Periodic or Oscillatory Motion

$x = A\cos(\omega t + \varphi)$

$y = A\sin(\omega t + \varphi)$

$y = a\cos(bx + c) + d$

A = amplitude (m)

ω = angular frequency (rad/s)

t = time (s)

φ = phase constant

φ - tells us what the displacement was at $t = \emptyset$

$(\omega t + \varphi)$ - the phase, useful for comparing 2 systems of motion

Position is periodic, repeats every 2π radians

$1\,\text{cycle} = \omega t = 2\pi\ \text{rad}$

$\omega T = 2\pi$ or $\boxed{T = \frac{2\pi}{\omega}}$ $\frac{1}{s^{-1}} = (s)$

$f = \frac{1}{T} = \frac{\omega}{2\pi}$ $s^{-1} \rightarrow$ (Hz)

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

$$x = A\cos(\omega t + \varphi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$

$$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \varphi)$$

Note: $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$$x = A\cos(\omega t + \varphi)$$

so...

$$a = -\omega^2 x$$

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$

A being amplitude and therefore $\max(X)$

Initial Conditions are key

$t = \emptyset \quad x = x_0 \quad v = v_0$

$$x = A\cos(\omega t + \varphi) \qquad v = -\omega A\sin(\omega t + \varphi)$$

$\omega t = \emptyset$, cycle has not begun

so...

$$x_0 = A\cos\varphi \qquad v_0 = -\omega A\sin\varphi$$

$$\frac{v_0}{x_0} = -\omega\tan\varphi$$

$$\boxed{\tan\varphi = -\frac{v_0}{\omega x_0}} \quad \text{or} \quad \varphi = \cos^{-1}\left(\frac{x_0}{A}\right)$$

$$\boxed{A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}}$$

$$x_0^2 + \left(\frac{V_0}{\omega}\right)^2 = A^2\cos^2\varphi + A^2\sin^2\varphi$$

$$A = \sqrt{x_0^2 + \left(\frac{V_0}{\omega}\right)^2}$$

Ex: Oscillating body with a displacement in respect to time of

phase = radians
t = seconds

$$x = (4_m)\cos\left(\pi t + \frac{\pi}{4}\right)$$

a) Determine the amplitude, frequency and period

$$x = A\cos(\omega t + \varphi)$$

$$A = \boxed{4_m}$$

$$\omega = \pi \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \boxed{\frac{1}{2}\text{ Hz}}$$

$$T = \frac{1}{f} = \boxed{2_s}$$

b) Calculate the velocity and acceleration at any time,

Differentiate

$$V = \frac{dx}{dt} = -4_m\omega\sin\left(\pi t + \frac{\pi}{4}\right)$$

$$\boxed{V = -4\pi_{m/s}\sin\left(\pi t + \frac{\pi}{4}\right)}$$

$$a = \frac{dv}{dt} = -4\pi_{m/s}\,\omega\cos\left(\pi t + \frac{\pi}{4}\right)$$

$$\boxed{a = -4\pi^2_{m/s^2}\cos\left(\pi t + \frac{\pi}{4}\right)}$$

c) What is the position, velocity, acceleration at $t = 1_s$

$$X = 4_m \cos\left(\frac{5\pi}{4}\right) = \boxed{-2.83_m}$$

$$v = -4\pi_{m/s} \sin\left(\frac{5\pi}{4}\right) = \boxed{8.89_{m/s}}$$

$$a = -4\pi^2_{m/s^2} \cos\left(\frac{5\pi}{4}\right) = \boxed{27.9_{m/s^2}}$$

d) Determine the max V and a

$$V_{max} = \omega A \qquad A = 4_m$$

$$V_{max} = \boxed{4\pi_{m/s}}$$

$$a_{max} = \omega^2 A = \boxed{4\pi^2_{m/s^2}}$$

e) Displacement between $t = 0 - 1_s$

$$X_0 = 4_m \cos\left(\emptyset + \frac{\pi}{4}\right) = 2.83_m$$

$$X_{1_s} = -2.8_m$$

$$\Delta X = X - X_0 = \boxed{-5.66_m}$$

Not same as distance!

Velocity changed signs.

f) What is the phase @ $t = 2_s$

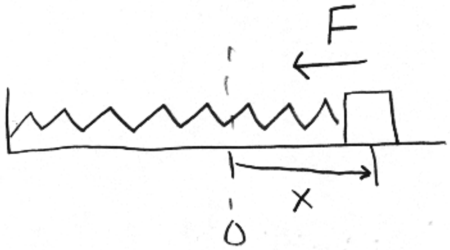
$$(\omega t + \varphi)$$

$$\left(\pi(2) + \frac{\pi}{4}\right) = \boxed{\frac{9}{4}\pi_{rad}}$$

Mass & Spring

Hooke's Law $\boxed{F = -kx}$

indicates that it is a restoring force.



$$F = -kx = ma$$

$$\boxed{a = -\frac{k}{m}x}$$

- acceleration is proportional to the displacement of mass and opposite the direction

$x_{max} = A$ then $a_{max} = \frac{-kA}{m}$

remember... $a_{max} = \omega^2 A$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

$$\boxed{\omega^2 = \frac{k}{m}}$$

$$\frac{a}{A} = \omega^2 = -\frac{a}{x}$$

then...

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

☆ How to prove this second-order differential

- What function of $x(t)$ satisfies this?

$$x(t) = A\cos(\omega t + \varphi)$$

$$\frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \varphi)$$

$$x = A\cos(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Yep!

- So whenever the force acting on a particle is linearly proportional to the displacement and in the opposite direction, the particle will exhibit SHM.

$$\omega^2 = \frac{k}{m}$$

$$T = \frac{2\pi}{\omega}$$

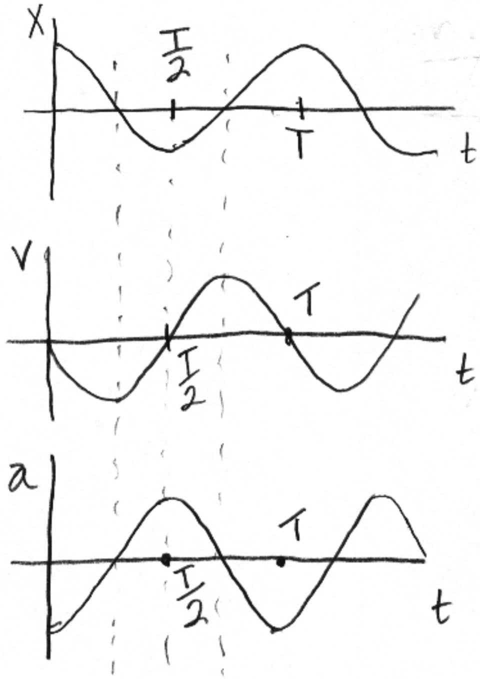
so...

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

and...

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Ex:



$x = A\cos\omega t$

$v = -\omega A \sin\omega t$

$a = -\omega^2 A\cos\omega t$