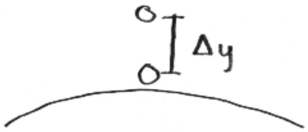
Ex: Change in Potential Energy

$$\Delta U = mg\Delta y$$



$$\Delta U = -GM_e m\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = GM_e m\left(\frac{r_f - r_i}{r_i r_f}\right)$$

- if Δy occurs close to the Earth's surface

$$r_f - r_i = \Delta y \quad \text{and} \quad r_i r_f = R_e^2$$

$$g = \frac{GM_e}{R_e^2} \quad \text{so...} \quad \Delta U = \frac{GM_e m}{R_e^2}\Delta y = mg\Delta y \ \checkmark$$

Energy and Motion of Planets

$$E = K + U$$

- Works when m is moving around massive M

$$\boxed{E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)}$$

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

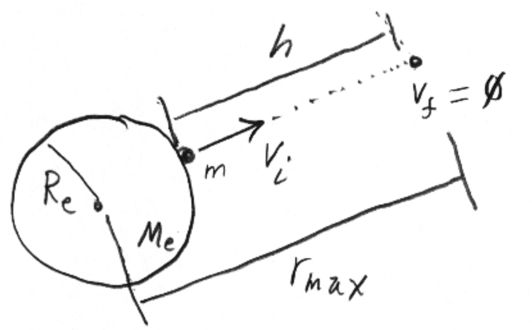
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$ ← Newton's 2nd Law

multiply both sides by r and divide by 2 → $$\frac{GMm}{2r} = \frac{1}{2}mv^2$$

Substitute → $$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$\boxed{E = -\frac{GMm}{2r}}$$ ← Total Energy of circular orbit

Escape Velocity



$$K + U = E$$

$$\frac{1}{2} m v_i^2 - \frac{G M_e m}{R_e} = -\frac{G M_e m}{r_{max}}$$

$$v_i^2 = 2 G M_e m \left(\frac{1}{R_e} - \frac{1}{r_{max}} \right)$$

with v_i we can find altitude because $h = r_{max} - R_e$

$v_f = \emptyset$ when we just escape Earth's gravity. This makes v_i the absolute minimum speed

Set $r_{max} = \infty$ $\quad \frac{1}{\infty} = \emptyset$

$v_i = v_{esc}$ $\quad \boxed{v_{esc} = \sqrt{\frac{2GM_e}{R_e}}}$

v_{esc} corresponds to the minimum amount of K needed to have $E = \emptyset$ at $r = \infty$

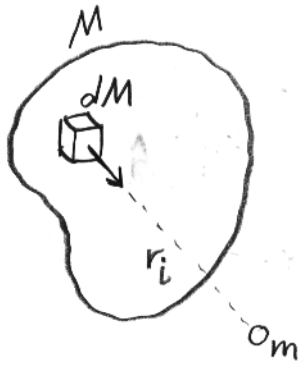
If $v_i > v_{esc}$ the object will have residual K at $r = \infty$

EX: Calculate the v_{esc} of a 5,000 kg rocket. How much K does it need?

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = 1.12 \times 10^4 \, m/s \approx 25{,}000 \, mi/h$$

$$K = \frac{1}{2} m v_{esc}^2 = \boxed{3.14 \times 10^{11} \, J}$$

Gravitational Force between an Extended Body



$$U = -Gm\int \frac{dM}{r}$$ from $$U = -\frac{GMm}{r}$$

Force can be obtained as

$$F = -\frac{dU}{dr}$$ this can be difficult integration

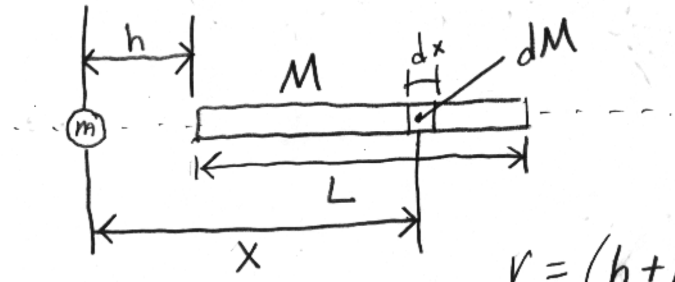
Use the vector equation of Force

$$F = -\frac{GMm}{r^2}\hat{r} \qquad F = -Gm\int \frac{dM}{r^2}\hat{r}$$

$\hat{r}$ is the unit vector directed from dM to m

☆ Use this with simple geometry

Ex: 7.5 in packet (uniform bar)



$r = (h+L)$

$$\lambda = \frac{M}{L}$$

$$dM = \lambda\, dx$$

$$dM = \frac{M}{L}dx$$

$$F = Gm\int \frac{dM}{r^2}$$

$$F = GM\int_h^{L+h} \frac{M}{L}\cdot\frac{dx}{X^2}$$

$$F = \frac{GMm}{L}\left[-\frac{1}{X}\right]_h^{L+h}$$

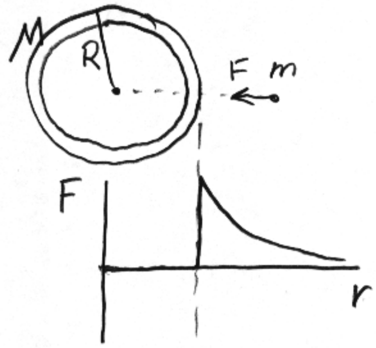
$$F = \frac{GMm}{L}\left(-\frac{1}{(L+h)} + \frac{1}{h}\right)$$

$$F = \frac{GMm}{\cancel{L}}\,\frac{\cancel{L}}{h(L+h)}$$

$$\boxed{F = \frac{GMm}{h(L+h)}}$$

Gravity Force between a particle and a sphere

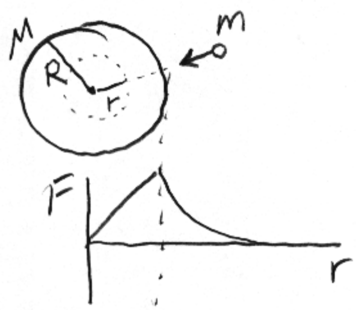
Spherical Shell



$r > R \quad F = -\frac{GMm}{r^2}\hat{r}$

$\boxed{r < R \quad F = \emptyset}$ (everywhere inside the shell)

Solid Sphere



$r > R \quad F = -\frac{GMm}{r^2}\hat{r}$

$r < R \quad \frac{M_{inside}}{M} = \frac{\rho V_{inside}}{\rho V} = \frac{\frac{4}{3}\pi r^3 \rho}{\frac{4}{3}\pi R^3 \rho} = \frac{r^3}{R^3}$

so... $M_{inside} = \frac{r^3}{R^3}M \rightarrow F = -\frac{Gm\left(\frac{r^3}{R^3}\right)M}{r^2}$

$$\boxed{F = \frac{GmM}{R^3}r \quad \text{when} \quad r < R}$$