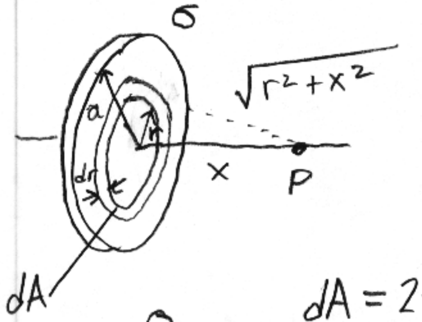
Potential of a uniformly charge disk.




• Divide the disk into a series of charged rings with width dr

$$\sigma = \frac{Q}{A} \qquad dA = 2\pi r\, dr$$
(circumference x width)

$$dq = \sigma dA = \sigma 2\pi r dr$$

Potential at point P due to the ring is

$$dV = \frac{k\, dq}{\sqrt{r^2+x^2}} = \frac{k \sigma 2\pi r\, dr}{\sqrt{r^2+x^2}}$$

To find total sum all the rings from Ø to a

$$V = k\pi\sigma \int_0^a \frac{2r\,dr}{\sqrt{r^2+x^2}} = k\pi\sigma \int_0^a (r^2+x^2)^{-\frac{1}{2}}\, 2r\, dr$$

$$\boxed{V = 2\pi k\sigma\left[(x^2+a^2)^{\frac{1}{2}} - x\right]}$$

$$\left\{ \begin{array}{l} x^n dx \\ x^{n+1}/n+1 \\ n=-\frac{1}{2} \\ x = r^2+x^2 \end{array} \right.$$

## The Potential of a sphere

Insulating solid sphere of radius(R) and charge(Q)
uniform charge density

1. Find electric potential at $r>R$ take $V=Ø$ @ $r=\infty$

$$E = k\frac{Q}{r^2} \quad r>R \quad \text{so} \quad V_B = -\int_\infty^r E\,dr = -kQ\int_\infty^r \frac{dr}{r^2}$$

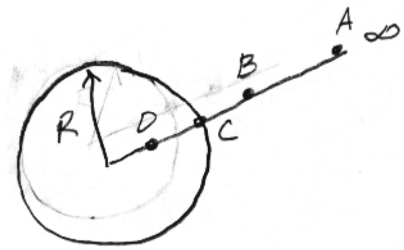
$$\boxed{V_B = k\frac{Q}{r}}$$

same as potential due to a point charge!

2. Find electric potential inside $r<R$

$$E = \frac{kQ}{R^3} r \quad (r<R)$$

This was proven with Gauss' Law




$$V_D - V_c = -\int_R^r E\,dr = -\frac{kQ}{R^3}\int_R^r r\,dr$$

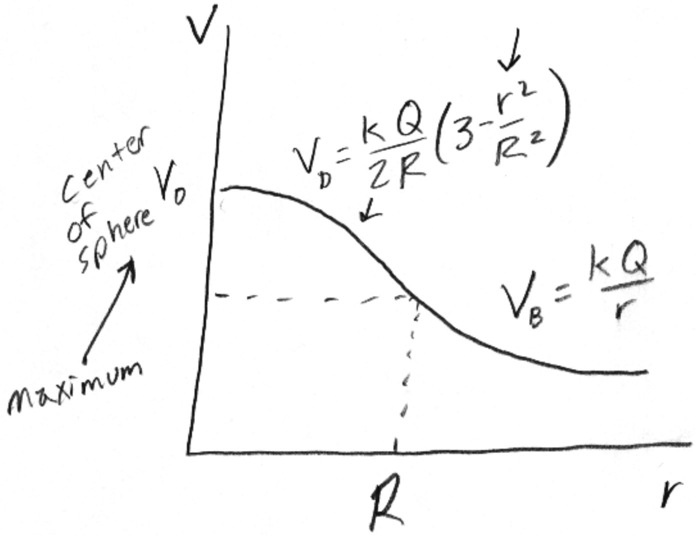
$$V_D - V_C = -\frac{kQ}{R^3}\left[r^2\right]_R^r$$

$$V_D - V_c = \frac{kQ}{R^3}\left(R^2 - r^2\right)$$

$$V_D - \frac{kQ}{R} = \frac{kQ}{R^3}\left(R^2 - r^2\right)$$

$$V_D = \frac{kQ}{R^3}R^2 - \frac{kQr^2}{R^3} + \frac{kQ}{R}$$

$$\boxed{V_D = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$




Uniformly charged (insulative) sphere only!

$$V = -\int E \cdot dr$$

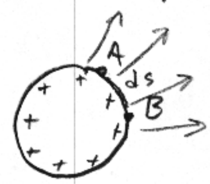
$$\boxed{E = -\frac{dV}{dr}}$$

# Potential of a Charged Conductor

(Sphere & Cylinder)

★ Every point on the surface of a charged conductor in static equilibrium is at the same potential!

$$\Delta V = -\int_A^B E \cdot ds = 0$$

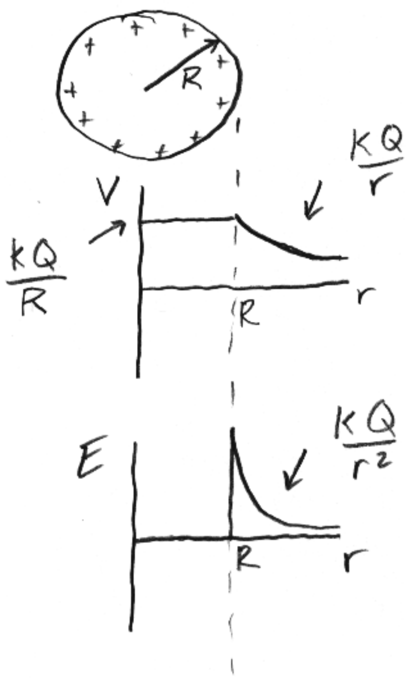



E is perpendicular always to ds (arc length)

The surface of any charged conductor is an equipotential surface.

★ Since the E-field is zero inside the conductor, we conclude that the potential is constant everywhere inside the conductor.

★ No work is required to move a charge inside a conductor



# Summary of Electrostatics

## Coulombs Law

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad \frac{1}{4\pi\varepsilon_0} = k$$

## Uniform E-field

$$F_e = Eq$$

## Electric Field

Point charge

$$E = \frac{kQ}{r^2}$$

uniform field

$$E = \frac{\sigma}{2\varepsilon_0}$$

uniform field from 2 plates.

$$E = \frac{\sigma}{\varepsilon_0}$$

## Charge Densities

$$\lambda = \frac{Q}{L} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$$

## Flux

$$\Phi = \oint E \cdot dA$$

in (−)

out (+)

## Gauss' Law

$$\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

$Q_{in}$ = charge inside gaussian surface

## Potential

$$V = -\int E \cdot dr$$

Point charge

$$V = k \sum_i \frac{q_i}{r_i}$$

## Potential Energy

$$U_E = -qEr$$

Point charge

$$U_E = k \sum_i \frac{q_i q_j}{r_{ij}}$$

# How they all relate

$$F = Eq$$

$$U_E = -qE \cdot r \qquad\qquad V = Er$$

$$\begin{tikzcd}
F_e \arrow[r, shift left, "\div \text{ by } q"] \arrow[d, shift right, "\div \text{ by } r"'] & E \arrow[l, shift left, "\times \text{ by } q"] \arrow[d, shift right, "\times \text{ by } r"'] \\
U_e \arrow[u, shift right, "\times \text{ by } r"'] \arrow[r, shift left, "\div \text{ by } q"] & V \arrow[l, shift left, "\times \text{ by } q"] \arrow[u, shift right, "\div \text{ by } r"']
\end{tikzcd}$$

$$U_E = Vq$$

The Calculus...

$$U_E = -\int F \cdot dr \qquad\qquad V = -\int E\, dr$$

$$F = -\frac{dU_E}{dr} \qquad\qquad E = -\frac{dV}{dr}$$