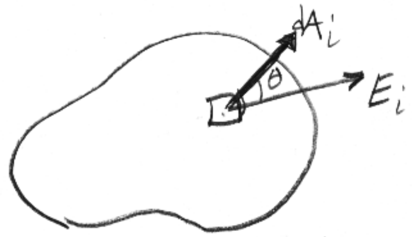
Consider an area A divided up into small elements each with area dA.



$$d\Phi_i = E_i\, dA_i$$

electric field makes an angle with the surface area

* This is the scalar product of 2 vectors $(A \cdot B = AB\cos\theta)$

As the number of elements (dA) approaches infinity the sum becomes an integral

dot product so $\cos\theta$

$$\Phi \equiv \lim_{dA_i \to 0} \sum E_i\, dA = \int E \cdot dA$$

This is usually the flux through a closed surface.

- lines leaving the surface will be (+) flux
- lines entering the surface will be (−) flux

due to $\cos\theta$

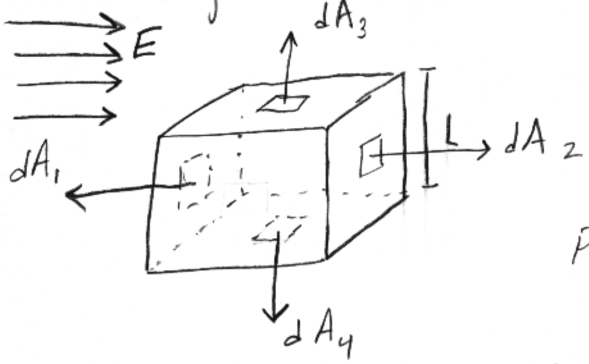
(Net flux) • Total flux will be the # of lines leaving the volume surrounding the surface minus the # of lines entering.

$$\boxed{\Phi_c = \oint E \cdot dA}$$

closed surface

$\oint$ surface integral
- definate integral
- this is denoting a closed surface for us!

Ex: Flux through a Cube



Perpendicular to A_4 and A_3 so $E\,dA\cos(90°) = \emptyset$

$A = L^2$

Parallel to A_1 and A_2

$$\Phi_c = \int_1 E \cdot dA_1 + \int_2 E \cdot dA_2$$

$$\Phi_1 = \int_1 E \cdot dA_1 \quad \theta = 180°$$

$$\Phi_1 = -E \int_1 dA_1$$

$$\Phi_1 = -EL^2$$

$$\Phi_2 = \int_2 E \cdot dA_2 \quad \theta = 0°$$

$$\Phi_2 = EL^2$$

$$\boxed{\Phi_c = \emptyset}$$

If the same # of lines enter as exit.

Gauss' Law

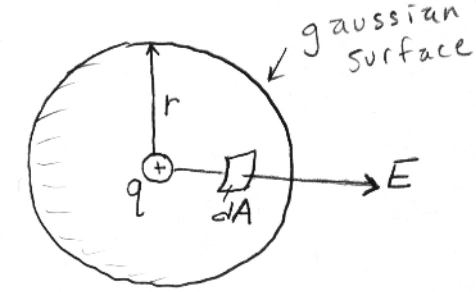
A. Remember, flux is the # of field lines leaving the volume surrounding the surface.

We will describe the general relation between net flux through a closed surface (gaussian surface) and the charge enclosed by the surface.

The simplest example is the sphere

A point charge inside a sphere

Surface Area $= 4\pi r^2$



The field lines project radially outward so that at each segment dA $\theta = 0^\circ$ (normal) to surface

$E = \dfrac{kq}{r^2}$ at the surface of the sphere

$$\Phi_c = \oint E \cdot dA$$

E is constant over the surface

$\oint dA = A$

$$\Phi_c = E \oint dA$$

$$\Phi_c = EA = \frac{kq}{r^2} 4\pi r^2 = 4\pi kq$$

Net flux through the surface.

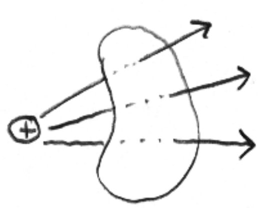
$k = \dfrac{1}{4\pi\varepsilon_0}$

$$\Phi_c = \frac{4\pi q}{4\pi\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$\boxed{\Phi_c = \frac{q}{\varepsilon_0}}$$

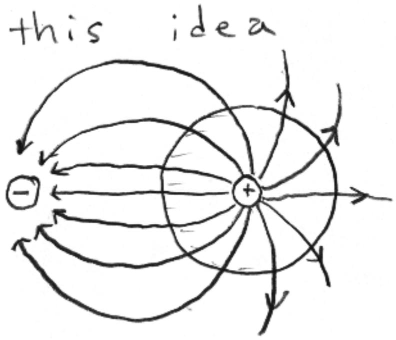
flux is independent of r

Now consider a charge outside a closed surface



$\Phi_c = \emptyset$ when it contains no charge

Or even this idea



So, this is really the Law

$$\Phi_c = \oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

Q_{in} = charge inside the Surface

$$\boxed{\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}}$$

- Gauss' Law states that the net flux through any closed gaussian surface is equal to the net charge inside the surface $/\varepsilon_0$

☆ E represents the total electric field which ☆ includes effects from outside charges as well

3 shapes we must know how to use

1. Sphere
2. Cylinder
3. plane

☆ These shapes are determined by the charge distribution

Example of point charge. (a)

Calculate the electric field due to a isolated point charge

- We should choose a gaussian sphere due to the charge distribution

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

E constant

$\int dA = A$

$SA = 4\pi r^2$

$$E \oint dA = \frac{q}{\varepsilon_0}$$

$E = k\frac{q}{r^2}$

$$E\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} = k\frac{q}{r^2}$$

If we place a particle q_0 in this E, we have $F_E = k\frac{q q_0}{r^2}$

Coulomb's Law

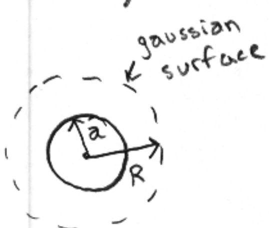
Example 2: A sphere of charge

An insulated sphere of radius (a) has a uniform charge density ρ and total charge Q

a) Calculate the E-field intensity at a point outside the charged sphere $r > a$

- We should again choose a gaussian sphere to evaluate

Also, $E = k\frac{Q}{r^2}$ when $r > a$ so this means that the the field is equivalent to a point charge.



b) Find the E-field at a point inside the sphere $r < a$

We draw the gaussian sphere inside the charged sphere to find the E-field

$q_{in} < Q$ We see that the enclosed charge will be less than the total.

$V = \frac{4}{3}\pi r^3$

Charge density

$\rho = \frac{Q}{V}$ so for inside $\rho = \frac{q_{in}}{V_{in}}$ $\quad q_{in} = \rho V_{in}$

$q_{in} = \rho\frac{4}{3}\pi r^3$ so...

$$\oint E\,dA = \frac{q_{in}}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{q_{in}}{\varepsilon_0}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$E = \frac{q_{in}}{4\pi r^2\varepsilon_0} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi r^2\varepsilon_0} = \frac{\rho}{3\varepsilon_0}r$$

$$E = \frac{\frac{Q}{\frac{4}{3}\pi a^3}}{3\varepsilon_0} r = \frac{Q}{4\pi\varepsilon_0 a^3} r = \boxed{\frac{kQ}{a^3} \cdot r}$$

→ wait, this looks familiar

$$F_g = \frac{GMm}{R^3} \cdot r$$

add this in

inside a solid sphere

some charge

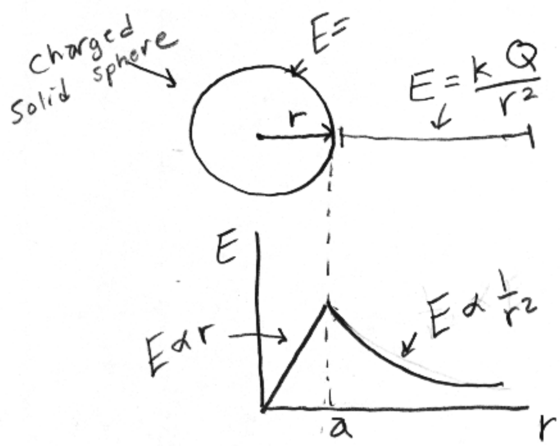
$$F_E = \frac{kQq}{a^3} \cdot r$$

$F_E \propto r$ ← distance inside charged sphere

—or—

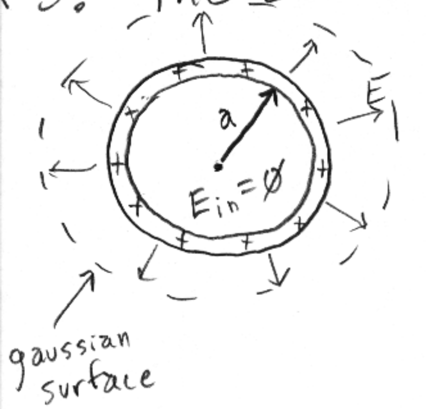
$$E = \frac{kQ}{a^3} \cdot r \qquad \text{if } r = \emptyset \quad E = \emptyset$$

$E \propto r$



analogous to Force of gravity inside a solid sphere

EX 3: The E field of a thin spherical shell radius (a)



a) E field when $r > a$ (outside sphere)

- This acts the same as a solid charge sphere

$$E = k\frac{Q}{r^2}$$

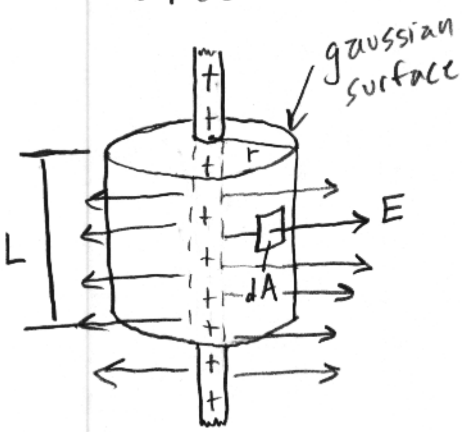
b) E field when $r < a$

$E = \emptyset$ due to symmetry

EX 4: A Cylindrically symmetric Charge Distribution

- rods
- wires

*We use a gaussian cylinder

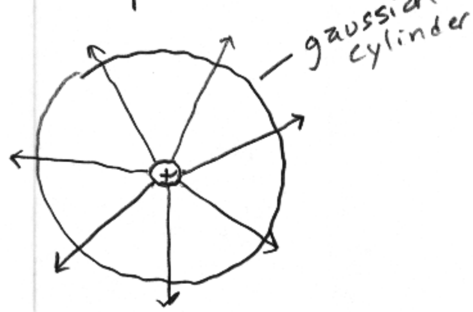


a) Find the E-field at r from the uniform line of charge

$$\lambda = \text{constant}$$

E-lines are perpendicular to the side of the cylinder and parallel to the ends.

Top View



We see that flux is zero at the ends.