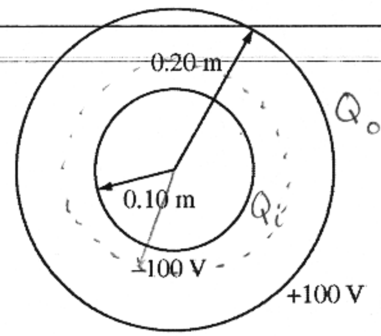
Name: KEY

Electrostatics (Gauss) Recitations Part 1




2012 E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of −100 V , and the outer shell is set at an electric potential of +100 V, with each potential defined relative to the conventional reference point. Let $Q_i$ and $Q_o$ represent the net charge on the inner and outer shells, respectively, and let $r$ be the radial distance from the center of the shells. Express all algebraic answers in terms of $Q_i$ , $Q_o$ , $r$, and fundamental constants, as appropriate.

(a) Using Gauss's Law, derive an algebraic expression for the electric field $E(r)$ for 0.10 m < $r$ < 0.20 m .

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0} \qquad E = \frac{Q_i}{4\pi r^2 \varepsilon_0} = \boxed{\frac{kQ_i}{r^2}}$$

$$E\, 4\pi r^2 = \frac{Q_i}{\varepsilon_0}$$

(b) Determine an algebraic expression for the electric field $E(r)$ for $r$ > 0.20 m .

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0} \qquad \boxed{E = \frac{Q_i + Q_o}{4\pi\varepsilon_0 r^2}}$$

$$E = \frac{Q_{in}}{4\pi r^2 \varepsilon_0}$$

(c) Determine an algebraic expression for the electric potential $V(r)$ for $r$ > 0.20 m .

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i} \qquad \boxed{V = \frac{k(Q_i + Q_o)}{r}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_i}{r} + \frac{Q_o}{r}\right)$$

(d) Using the numerical information given, calculate the value of the total charge $Q_T$ on the two spherical shells

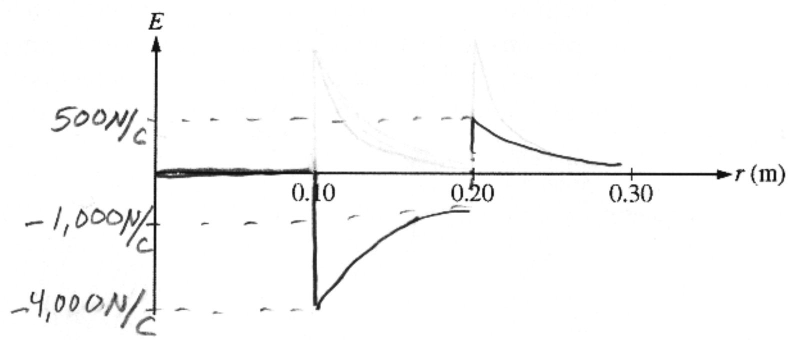
( ) $Q_T = Q_i + Q_o$ .

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_T}{r}$$

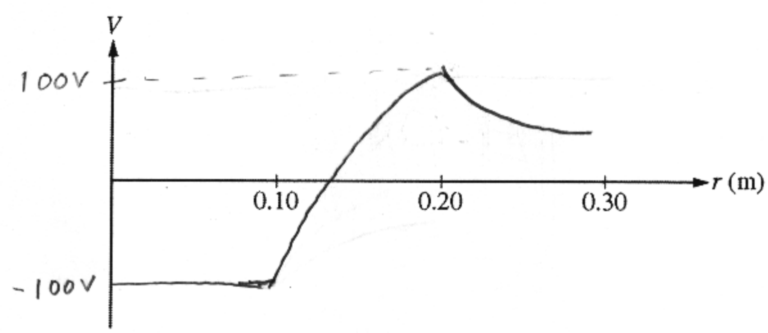
$$Q_T = V 4\pi\varepsilon_0 r$$

$$Q_T = (100\text{V})\, 4\pi\varepsilon_0 (0.20\text{m}) = \boxed{2.2 \times 10^{-9}\text{C}}$$

(e) On the axes below, sketch the electric field $E$ as a function of $r$. Let the positive direction be radially outward.




(f) On the axes below, sketch the electric potential $V$ as a function of $r$.



E&M. 1.

A nonconducting, thin, spherical shell has a uniform surface charge density $s$ on its outside surface and no charge anywhere else inside.

(a) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Describe the Gaussian surface that you use.

Gaussian Sphere

$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

$E(\cancel{4\pi r^2}) = \frac{s\,\cancel{4\pi r^2}}{\varepsilon_0}$

$E = \frac{s}{\varepsilon_0} = \emptyset$

$s = \frac{Q}{A}$ $\quad Q_{in} = sA$

$Q_{inside} = \emptyset$

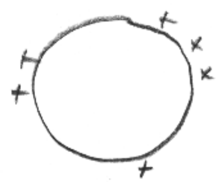
$s = \emptyset$

(b) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell?

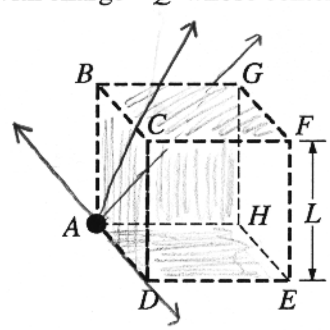
____Yes __✓__No _______It cannot be determined from the information given.

Justify your answer.

The symmetry inside is now changed




Now consider a small conducting sphere with charge $+Q$ whose center is at corner $A$ of a cubical surface, as shown below.




(c) For which faces of the surface, if any, is the electric flux through that face equal to zero?

__✓__ABCD _____CDEF _____EFGH __✓__ABGH _____BCFG __✓__ADEH

Explain your reasoning.

The E-field lines are parallel with these surfaces.

(d) At which corner(s) of the surface does the electric field have the least magnitude?

A is inside of the conductor's surface

(e) Determine the electric field strength at the position(s) you have indicated in part (d) in terms of $Q$, $L$, and fundamental constants, as appropriate.

$E = \emptyset$

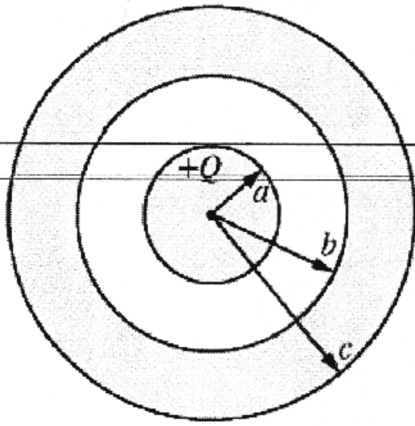
(f) Given that one-eighth of the sphere at point $A$ is inside the surface, calculate the electric flux through face $CDEF$.

$Q_{in} = \frac{1}{8}Q$

$\Phi_{net} = \frac{Q}{8\varepsilon_0}$

$\Phi_{CDEF} = \frac{1}{3} \cdot \frac{Q}{8\varepsilon_0}$

$$\boxed{\Phi_{CDEF} = \frac{Q}{24\varepsilon_0}}$$




E&M. 1.
A metal sphere of radius $a$ contains a charge $+Q$ and is surrounded by an uncharged, concentric, metallic shell of inner radius $b$ and outer radius $c$, as shown above. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the induced charge on each of the following and explain your reasoning in each case.

i. The inner surface of the metallic shell

$\boxed{-Q}$, the inner sphere is $+Q$ and will induce a $-Q$ charge on the inner surface of the shell.

ii. The outer surface of the metallic shell

$\boxed{+Q}$, the shell is uncharged, therefore its $Q = \emptyset$. So if the inner surface is $-Q$ the outer should cancel it out.

(b) Determine expressions for the magnitude of the electric field $E$ as a function of $r$, the distance from the center of the inner sphere, in each of the following regions.

i. $r < a$

$E = \emptyset$

ii. $a < r < b$

$E = \frac{kQ}{r^2}$ (point charge)

iii. $b < r < c$

$E = \emptyset$

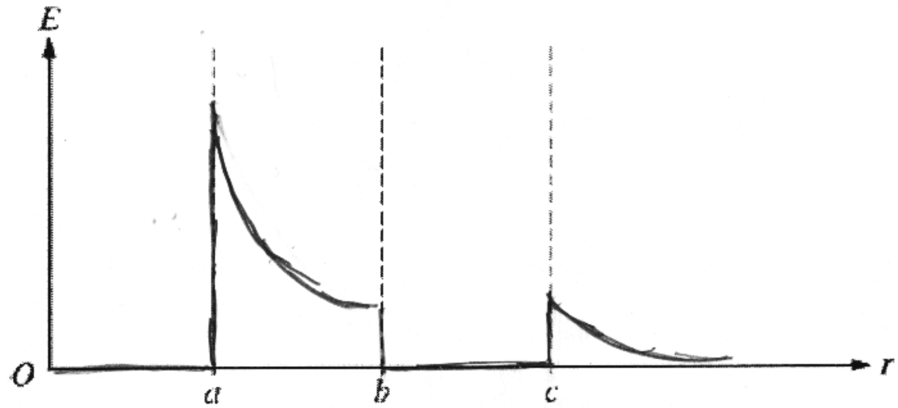
$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

iv. $c < r$

$$E(4\pi r^2) = \frac{Q_{in}}{\varepsilon_0}$$

$$E = \frac{kQ}{r^2}$$

(c) On the axes below, sketch a graph of $E$ as a function of $r$.




(d) An electron of mass $e$ $m$ carrying a charge $-e$ is released from rest at a very large distance from the spheres. Derive an expression for the speed of the particle at a distance $10r$ from the center of the spheres.

$10r$ is indefinite

$10r = 10c$

From $r=\infty$ to $r=10c$

COE

$$\cancel{K_0} + \cancel{U_0} = K + U$$

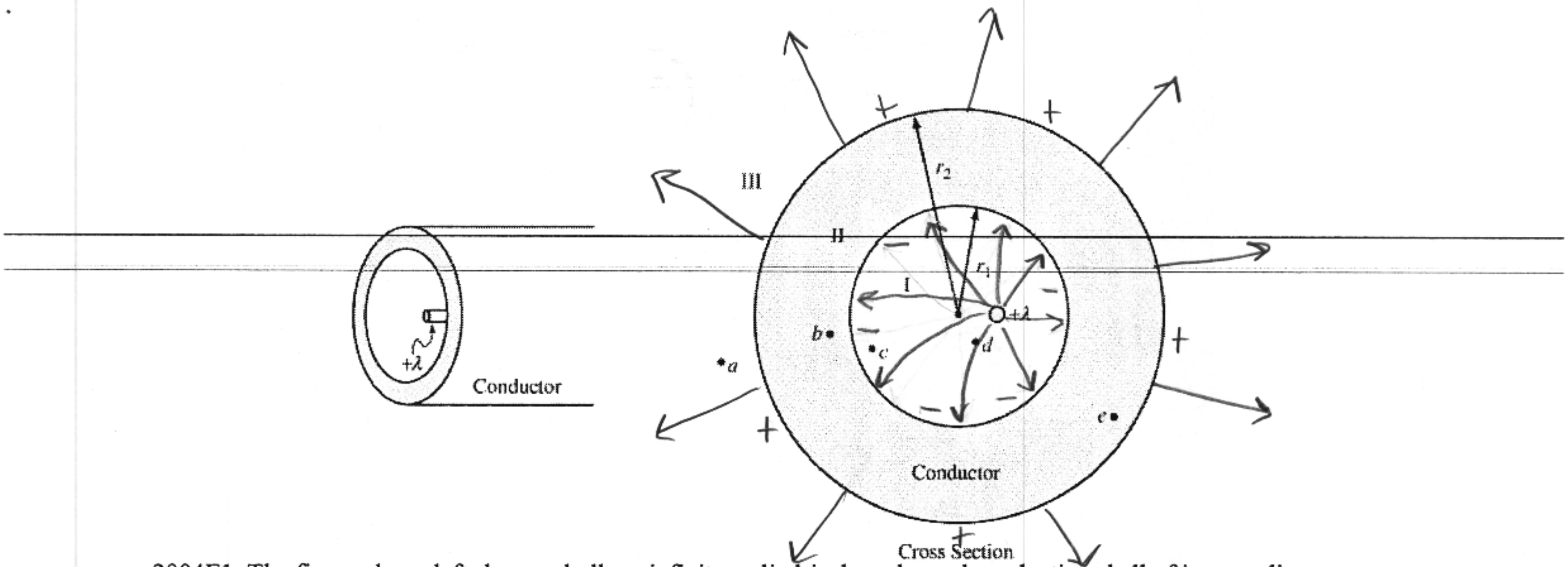
$r=\infty$

$$K + U = \emptyset$$

$$\frac{1}{2} m_e v_f^2 = \frac{kQe}{10c}$$

$$v^2 = \frac{2kQe}{(10c)m_e}$$

$$\boxed{v = \sqrt{\frac{2kQe}{(10c)m_e}}}$$




2004E1. The figure above left shows a hollow, infinite, cylindrical, uncharged conducting shell of inner radius $r_1$ and outer radius $r_2$. An infinite line charge of linear charge density $+\lambda$ is parallel to its axis but off center. An enlarged cross section of the cylindrical shell is shown above right.

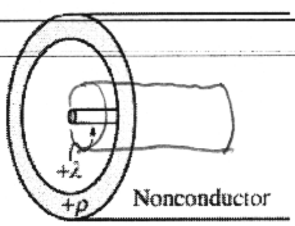
a. On the cross section above right,

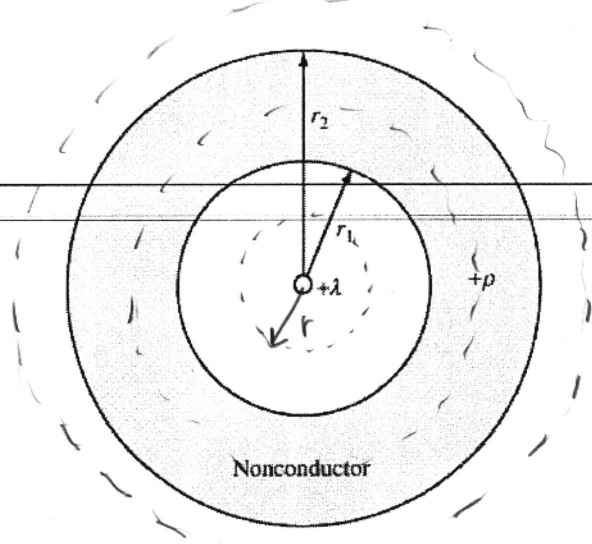
i. sketch the electric field lines, if any, in each of regions I, II, and III and

ii. use + and - signs to indicate any charge induced on the conductor.

b. In the spaces below, rank the electric potentials at points $a$, $b$, $c$, $d$, and $e$ from highest to lowest (1 = highest potential). If two points are at the same potential, give them the same number.

4 $V_a$ 3 $V_b$ 2 $V_c$ 1 $V_d$ 3 $V_e$




Cross Section

$\lambda = \frac{Q}{L}$ $\quad A = 2\pi r L$

c. The shell is replaced by another cylindrical shell that has the same dimensions but is nonconducting and carries a uniform volume charge density $+\rho$. The infinite line charge, still of charge density $+\lambda$, is located at the center of the shell as shown above. Using Gauss's law, calculate the magnitude of the electric field as a function of the distance $r$ from the center of the shell for each of the following regions. Express your answers in terms of the given quantities and fundamental constants.

gaussian cylinder

i. $r < r_1$

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

$$E \cdot (2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

ii. $r_1 \le r \le r_2$

$$Q_{in} = +Q_{wire} + Q_{shell}$$

$$Q_{shell} = \rho(\pi r^2 L - \pi r_1^2 L)$$

$$E_{tot} = E_{wire} + E_{shell}$$

$$E_{shell} = \frac{\rho}{2\varepsilon_0 r}(r^2 - r_1^2)$$

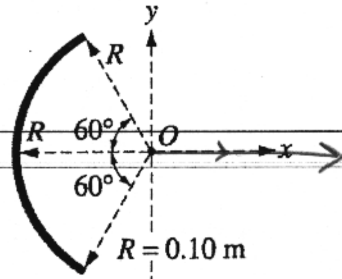
$$\boxed{E_{tot} = \frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\rho}{2\varepsilon_0 r}(r^2 - r_1^2)}$$

iii. $r > r_2$

$$E_{tot} = E_{wire} + E_{shell}$$

$$\boxed{E_{tot} = \frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\rho}{2\varepsilon_0 r}(r_2^2 - r_1^2)}$$

$120^\circ$ of $360^\circ$
$\frac{1}{3}$ of a circle




2002E1. A rod of uniform linear charge density $\lambda = +1.5 \times 10^{-5}$ C/m is bent into an arc of radius R = 0.10 m. The arc is placed with its center at the origin of the axes shown above.

a. Determine the total charge on the rod.

$$Q = \lambda L = \lambda \tfrac{1}{3}(2\pi r) = \frac{2\pi r \lambda}{3} = \boxed{3.1 \times 10^{-6}\,C}$$

b. Determine the magnitude and direction of the electric field at the center O of the arc.

$dq = \lambda dL$ $\quad E = \frac{kq}{r^2}$ $\quad$ y-components cancel $\quad dL = r\,d\theta$ $\quad dq = \lambda r\,d\theta$

x-combine $E = 2E\cos\theta$ $\quad E = k\int \frac{dq}{r^2}$ $\quad E = k\int \frac{\lambda r\,d\theta}{r^2}\cos\theta$

$$E = \frac{\lambda k}{r}\int_{120^\circ}^{240^\circ} \cos\theta\,d\theta = \frac{\lambda k}{r}\Big[\sin\theta\Big]_{120^\circ}^{240^\circ}$$

$$E = \frac{\lambda k}{r}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right] = \frac{-\sqrt{3}\,k\lambda}{r}$$

$$\boxed{|E| = 2.3 \times 10^{6}\,N/C}$$

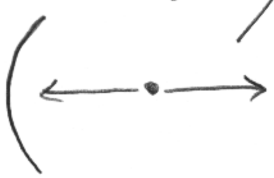
c. Determine the electric potential at point O.

$$V = \frac{kq}{r} = \boxed{2.8 \times 10^{5}\,V}$$

A proton is now placed at point O and held in place. Ignore the effects of gravity in the rest of this problem.

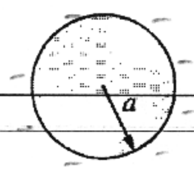
d. Determine the magnitude and direction of the force that must be applied in order to keep the proton at rest.

$$F = Eq = \boxed{-3.7 \times 10^{-13}\,N}$$



e. The proton is now released. Describe in words its motion for a long time after its release.

The proton moves in the +x direction with decreasing acceleration.

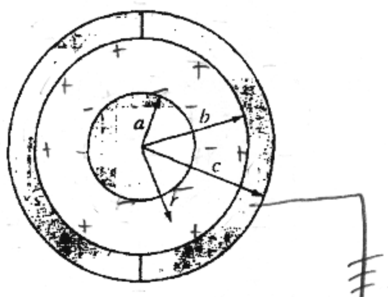



1999E1. An isolated conducting sphere of radius a = 0.20 m is at a potential of -2,000 V.

a. Determine the charge $Q_0$ on the sphere.

$V = \frac{kQ}{r}$ $\quad Q = \frac{Vr}{k} = \boxed{-4.4 \times 10^{-8} C}$

important




The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius b = 0.40 m and outer radius c = 0.50 m, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

b. Determine the magnitude of the electric field in the following regions as a function of the distance r from the center of the inner sphere.

i. r <a

$E = \emptyset$

ii. a < r < b

$E = \frac{kQ}{r^2}$

iii. b < r < c

$E = \emptyset$

iv. r > c

$E = \emptyset$ No charge outside

c. Determine the magnitude of the potential difference between the sphere and the conducting shell.

$\Delta V = -\int_a^b E \cdot dr = -kQ_0 \int_a^b \frac{1}{r^2} dr = -kQ_0 \left[-\frac{1}{r}\right]_a^b = kQ_0 \left(\frac{1}{b} - \frac{1}{a}\right) = \boxed{1000V}$

d. Determine the capacitance of the spherical capacitor.

$V = \frac{Q}{C}$ $\quad C = \frac{Q_0}{V} = \boxed{4.4 \times 10^{-11} F}$


Insulator

$\rho = \text{constant}$ $\qquad \rho = \frac{Q}{V}$

1987E1. A total charge Q is distributed uniformly throughout a spherical volume of radius R. Let r denote the distance of a point from the center of the sphere of charge. Use Gauss's law to derive an expression for the magnitude of the electric field at a point

a. ~~outside the sphere, r > R;~~

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} = \boxed{\frac{kQ}{r^2}}$$

b. inside the sphere, r < R.

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0} \qquad E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{\frac{4}{3}\pi R^3 3\varepsilon_0}$$

$$E(4\pi r^2) = \frac{\rho V_{in}}{\varepsilon_0}$$

$$E\cancel{4\pi r^2} = \frac{\rho \frac{4}{3}\pi r^{\cancel{3}}}{\varepsilon_0} \qquad E = \frac{Q}{4\pi\varepsilon_0 R^3} r = \boxed{\frac{kQ}{R^3} r}$$

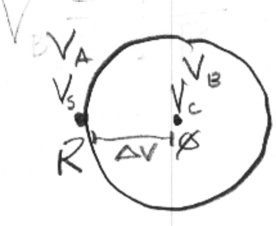
The electrostatic potential is assumed to be zero at an infinite distance from the sphere.

c. What is the potential at the surface of the sphere?

$$\Delta V = V_B - V_A = -\int_{\infty}^{R} E \cdot dr$$

$$V_B = -kQ \int_{\infty}^{R} \frac{1}{r^2} \, dr = kQ\left[\frac{1}{r}\right]_{\infty}^{R} = \boxed{\frac{kQ}{R}}$$

d. Determine the potential at the center of the sphere.




$$\Delta V = V_B - V_A$$

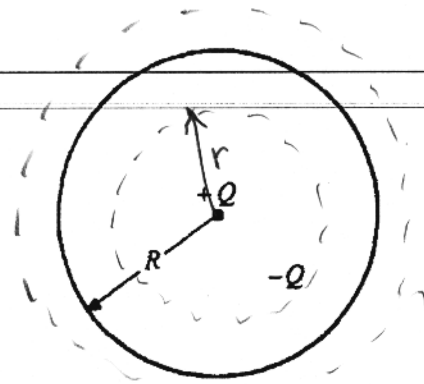
$$V_{center} = \Delta V + V_A$$

$$\Delta V = -\int_R^0 E \cdot dr$$

$$\Delta V = -\frac{kQ}{R^3} \int_R^0 r \, dr = -\frac{kQ}{R^3}\left[\frac{r^2}{2}\right]_R^0 = -\frac{kQ}{R^{\cancel{3}}}\left(\emptyset - \frac{R^{\cancel{2}}}{2}\right)$$

$$\Delta V = \frac{kQ}{2R} \qquad V_{center} = \frac{kQ}{2R} + \frac{kQ}{R} = \boxed{\frac{3kQ}{2R}}$$

$V = \frac{4}{3}\pi r^3$




$\rho = \frac{Q}{V}$

$Q_{in} = \rho V_{in} + Q$

1989E1. A negative charge - Q is uniformly distributed throughout the spherical volume of radius R shown above. A positive point charge + Q is at the center of the sphere. Determine each of the following in terms of the quantities given and fundamental constants.

$Q_{in} = \left(\frac{-Q \frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right) + Q$

$Q_{in} = \left(-Q \frac{r^3}{R^3}\right) + Q$

$Q_{in} = -Q\left(\frac{r^3}{R^3} - 1\right)$

a. The electric field E outside the sphere at a distance r > R from the center

$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

$Q_{in} = -Q + Q = \emptyset$

$\boxed{E = \emptyset}$

b. The electric potential V outside the sphere at a distance r > R from the center

$\Delta V = -\int E \cdot dr$ $\quad E = \emptyset \quad Q = \emptyset$

$\Delta V = \text{constant}$

$V = \frac{kQ}{r} = \boxed{\emptyset}$

c. The electric field inside the sphere at a distance r < R from the center

$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

$E(4\pi r^2) = \frac{Q_{in}}{\varepsilon_0}$

$E = \frac{-Q\left(\frac{r^3}{R^3} - 1\right)}{4\pi r^2 \varepsilon_0}$

$E = \frac{kQ\left(1 - \frac{r^3}{R^3}\right)}{r^2}$

$\boxed{E = kQ\left(\frac{1}{r^2} - \frac{r}{R^3}\right)}$

d. The electric potential inside the sphere at a distance r < R from the center

$\Delta V = V_B - V_A$ $\quad \Delta V = V_r - V_{center}$ $\quad V_{center} = \frac{k(+Q)}{R}$

$V_r = \Delta V + V_{center}$

$\Delta V = -\int_{\infty}^{R} E \cdot dr = \emptyset$ $\quad \Delta V = -\int_{R}^{r} E \cdot dr = -kQ\left[\int_R^r \frac{1}{r^2} \cdot dr - \int_R^r \frac{r}{R^3} \cdot dr\right]$

$\Delta V = -kQ\left[-\frac{1}{r} - \frac{r^2}{2R^3}\right]_R^r = kQ\left(\left(\frac{1}{r} - \frac{1}{R}\right) + \left(\frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right)\right) = kQ\left[\frac{1}{r} + \frac{r^2}{2R^3} - \frac{3}{2R}\right]$