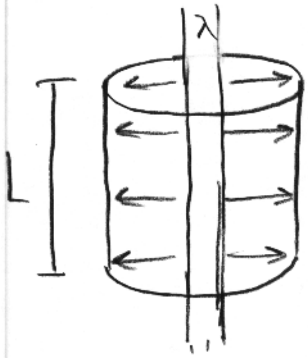


$$\Phi_c = \oint E \cdot dA$$

$$= E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

$A = 2\pi r L$

$\oint dA = A$

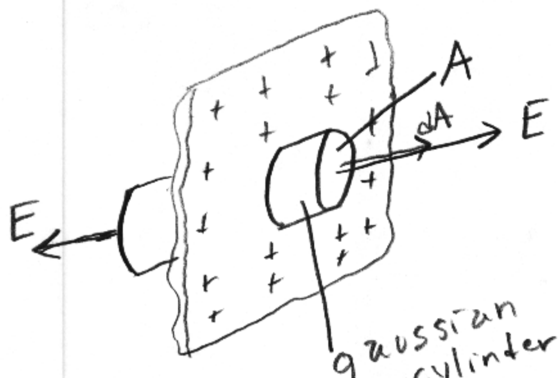
$$E = \frac{\lambda}{2\pi r \varepsilon_0} = 2k\frac{\lambda}{r}$$

$k = \frac{1}{4\pi\varepsilon_0}$

$E \propto \frac{1}{r}$ for a cylindrical charge distribution

EX5: A Nonconducting Plane Sheet of Charge

uniform charge per unit area σ



No flux through sides of the cylinder.

Flux out of both ends

$\Phi = 2EA$

$A = \pi r^2$

$dA = A$

$\sigma = \frac{Q}{A}$

$Q = \sigma A$

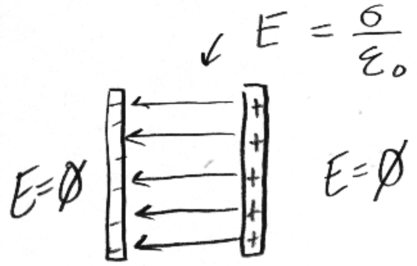
$$\oint 2E \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$2E(\pi r^2) = \frac{\sigma(\pi r^2)}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

- This shows that the field is uniform everywhere (No relation to "r")

$E \not\propto r$



2 plates.

Conductors in Equilibrium

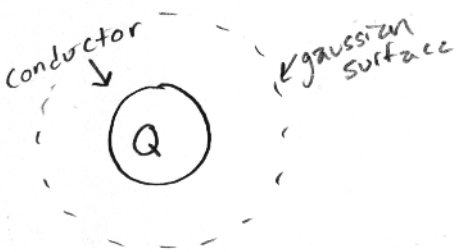
Follow these Rules

1. E-field is zero everywhere inside the conductor.

2. Charge will reside on the surface of the conductor

3. Electric field just outside the conductor is normal to the surface and is equal to $\frac{\sigma}{\varepsilon_0}$. σ = charge per unit area

4. Charge tends to accumulate at sharp points



E_n = field just outside the conductor

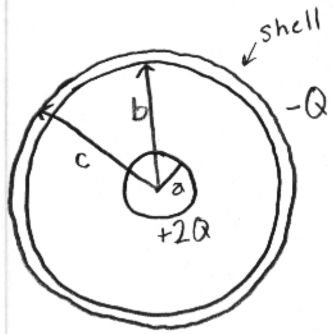
$$\Phi_c = \oint E \cdot dA = E_n A = \frac{q_{in}}{\varepsilon_0}$$

$$\sigma = \frac{Q}{A} \quad Q = A\sigma$$

$$E_n \cancel{A} = \frac{\cancel{A}\sigma}{\varepsilon_0}$$

$$\boxed{E_n = \frac{\sigma}{\varepsilon_0}}$$

Example 6: Sphere inside a Spherical Shell



inner sphere has a charge (+2Q) and radius (a)

outer shell has a charge (-Q) and inner radius (b) outer radius (c)

We want to find the E-field at 4 locations.

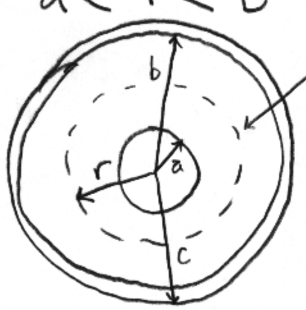
1. $r < a$

$\boxed{E = \emptyset}$ There is no charge inside a conductor in electrostatic equilibrium

Not true of solid insulative spheres!

+2Q is all on its surface.

2. $a < r < b$



$q_{in} = +2Q$ (This region is all due to the inner sphere)

$$\Phi = EA = E\,4\pi r^2 = \frac{q_{in}}{\varepsilon_0} = \frac{+2Q}{\varepsilon_0}$$

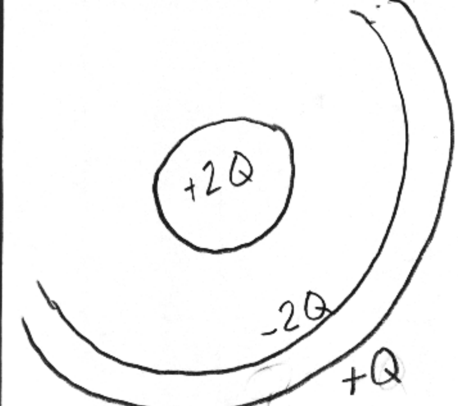
$$E = \frac{2Q}{4\pi r^2 \varepsilon_0} = \boxed{\frac{2kQ}{r^2}}$$

3. $b < r < c$

This is inside the outer conductive shell

$\boxed{E = 0}$ This is still a conductor in electrostatic equilibrium

This means the charge on the inner surface of the shell must cancel the sphere's charge (+2Q)



This makes the outer surface of the shell need to be (+Q) to achieve an overall charge of −Q

4. $r > c$ The overall charge of the entire system inside

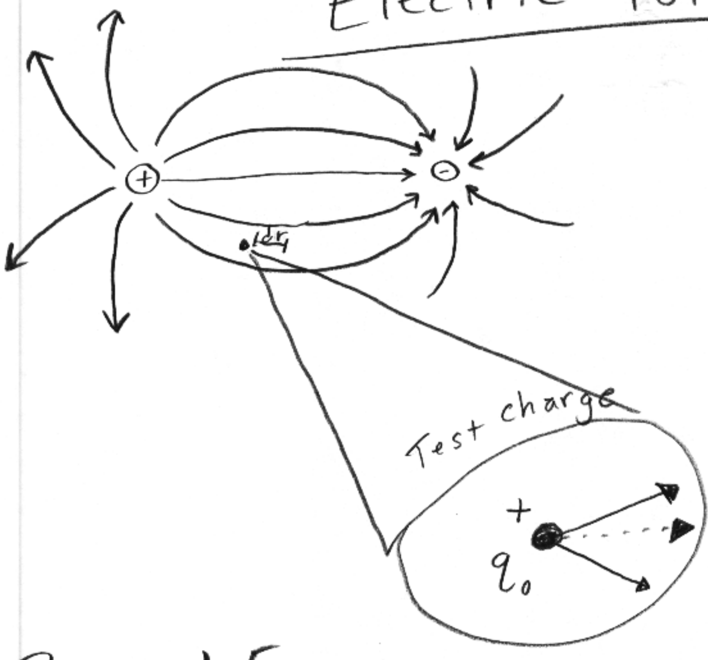
$$+2Q + (-Q) = Q$$

$$\boxed{E = \frac{kQ}{r^2}}$$ ← Treat like a point mass

★ Typical Electric Fields ★

Charge Distribution	E-field	Location
Insulating shere of radius (R) and uniform charge density	$E = \frac{kQ}{r^2}$	⟶ $r > R$
	$E = \frac{kQ}{R^3} r$	⟶ $r < R$
Thin spherical shell of radius (R)	$E = \frac{kQ}{r^2}$	⟶ $r > R$
	$E = \emptyset$	⟶ $r < R$
Line of charge of infinite length and charge density of λ	$E = \frac{2k\lambda}{r}$	⟶ anywhere outside the line of charge
Noncondvcting, charged plane of charge density σ	$E = \frac{\sigma}{2\varepsilon_0}$	⟶ Everywhere outside the plane
Conductor of charge density σ	$E = \frac{\sigma}{\varepsilon_0}$	⟶ Just outside the conductor
	$E = \emptyset$	⟶ Inside the conductor

Electric Potential



$F_E = E q_0$ $\qquad$ $W = F \cdot d$

Potential Energy

$$dW = F \cdot dr = E q_0 \cdot dr$$

For conservative Forces

$$dU = -E q_0 \cdot dr$$

$$W = -\Delta U$$

$(U_B - U_A)$ $\quad$ $\boxed{\Delta U = -q_0 \int_A^B E \cdot dr}$ change in potential energy

- Since the force is conservative the integral is path independent (line integral)

Potential Difference (Not the same as Potential Energy)

$$V_B - V_A = \frac{U_B - U_A}{q_0} = -\int_A^B E \cdot dr$$

$V = \frac{U_E}{q}$

We will come back to this

- The potential difference is the work per unit charge an external agent must perform to move a test charge from A to B without changing kinetic energy.

- We usually choose potential to be zero for a point infinitely far away from the charge producing the E-field.

So... $V_A = \emptyset$ then from ∞ to point P

$$V_p = -\int_{\infty}^{p} E \cdot dr$$

SI unit

$1V = 1\,J/C$ (Volt) this means 1 J is needed to take 1C through a potential difference of 1 V

E-field can also be $1\,N/C = 1\,V/m$

A common unit of energy in nuclear Physics is the electron volt (eV)

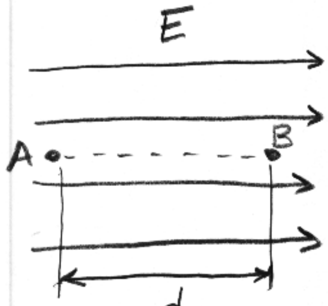
- The energy that 1 electron (or proton) gains when moved through 1V of potential difference.

$$\boxed{1\,eV = 1.6 \times 10^{-19}\,J}$$

☆ A cathode ray get electrons to $5 \times 10^{7}\,m/s$. ☆
$1.1 \times 10^{-15}\,J$ of kinetic energy or $7.1 \times 10^{3}\,eV$

This device needs $\gtrsim$ 7,000 Volts to accelerate the electron

Potential Differences in Uniform E-fields



(must be parallel to the field)

constant

$$\Delta V = -\int_A^B E\,dr = -E\int_A^B dr = -Ed$$

The (−) shows B is lower potential than A

$$\boxed{\Delta V = -Ed}$$

Now suppose a test charge q_0 moves from A to B

$$\Delta U = q_0 \Delta V = -q_0 E d$$

look what we find

$E = \frac{kq}{r^2}$

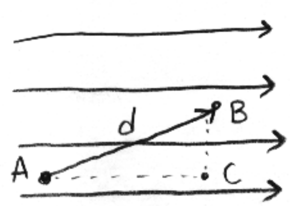
$k = \frac{1}{4\pi\varepsilon_0}$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$$

$$\boxed{U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

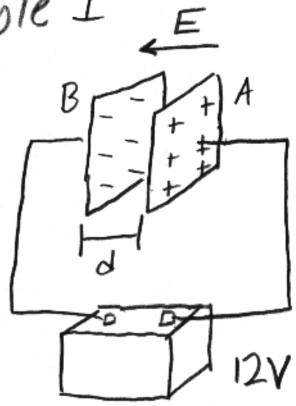
A positive charge will lose electric potential energy as it moves

A negative charge will gain electric potential energy



B and C have the same potential

Example 1



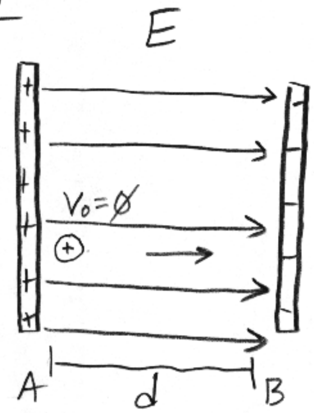
What is the Electric field between the plates

$$\Delta V = E \cdot d$$

Potential difference between the battery terminals should equal the potential difference between the plates

$$E = \frac{V_B - V_A}{d} = \frac{12V}{0.3\times10^{-2}m} = \boxed{4\times10^{3} V/m}$$

EX 2



Find ΔV

$$\Delta V = -Ed = \boxed{-4\times10^{4} V}$$

moving to lower potential

Find ΔU of the proton

$$\Delta U = q_0 \Delta V = e\,\Delta V = \boxed{-6.4\times10^{-15} J}$$

Show potential Energy decreases

Find the speed of the proton

$$\Delta K + \Delta U = \emptyset$$

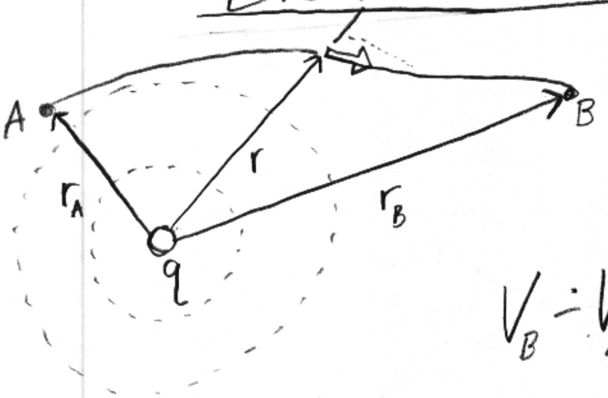
$$(K_f - \emptyset) + (\emptyset - U_i) = \emptyset$$

$$\frac{1}{2} m_p v^2 - 6.4\times10^{-15} J = \emptyset$$

$$v = \sqrt{\frac{2(6.4\times10^{-15} J)}{(1.67\times10^{-27} kg)}}$$

$$\boxed{v = 2.77\times10^{6} m/s}$$

Electrical Potential due to point Charges



$$V_B - V_A = -\int_A^B E \cdot dr \qquad E = \frac{kq}{r^2}$$

$$V_B - V_A = -\int E\,dr = -kq\int_{r_A}^{r_B} \frac{dr}{r^2} = \left.\frac{kq}{r}\right]_{r_A}^{r_B}$$

$$V_B - V_A = kq\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad V_A = \varnothing \quad \text{when} \quad r_A = \infty$$

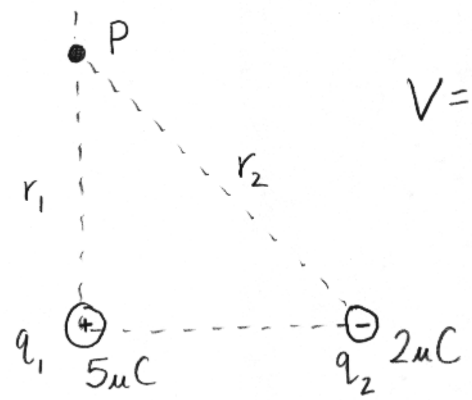
$$V_B - \varnothing = kq\left[\frac{1}{r_B} - \varnothing\right]$$

$$\boxed{V = \frac{kq}{r}}$$ Potential of a point charge

$$\boxed{V = k\sum_i \frac{q_i}{r_i}}$$ Potential of several point charges

$$\boxed{U = q_2 V_1 = \frac{k\,q_1 q_2}{r}}$$ electric potential energy

Ex:



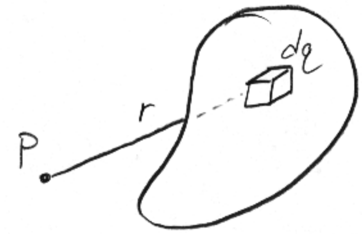
$$V = k\sum_i \frac{q_i}{r_i} \qquad V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V = 9\times10^9\,N\cdot\frac{m^2}{C^2}\left(\frac{5\times10^{-6}C}{4m} + \frac{2\times10^{-6}C}{5m}\right)$$

$$\boxed{V = 7.65\times10^3\,V}$$

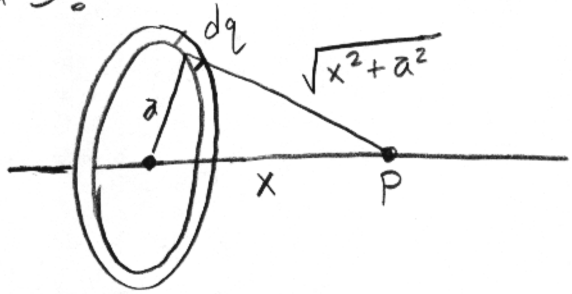
Electric Potential Due to Continuous Charge Distributions

$$dV = k\frac{dq}{r}$$



$$\boxed{V = k\int \frac{dq}{r}}$$

Ex 3:



Total charge Q

$$V = k\int \frac{dq}{r} = k\int \frac{dq}{\sqrt{x^2+a^2}}$$

$$V = \frac{k}{\sqrt{x^2+a^2}}\int dq = \boxed{\frac{kQ}{\sqrt{x^2+a^2}}}$$

Ex 4: Potential of a