APC Kinematic Recitations 2

Name: KEY

1. The position of an object moving along a straight line is given by $x = 3 - 2t^2 + 3t^3$ where x is in meters and t in seconds. **SHOW ALL WORK and/or EXPLAIN IN DETAIL!**

a) Derive the expressions for the velocity and acceleration of the object as a function of time. (v = -4t + 9t², a = -4 + 18t)

$\frac{dx}{dt} = v_{(t)} = -4t + 9t^2$ / $\frac{d^2x}{dt^2} = \frac{dv}{dt} = a_{(t)} = -4 + 18t$

b) Find the position of the object at t = 0, t = 2s, t = 4s. (3m, 19m, 163m)

$X_{(0)} = 3m$

$X_{(2)} = 3 - 2(2)^2 + 3(2)^3 = 19m$

$X_{(4)} = 3 - 2(4)^2 + 3(4)^3 = 163m$

c) Find the displacement or the object between t = 2s and t = 4s; between t = 0s and t = 4s. (144m, 160m)

$\Delta X_{(2-4)} = 144m$

$\Delta X_{(0-4)} = 160m$

d) Find the average velocity between t = 2s and t = 4s; between t = 0s and t = 4s; between t = 1s and t = 3s. (72m/s, 40m/s, 31m/s)

$\bar{V} = \frac{\Delta X_{(2-4)}}{t} = \frac{144m}{2s} = \boxed{72m/s}$

$\bar{V} = \frac{\Delta X_{(0-4)}}{t} = \frac{160m}{4s} = \boxed{40m/s}$

$\bar{V} = \frac{\Delta X_{(1-3)}}{t} = \frac{62m}{2s} = \boxed{31m/s}$

e) What is the instantaneous velocity at t = 2s? at t = 5s? (28m/s, 205m/s)

$V_{(2)} = 28m/s$

$V_{(5)} = 205m/s$

f) At what time(s) is/are the instantaneous velocities zero? (0s, 0.44s)

$\emptyset = -4t + 9t^2$

$t_2 = 0s$

$4t = 9t^2$

$\boxed{t_1 = \frac{4}{9} = 0.44s}$

g) When does the instantaneous velocity have a maximum or a minimum value? (0.22s)

$\frac{dv}{dt} = \emptyset = a$ $\quad t = \frac{4}{18} = 0.22s$

$\emptyset = -4 + 18t$

h) Find the change in velocity between t = 2s and t = 5s. (See part e) (177m/s)

$\Delta V_{(2-5)} = 205 m/s - 28 m/s = 177 m/s$

i) Find the average acceleration between t = 2s and t = 5s; between t = 1s and t = 3s. ($59 m/s^2$, $32 m/s^2$)

$\bar{a} = \frac{\Delta V_{(2-5)}}{t} = \frac{177 m/s}{3s} = 59 m/s^2$

$\bar{a} = \frac{\Delta V_{(1-3)}}{t} = \frac{64 m/s}{2s} = 32 m/s^2$

j) When is the instantaneous acceleration of the object zero? (0.22s)

$\emptyset = -4 + 18t$

$t = \frac{4}{18} = 0.22s$

k) Find the instantaneous acceleration of the object at t = 2s; t = 5s. ($32 m/s^2$, $86 m/s^2$)

$a_{(2)} = 32 m/s^2$

$a_{(5)} = 86 m/s^2$

2. The position of a body moving along a straight line is given by $x = 16t - 6t^2$ where x is in meters and t in seconds.

a) Find the position of the body at t = 1s. (10m)

$X_{(1)} = 10m$

b) At what times does the body pass the origin? (0s, 2.67s)

$\emptyset = 16t - 6t^2 \qquad t = \frac{16}{6} = 2.67s, \quad t = \emptyset$

c) Calculate the average velocity of the body between t = 0 and 2 seconds. (4m/s)

$\bar{V} = \frac{\Delta X}{t} = \frac{[16(2) - 6(2)^2] - [16(0) - 6(0)^2]}{2s} = \frac{8m/s}{2s} = 4m/s$

d) Find the velocity of the object at any time t. (v = 16 - 12t)

$\frac{dX}{dt} = V_{(t)} = 16 - 12t$

e) What is velocity at t = 0? at t = 2s? (16m/s, -8m/s)

$V_{(0)} = 16m/s$

$V_{(2)} = -8m/s$

f) At what times and positions will the body be at rest? (1.33s, 10.6m)

$\emptyset = 16 - 12t \qquad X_{(1.33)} = 16(1.33) - 6(1.33)^2$

$t = \frac{16}{12} = 1.33s \qquad X_{(1.33)} = 10.6m$

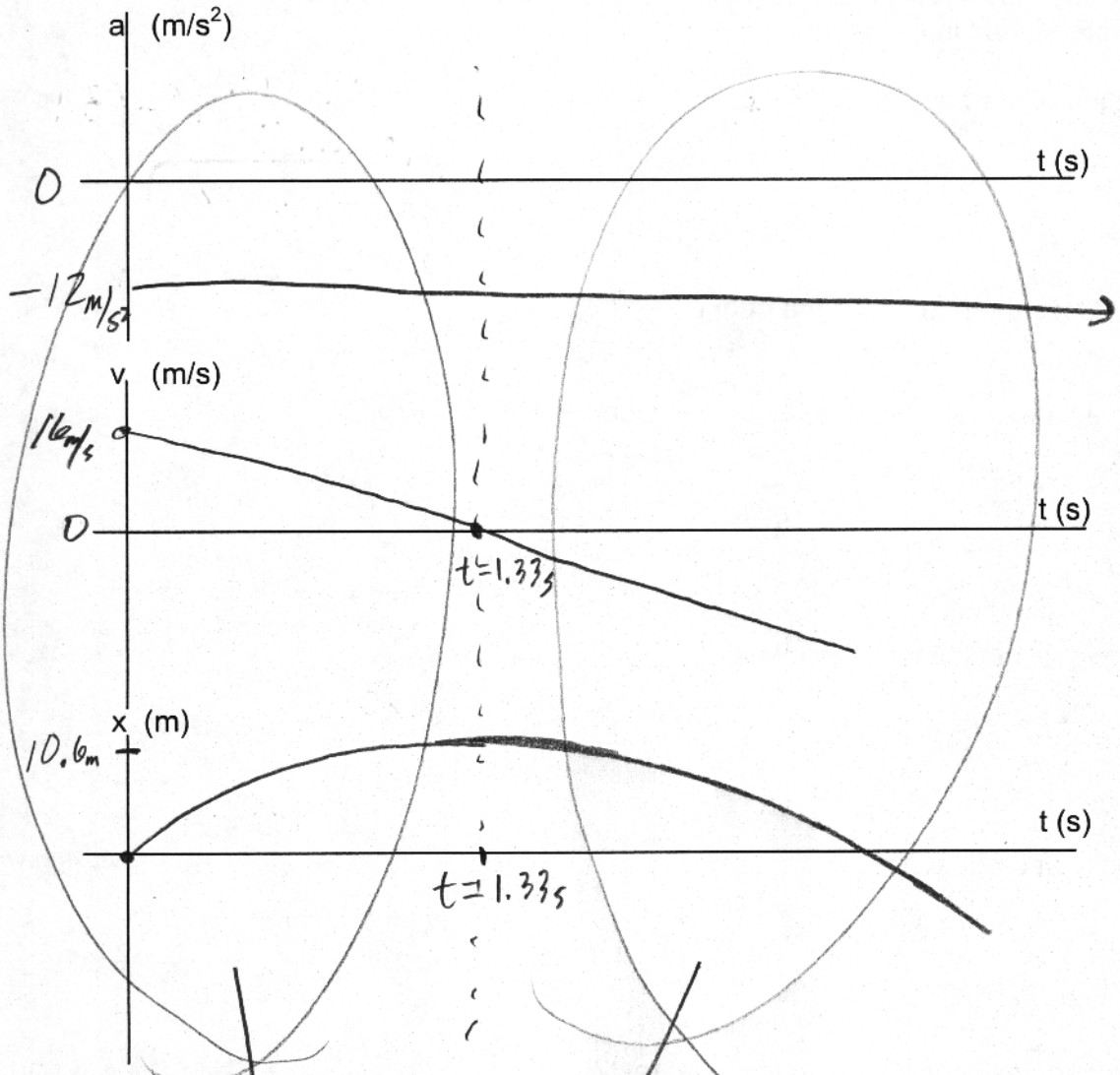
g) Find the acceleration of the body at any time t. ($-12m/s^2$)

$\frac{dv}{dt} = a = -12m/s^2$

h) When is the acceleration of the body zero? (never)

$a = -12m/s^2$ (constant)

i) Using the data calculated from a-h, plot a vs t, v vs t, and x vs t.



$a = -12$

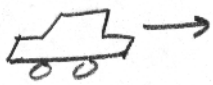
$v = 16 - 12t$

$x = 16t - 6t^2$

j) During what time interval(s) is the body "speeding up" (i.e. accelerating)? ($t > 1.33s$)

k) During what time interval(s) is the body "slowing down" (i.e. decelerating)? ($0 < t < 1.33s$)

3. A motorist travels at a constant speed of 39.0 m/s through a school zone; exceeding the posted speed limit. A policeman, waits 7.0 s before giving chase at an acceleration of 3.5 m/s^2. Find the time required to catch the car, from the instant the car passes the policeman.



$V_1 = 39 m/s$

$X_1 = X_{0_1} + V_1 t$

$X_1 = 39t$

$\emptyset = 1.75t^2 - 63.5t + 85.75$

$t_1 = 34.8 s$

$V_{2_0} = \emptyset$

$a_2 = 3.5 m/s^2$

$X_2 = X_{0_1} + V_{2_0} t + \frac{1}{2} a_2 t^2$

$X_2 = \frac{1}{2}(3.5)(t-7)^2$

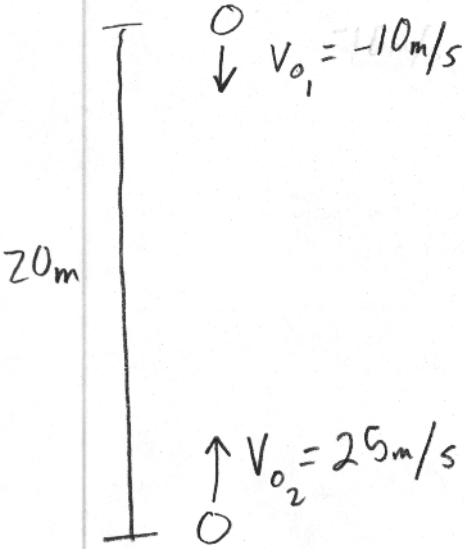
$X_1 = X_2$

$39t = 1.75(t-7)^2$

$39t = 1.75(t^2 - 14t + 49)$

$39t = 1.75t^2 - 24.5t + 85.75$

4. A ball is tossed downward from a cliff with a speed of 10m/s. At the same moment another ball is tossed upward from the ground below with a speed of 25m/s. If the cliff is 20m tall, where do the balls collide relative to the ground?



$X_1 = X_{0_1} + V_{0_1} t - \frac{1}{2} g t^2$

$X_1 = 20 - 10t - 5t^2$

$X_1 = X_2$

$20 - 10t - 5t^2 = 25t - 5t^2$

$X_2 = 25t - 5t^2$

$X_2 = X_{0_2} + V_{0_2} t - \frac{1}{2} g t^2$

$20 - 10t = 25t$

$20 = 35t$

$t = \frac{20}{35} = 0.57 s$

$X_2 = 25(0.57) - 5(0.57)^2$

$X_2 = 14.25 - 1.62 = \boxed{12.63 m}$

5. A particle has an acceleration of $a_{(t)} = 6t + 2$.

$V_o = 0\ m/s$

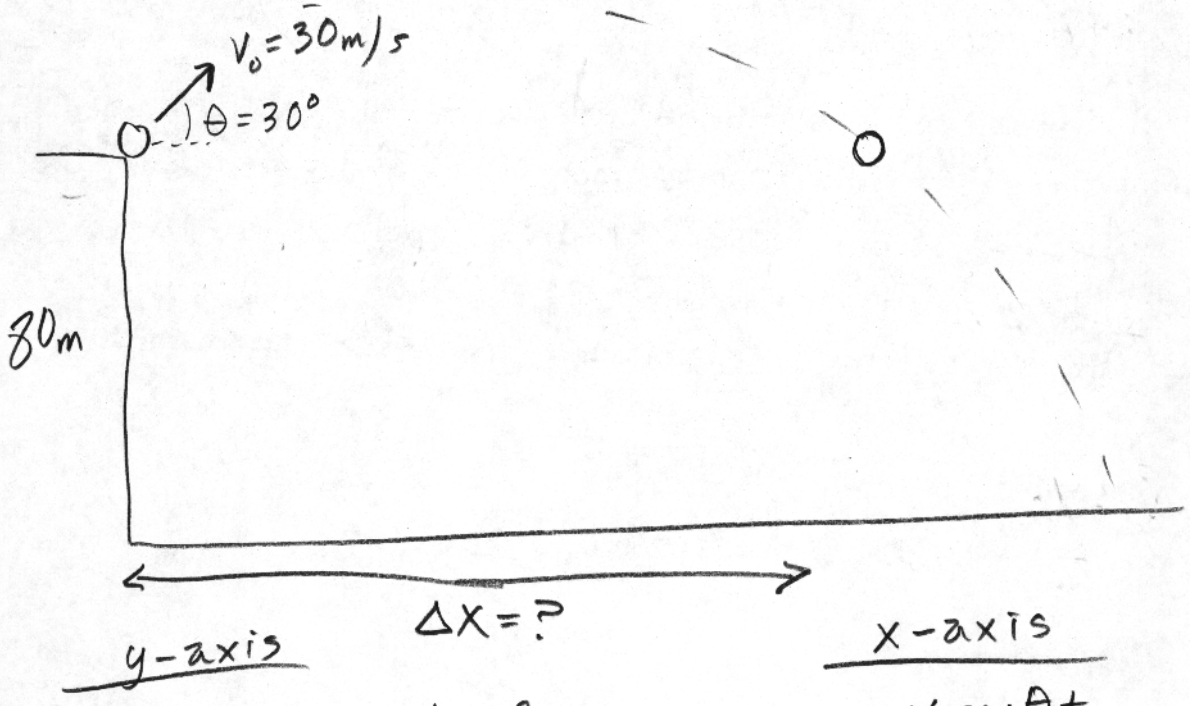
How far will the particle travel in the first 5 seconds of travel if $X_o = 3m$?

$$\int a\,dt = V_{(t)} = \int (6t+2)\,dt = 3t^2 + 2t$$

$$\int v\,dt = x = \int (3t^2 + 2t)\,dt = t^3 + t^2 + 3$$

$$X_{(5)} = (5)^3 + (5)^2 + 3 = \boxed{153m}$$

6. A ball is launched off a cliff with a velocity of $30 m/s$ at an angle of $30°$ above the horizontal. If the cliff is $80m$ tall, what will be the ball's horizontal displacement from the edge of the cliff at $\frac{2}{3}$ of the ball's total time in the air?



y-axis

$$y = y_o + V_o \sin\theta\, t - \tfrac{1}{2} g t^2$$

$$y = 80 + 15t - 5t^2$$

$$t_{\text{in air}} = 5.77_s$$

x-axis

$$\Delta X = V_o \cos\theta\, t$$

$$\Delta X = V_o \cos\theta\, \tfrac{2}{3} t$$

$$\boxed{\Delta X = 99.9_m}$$