page with correct justification	*1 point*

A point is deducted for each incorrect vector drawn, with a maximum 2 point deduction

(c) 1 point

The normal force exerted by the wall is the centripetal force.

$F_N = F_C$

For a correct answer — 1 point

$F_N = mv^2/R$

Note: Since the statement of part (c) says to "determine" an expression, credit is given for just stating the correct answer.

(d) 3 points

For correctly using the frictional force in an expression of Newton's second law — 1 point

$-F_f = ma_t$

$-\mu F_N = ma_t$

For substituting the normal force from part (c) into a correct expression — 1 point

$-\mu mv^2/R = ma_t$

For an answer consistent with part (c) — 1 point

$a_t = -\mu v^2/R$

Note: Since the question asks for the magnitude of the acceleration, the negative sign is not needed but students are not penalized for including it.

Question 2 (continued)

Distribution of points

(e) 3 points

For substituting dv/dt for a into the answer from part (d), <u>or</u> substituting dv/dt for a and the friction force for F_{net} into Newton's second law — 1 point

$$\frac{dv}{dt} = -\frac{\mu v^2}{R} \quad \text{or} \quad m\frac{dv}{dt} = -F_f$$

For including the negative sign — 1 point

Substituting for F_f produces the same relationship as the first equation above.

For separation of variables and using correct limits — 1 point

$$\frac{1}{v^2}dv = -\frac{\mu}{R}dt$$

$$\int_{v_0}^{v} \frac{1}{v^2}dv = \int_0^t -\frac{\mu}{R}dt$$

Integrate the equation to solve for v.

$$\left[-\frac{1}{v}\right]_{v_0}^{v} = \left[-\frac{\mu t}{R}\right]_0^t$$

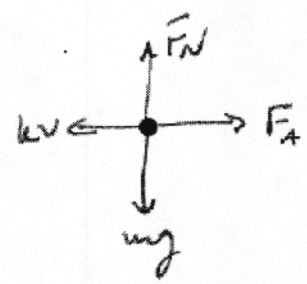
$$\frac{1}{v} - \frac{1}{v_0} = \frac{\mu t}{R}$$

$$v = \frac{Rv_0}{R + \mu v_0 t} \quad \text{or} \quad \frac{v_0}{1 + \mu v_0 t/R}$$

Question 2

15 points total — **Distribution of points**

(a) 4 points



For correctly showing and labeling the applied force directed to the right	1 point
For correctly showing and labeling the downward gravitational force	1 point
For correctly showing and labeling the upward normal force	1 point
For correctly showing and labeling the drag force directed to the left	1 point
One earned point was deducted for having any extraneous vectors	

(b) 2 points

$F_{net} = ma$

For the correct substitution into Newton's second law — 1 point

$F_A - kv = ma$

For a correct differential equation — 1 point

$$F_A - kv = m\frac{dv}{dt}$$

(c) 1 point

Set $\frac{dv}{dt} = 0$ in the equation from part (b)

$F_A - kv = 0$

For the correct expression for the terminal velocity — 1 point

$$v_T = \frac{F_A}{k}$$

Question 2 (continued)

Distribution of points

(d) 5 points

Use the differential equation from part (b)

$$F_A - kv = m\frac{dv}{dt}$$

For demonstrating separation of variables — 1 point

$$\frac{1}{m}dt = \frac{1}{F_A - kv}dv$$

For demonstrating that the equation must be integrated — 1 point

$$\int \frac{1}{m}dt = \int \frac{1}{F_A - kv}dv$$

For demonstrating substitution using initial and final values (or evaluating the constant of integration using the boundary conditions) — 1 point

$$\int_0^t \frac{1}{m}dt = \int_0^{v(t)} \frac{1}{F_A - kv}dv$$

$$\left[\frac{t}{m}\right]_0^t = -\frac{1}{k}\left[\ln(F_A - kv)\right]_0^{v(t)}$$

For attempting to solve for $v(t)$ — 1 point

$$-\frac{kt}{m} = \ln\left(\frac{F_A - kv(t)}{F_A}\right)$$

$$e^{-kt/m} = \frac{F_A - kv(t)}{F_A} = 1 - \frac{kv(t)}{F_A}$$

$$\frac{kv(t)}{F_A} = 1 - e^{-kt/m}$$

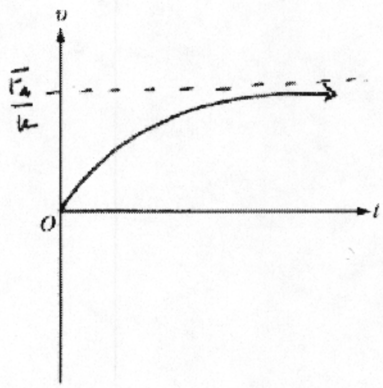
For a correct answer — 1 point

$$v(t) = \frac{F_A}{k}\left(1 - e^{-kt/m}\right)$$

Question 2 (continued)

Distribution of points

(e) 3 points



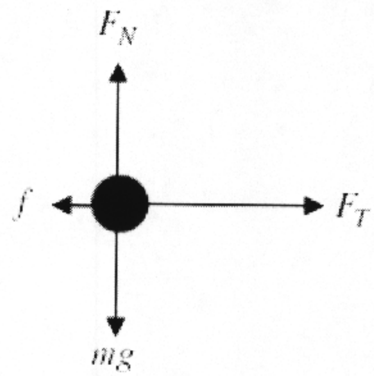
For a graph that begins at the origin, with a non-negative slope everywhere, and is concave downward	1 point
For a graph with a horizontal asymptote	1 point
For the correct label of the expression for the asymptote or maximum on the vertical axis	1 point

6

Question 1

15 points total | **Distribution of points**

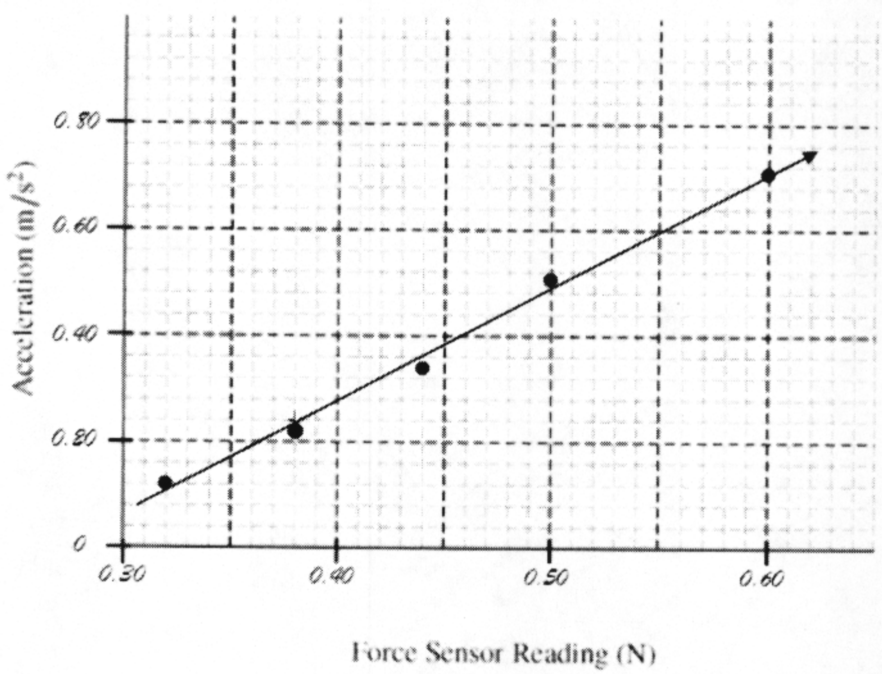
(a) 3 points



For correctly drawing and labeling the force of tension — 1 point
For correctly drawing and labeling the force of friction — 1 point
For correctly drawing and labeling both forces in the vertical direction — 1 point
Note: A maximum of two points may be earned if there are any extraneous vectors.

(b)

i. 3 points



For a correct scale that uses more than half the grid — 1 point
For correctly plotting the given data — 1 point
For drawing a straight line consistent with the given data — 1 point
Note: Full credit can be earned if the axes are switched.

Question 1 (continued)

Distribution of points

(b)

ii. 2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.70 - 0.16)}{(0.60 - 0.35)}\,\text{kg}^{-1} = 2.16\ \text{kg}^{-1}$$

Note: Linear regression gives slope $= 2.12\ \text{kg}^{-1}$.

For correctly calculating the mass of the cart using the slope — 1 point

$$m = \frac{1}{\text{slope}} = \frac{1}{\left(2.16\ \text{kg}^{-1}\right)}$$

Correct answer:

$m = 0.463\ \text{kg}$ (Note: linear regression gives $m = 0.472\ \text{kg}$)

iii. 1 point

For an answer with correct units consistent with the x-intercept of the graph from (b) i. — 1 point

$f = 0.272\ \text{N}$

(c)

i. 1 point

Applying Newton's second law and substituting the values from part (b)

$$\sum F = ma = F_a - f$$

$$a = \frac{F_a - f}{m} = \frac{0.45\ \text{N} - 0.272\ \text{N}}{0.463\ \text{kg}}$$

For an answer with correct units consistent with part (b), either from the graph or calculated using the mass and frictional force. — 1 point

$a = 0.376\ \text{m/s}^2$

Question 1 (continued)

Distribution of points

(c)

ii. 3 points

For using a correct equation to solve for the speed of the cart when the string breaks, with an acceleration consistent with part (c) i. — 1 point

$v_2 = v_1 + at$

$v_2 = 0 + \left(0.376\ \text{m/s}^2\right)(2.0\ \text{s})$

$v_2 = 0.752\ \text{m/s}$

For recognizing that the acceleration of the cart after the string breaks is due to the frictional force determined in part (b) — 1 point

Use a correct equation to solve for the time for the cart to stop after the string breaks

$v_2 = v_1 + at$

$0 = v_1 - \dfrac{f}{m}t$

For using the final velocity before the string breaks as the initial velocity for the cart stopping in the correct equation for time — 1 point

$t = \dfrac{mv_1}{f}$

$t = \dfrac{(0.463\ \text{kg})(0.752\ \text{m/s})}{(0.272\ \text{N})}$

Correct answer
$t = 1.28\ \text{s}$

Alternate solution — *Alternate points*

For setting the magnitude of the impulse before the string breaks equal to the magnitude of the impulse after the string breaks — *1 point*

$F_1 t_1 = F_2 t_2$

For correctly using the proper force (e.g., the tension force minus the friction force) for F_1 — *1 point*

For correctly using the proper force (e.g., the friction force) for F_2 — *1 point*

Correct answer
$t = 1.28\ \text{s}$

(d)

i. 1 point

For selecting "Equal to" — 1 point

ii. 1 point

For selecting "Greater than" — 1 point