The Law of Universal Gravitation

- Newton's Law of Gravity

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$

$$\boxed{G = 6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}}$$

★experimentally found value

- Inverse-square Law

$$\frac{1}{r^2}$$

$$F_{21} = -G\frac{m_1 m_2}{(r_{12})^2}\hat{r}_{12} \quad \text{vector form}$$

indicates m_2 is attracted to m_1

directed from m_1 to m_2

$$F_{21} = -F_{12}$$

- Object on Earth's surface

$$F_g = G\frac{M_e m}{(R_e)^2}$$

- G was found by Sir Henry Cavendish (1798)

- Weight

$$F_g = mg \qquad \cancel{m}g = G\frac{M_e \cancel{m}}{(R_e)^2}$$

$$\boxed{g = G\frac{M_e}{(R_e)^2}}$$

- Density of Earth $\quad R_e = 6.38\times10^{6}\,m^3$

$M_e = 5.98\times10^{24}\,kg$

$$\rho_e = \frac{M_e}{V_e} = \frac{M_e}{\frac{4}{3}\pi R_e^{2}} = 5.50\times10^{3}\,kg/m^3$$

- Gravitational Force and Altitude

$r = R_e + h \qquad h$ = height above Earth's surface

Kepler's 3 Laws

100 years before Newton!

1. All planet move in elliptical orbits w/ sun at the focal point.

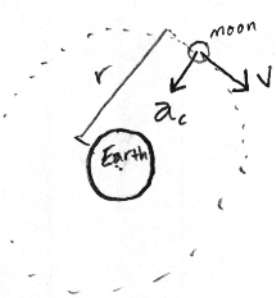
2. The radius from the sun to any planet sweeps out equal areas in equal time intervals.

$$\boxed{\frac{dA}{dt} = \text{constant}}$$

3. $\boxed{T^2 \propto a^3}$

Gravitation and the motion of Planets/Moons

- Centripetal Acceleration



$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \boxed{\frac{4\pi^2 r}{T^2}}$$

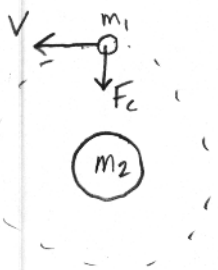
$$v = \frac{2\pi r}{T}$$

• Kepler's 3rd Law

- Can be predicted with the inverse-square law for a circular orbit.

$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_c = \frac{m_1 v^2}{r}$$

★ Gravitational force is equal to centripetal force



$$G\frac{\cancel{m_1} m_2}{r^2} = \frac{\cancel{m_1} v^2}{r} \qquad v = \frac{2\pi r}{T}$$

$$G\frac{m_2}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_2}\right) r^3 \qquad T^2 \propto a^3$$

★ when the orbit is circular $a = r$ at all points

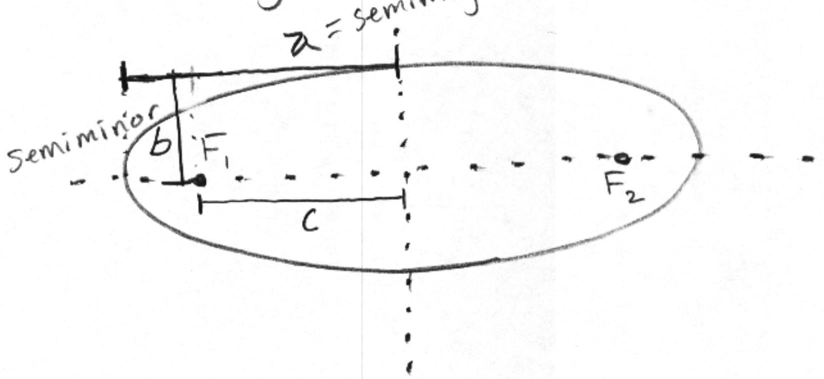
$$T^2 = Kr^3$$

K = constant when m_2 is the sun

$$K_s = \frac{4\pi^2}{GM_s} = 2.97 \times 10^{-19}\ s^2/m^3$$

Also works for elipses

- replace (r) by the semimajor axis (a)



• eccentricity

$$e = \frac{c}{a}$$

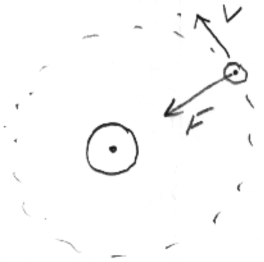
$e = 0$ (circular)

$0 < e < 1$ (elipse)

$e = 1$ (parabolic)

$e > 1$ (hyperbolic)

- Kepler's 2nd Law and Conservation of Angular Momentum.



$\Sigma \tau = \emptyset$

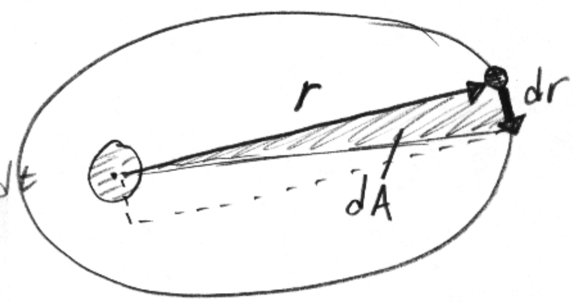
Force is parallel to axis

$\sin(0^\circ) = \emptyset$

$\tau = \frac{dL}{dt} \leftrightarrow F = \frac{mv}{t}$

$L = r \times p = m r \times v$ =constant

$\frac{dL}{dt} = 0$ so... L is constant

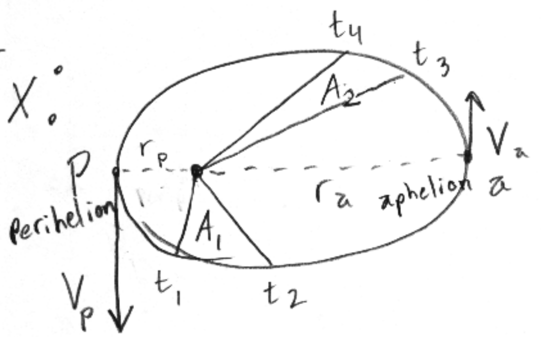


$dA = \frac{1}{2}|r \times dr| = \frac{1}{2}|r \times v\,dt| = \frac{L}{2m}dt$

$$\boxed{\frac{dA}{dt} = \frac{L}{2m} = \text{constant}}$$

L, m are constants

Ex:



Conservation of Angular Momentum

$L = mvr$

Assume: r_p, r_a, V_p are known

$m v_a r_a = m v_p r_p$

$$\boxed{V_a = \frac{r_p}{r_a} V_p}$$

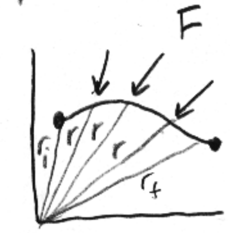
Gravity Fields

- acceleration of a gravity field is constant for a given location

$$g = \frac{GM_e}{r^2} \quad \text{when } r = R_e \quad g = 9.8\,m/s^2$$

Gravitational Potential Energy

- Remember gravity is a conservative force



- gravity is a central force (depends only on polar "r")

$$dW = F \cdot dr = F(r)\,dr$$

★ Work done by a perpendicular force is zero
- Example is work along a circular path by the centripetal force.

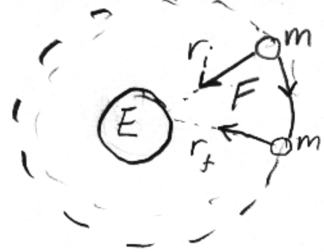
So we use the central force approach to sum the the radial segments of work.

$$\Delta U = -W \qquad W = \int_{r_i}^{r_f} F(r)\,dr$$

$$\Delta U = -\int_{r_i}^{r_f} F(r)\,dr$$

This can be used for potential energy of an orbit.

$$F = -\frac{GM_e m}{r^2}$$



$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_e m}{r^2}\, dr$$

$$U_f - U_i = GM_e m \int_{r_i}^{r_f} \frac{1}{r^2}\, dr \qquad \left(\int_{r_i}^{r_f} r^{-2} \rightarrow \frac{r^{-1}}{-1} \rightarrow -\frac{1}{r} \right)$$

$$= GM_e m \left[-\frac{1}{r} \right]_{r_i}^{r_f}$$

$$\boxed{U_f - U_i = -GM_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)} \quad \text{so...} \quad W_g = GM_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

This can be applied to any 2 particles as

$$\boxed{U = -\frac{G m_1 m_2}{r}}$$

when initial points are reference zero points