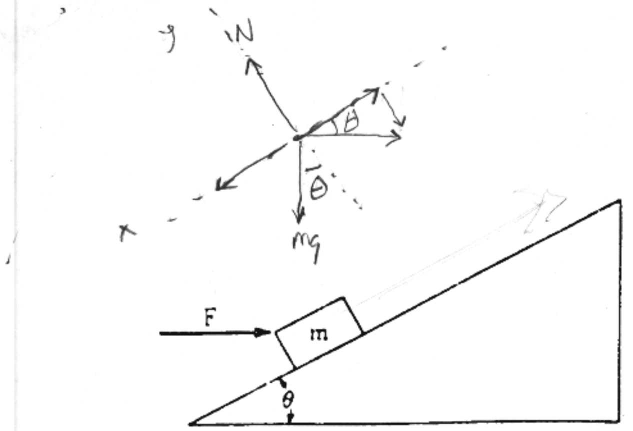


$\Sigma F_y = \emptyset$

$N - mg\cos\theta - F\sin\theta = \emptyset$

$N = mg\cos\theta + F\sin\theta$

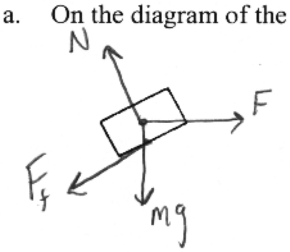
$\Sigma F_x = ma$

$\downarrow$

$F\cos\theta - mg\sin\theta - \mu N = ma$

1981M1. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.

a. On the diagram of the block below, draw and label all the forces that act on the block as it slides up the plane.



(b) $F\cos\theta - mg\sin\theta - \mu(mg\cos\theta + F\sin\theta) = ma$

$$\boxed{a = \frac{F\cos\theta - mg\sin\theta - \mu(mg\cos\theta + F\sin\theta)}{m}}$$

b. Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.

c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?

$a = \emptyset$

$V =$ constant

$\emptyset = F\cos\theta - mg\sin\theta - \mu(mg\cos\theta + F\sin\theta)$

$\emptyset = F\cos\theta - mg\sin\theta - \mu mg\cos\theta - \mu F\sin\theta$

$F\cos\theta - \mu F\sin\theta = \mu mg\cos\theta + mg\sin\theta$

$F(\cos\theta - \mu\sin\theta) = mg(\sin\theta + \mu\cos\theta)$

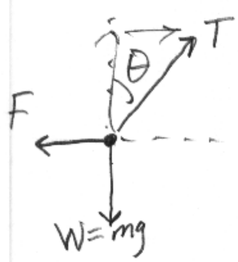
$$\boxed{F = mg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$

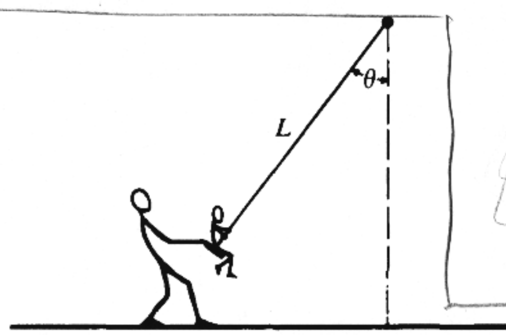
(+)

$F > \emptyset$

$\cos\theta > \mu\sin\theta$

$\boxed{\tan\theta > \mu \text{ or } \tan\theta < \frac{1}{\mu}}$





1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

a. the tension in the rope;

b. the horizontal force exerted by the adult.

The adult releases the swing from rest. In terms of W and θ determine

c. the tension in the rope just after the release (the swing is instantaneously at rest);

d. the tension in the rope as the swing passes through its lowest point.

a) $\Sigma F_x = \emptyset$

$T\sin\theta - F = \emptyset$

$T = \frac{F}{\sin\theta}$

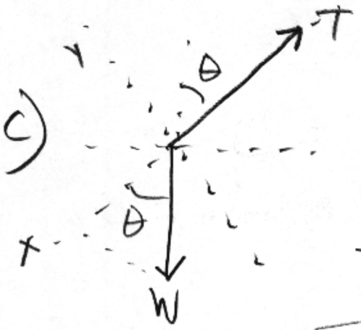
$\Sigma F_y = \emptyset$

$T\cos\theta - W = \emptyset$

$\boxed{T = \frac{W}{\cos\theta}}$

b) $\frac{W}{\cos\theta} = \frac{F}{\sin\theta}$

$\boxed{W\tan\theta = F}$

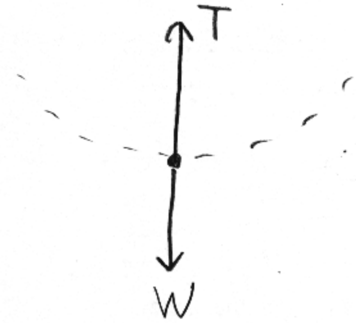
c) 

$$\Sigma F_c = m a_c$$

$$T - W\cos\theta = m\frac{v^2}{r}$$ at moment $v = \emptyset$

$$T - W\cos\theta = \emptyset$$

$$\boxed{T = W\cos\theta}$$

d) 

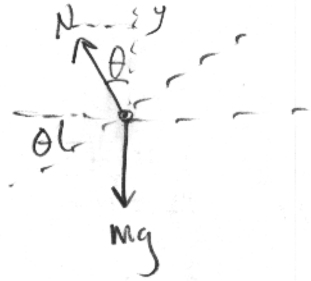
$$\Sigma F_c = m a_c$$

$$T - W = m\frac{v^2}{r}$$

$$T = \frac{mv^2}{L} + W$$

$$T = \frac{mv^2}{L} + mg$$

$$\boxed{T = m\left(\frac{v^2}{L} + g\right)}$$



a) $\Sigma F_{c_x} = ma_c$

$N\sin\theta = m\frac{v^2}{r}$

$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$

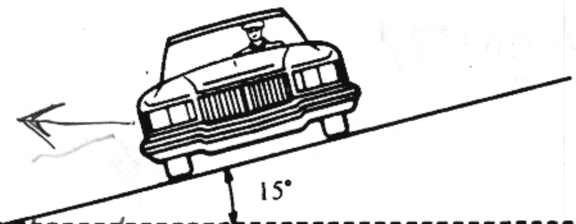
$\cancel{m}g\tan\theta = \cancel{m}\frac{v^2}{r}$

$\sqrt{rg\tan\theta} = v = 16\ m/s$

$\Sigma F_y = \emptyset$

$N\cos\theta - mg = \emptyset$

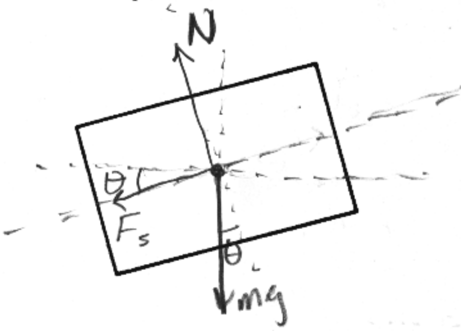
$N = \frac{mg}{\cos\theta}$



1988M1. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.

a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.



b. On the diagram below, in which the block represents the automobile, draw and label all of the forces on the automobile.

c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.

c) $\mu_s = ?$

$\Sigma F_{c_x} = ma_c$

$N\sin\theta + F_s\cos\theta = ma_c$

$N\sin\theta + \mu_s N\cos\theta = ma_c$

$N(\sin\theta + \mu_s\cos\theta) = m\frac{v^2}{r}$

$\Sigma F_y = \emptyset$

$N\cos\theta - F_s\sin\theta - mg = \emptyset$

$N\cos\theta = mg + F_s\sin\theta$

$N\cos\theta = mg + \mu_s N\sin\theta$

$N(\cos\theta - \mu\sin\theta) = mg$

$N = \frac{mg}{\cos\theta - \mu\sin\theta}$

$$N(\sin\theta + \mu\cos\theta) = m\frac{v^2}{r} \qquad N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$\frac{mg}{\cos\theta - \mu\sin\theta} \cdot \frac{\sin\theta + \mu\cos\theta}{1} = m\frac{v^2}{r}$$

$$\cancel{m}g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) = \cancel{m}\frac{v^2}{r}$$

$$\frac{v^2}{rg}(\cos\theta - \mu\sin\theta) = \sin\theta + \mu\cos\theta$$

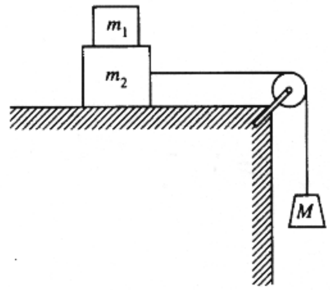
$$\frac{v^2}{rg}\cos\theta - \frac{v^2}{rg}\mu\sin\theta = \sin\theta + \mu\cos\theta$$

$$\mu\cos\theta + \frac{v^2}{rg}\mu\sin\theta = \frac{v^2}{rg}\cos\theta - \sin\theta$$

$$\mu\left(\cos\theta + \frac{v^2}{rg}\sin\theta\right) = \frac{v^2}{rg}\cos\theta - \sin\theta$$

$$\mu_s = \frac{\left(\frac{v^2}{rg}\cos\theta - \sin\theta\right)}{\left(\cos\theta + \frac{v^2}{rg}\sin\theta\right)} = \frac{(0.625\cos(15^\circ)) - [\sin(15^\circ)]}{[\cos(15^\circ)] + [0.625\sin(15^\circ)]}$$

$$= \frac{0.345}{1.13} = \boxed{0.31}$$



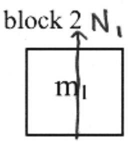
1998M3. Block 1 of mass m_1 is placed on block 2 of mass m_2 which is then placed on a table. A string connecting block 2 to a hanging mass M passes over a pulley attached to one end of the table, as shown above. The mass and friction of the pulley are negligible. The coefficients of friction between blocks 1 and 2 and between block 2 and the tabletop are nonzero and are given in the following table.

	Coefficient Between Blocks 1 and 2	Coefficient Between Block 2 and the Tabletop
Static	μ_{s1}	μ_{s2}
Kinetic	μ_{k1}	μ_{k2}

Express your answers in terms of the masses, coefficients of friction, and g, the acceleration due to gravity.

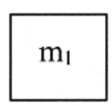
a. Suppose that the value of M is small enough that the blocks remain at rest when released. For each of the following forces, determine the magnitude of the force and draw a vector on the block provided to indicate the direction of the force if it is nonzero.

i. The normal force N_1 exerted on block 1 by block 2



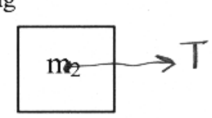
$\boxed{N_1 = m_1 g}$

ii. The friction force f_1 exerted on block 1 by block 2



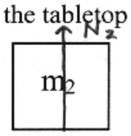
zero

iii. The force T exerted on block 2 by the string



$\boxed{T = Mg}$

iv. The normal force N_2 exerted on block 2 by the tabletop

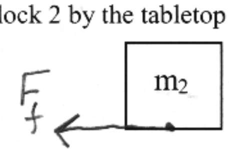


$\Sigma F = \emptyset$

$N_2 - (m_1 + m_2) g$

$\boxed{N_2 = (m_1 + m_2) g}$

v. The friction force f_2 exerted on block 2 by the tabletop



$\boxed{F_s = Mg}$

b. Determine the largest value of M for which the blocks can remain at rest.

c. Now suppose that M is large enough that the hanging block descends when the blocks are released. Assume that blocks 1 and 2 are moving as a unit (no slippage). Determine the magnitude *a* of their acceleration.

d. Now suppose that M is large enough that as the hanging block descends, block 1 is slipping on block 2. Determine each of the following.
i. The magnitude a_1 of the acceleration of block 1
ii. The magnitude a_2 of the acceleration of block 2

b) $T = Mg$ $\quad F_s = \mu_s N$

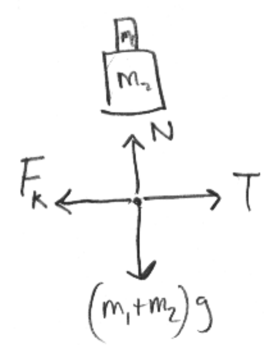
$$F_s = \mu_s((m_1+m_2)g)$$

$$F_s = T$$

$$M\cancel{g} = \mu_s[(m_1+m_2)\cancel{g}]$$

$$\boxed{M = \mu_s(m_1+m_2)}$$

c)



$$\Sigma F = (m_1+m_2)a$$

$$T - F_k = (m_1+m_2)a$$

$$T - \mu_k N = (m_1+m_2)a$$

$$T - \mu_k((m_1+m_2)g) = (m_1+m_2)a$$

$$\Sigma F = Ma$$

$$Mg - T = Ma$$

$$Mg - T = Ma$$

$$\cancel{T} - \mu_k((m_1+m_2)g) + Mg - \cancel{T} = (m_1+m_2)a + Ma$$

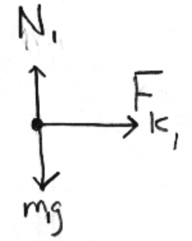
$$-\mu_k[(m_1+m_2)g] + Mg = a(m_1+m_2+M)$$

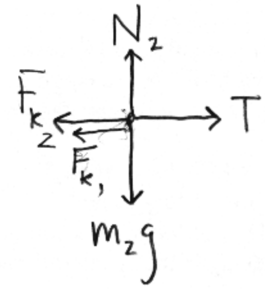
$$a = \frac{Mg - [\mu_k(m_1+m_2)g]}{m_1+m_2+M}$$

$$\boxed{a = \left[\frac{M - \mu_k(m_1+m_2)}{m_1+m_2+M}\right]g}$$

c) M_2 and M share a_2
m_1 has a_1

No 2 forces of friction





$$\Sigma F = m_1 a_2$$

$$F_{k_1} = m_1 a_1$$

$$\Sigma F = m_2 a_2$$

$$T - F_{k_2} - F_{k_1} = m_2 a_2$$

$$\Sigma F = M a_2$$

$$Mg - T = M a_2$$

i) $a_1 = \dfrac{F_{k_1}}{m_1}$

$$\boxed{a_1 = \frac{\mu_{k_1} \cancel{m_1} g}{\cancel{m_1}} = \mu_{k_1} g}$$

ii) $\cancel{T} - F_{k_2} - F_{k_1} + Mg \cancel{- T} = m_2 a_2 + M a_2$

$$-F_{k_2} - F_{k_1} + Mg = a_2(m_2 + M)$$

$$Mg - \mu_{k_2}(m_1 + m_2)g - \mu_{k_1} m_1 g = a_2(m_2 + M)$$

$$\boxed{a_2 = g\left[\frac{M - \mu_{k_1} m_1 - \mu_{k_2}(m_1 + m_2)}{M + m_2}\right]}$$