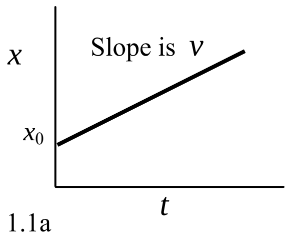
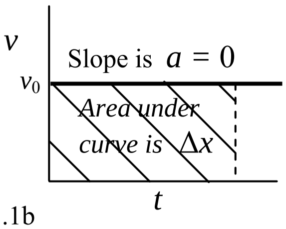
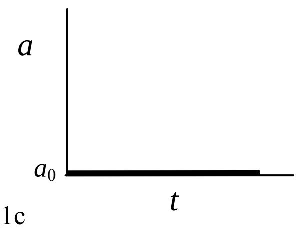
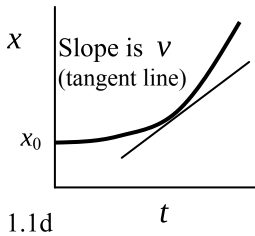
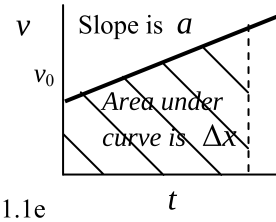
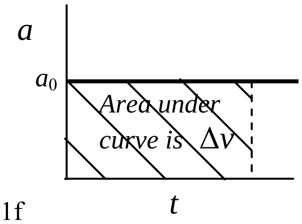
Graphing

	Displacement–Time Graph	*Velocity–Time Graph*	*Acceleration–Time Graph*
Constant Velocity	 Fig 1.1a	 Fig 1.1b	 Fig 1.1c
Uniform Acceleration	 Fig 1.1d	 Fig 1.1e	 Fig 1.1f

Slopes of Curves are an important analytical tool used in physics. Any equation that can be manipulated into the format $y = mx + b$ can be represented and analyzed graphically. As an example: $v = v_0 + at$ can be rearranged slightly into $v = at + v_0$. Compare this equation to the equation of a line. It is apparent that a is the slope and that v_0 is the y-intercept (Fig 1.1b and Fig 1.1e). What equation generates velocity in Fig. 1.1a and Fig 1.1d?

Area Under a Curve is another important graphical tool. Multiply the y-axis (height) by the x-axis (base) and determine if this matches any known equations. For example: Figures 1.1b and 1.1e are velocity-time plots. Simply multiply $v \times t$. This is a rearranged form of the equation $v = x/t$. The form obtained from the graph is $x = vt$, which means that displacement is the area under the velocity-time plot.

Calculus: ***Required in AP Physics C*** (optional for AP Physics B students)***. Calculus is taught in math class. These review sheets will focus on the Physics aspect of solutions. Calculus steps may not be shown. Solution will be up to the student.***

The equations outlined in the previous page work well in the following situations.

- Linear Functions: involving constant velocity and acceleration, as diagrammed in the above graphs (except Fig 1.1d).
- Nonlinear Functions: scenarios where the problem is seeking information about a change in a quantity, Δx or Δv.
- Nonlinear Functions: scenarios where the problem is seeking an average velocity in an interval.

Calculus is needed to find the slopes of nonlinear functions and the areas under nonlinear curves.

1. **Velocity: Slope of the displacement-time curve.**

$$v = \frac{dx}{dt} \qquad \text{example:} \qquad v = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at \qquad v = v_0 + at$$

2. **Acceleration: Slope of the velocity-time curve.**

$$a = \frac{dv}{dt} \qquad \text{example:} \qquad a = \frac{d}{dt}\left(v_0 + at\right) = a$$

3. **Velocity: Area under acceleration-time curve.** (Note: if $c = v_0$ cannot be found, then you can only solve for Δv)

$$v = \int a \ dt \qquad \text{example:} \qquad v = \int (a) \ dt = c + at \qquad v = v_0 + at$$

4. **Displacement: Area under the velocity-time curve.** (Note: if $c = x_0$ cannot be found, then you can only solve Δx)

$$x = \int v \ dt \qquad \text{example:} \qquad x = \int (v_0 + at) \ dt = c + v_0 t + \frac{1}{2}at^2 \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

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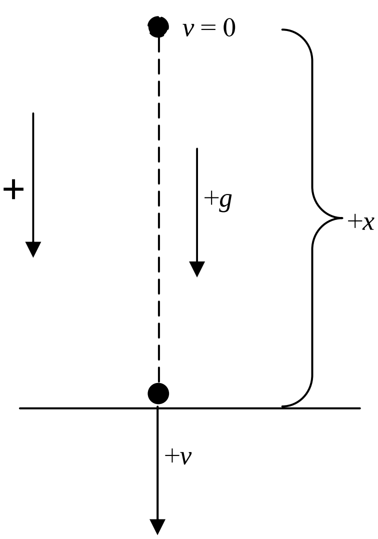
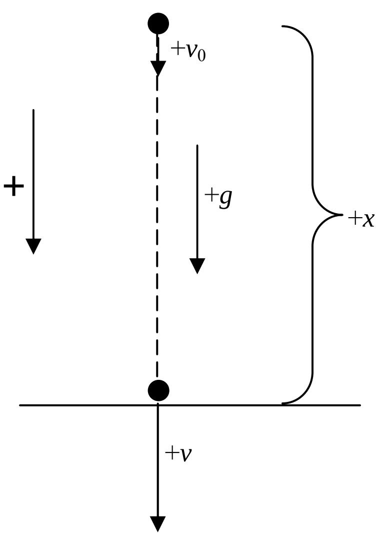
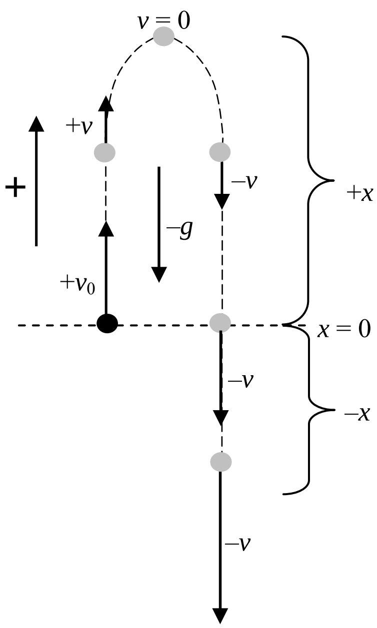
Falling Bodies: Objects moving vertically under the influence of gravity. Earth's surface gravity is $g = -9.8$ m/s^2. This speeds objects up (+ acceleration), but is directed downward (–). Objects can be thrown up, down, or just be dropped. They can land below, at the same height, or above the origin. They come to an instantaneous stop at the highest point in flight.

$$\boxed{v_y = v_{0y} + gt} \qquad \boxed{y = y_0 + v_{0y}t + \frac{1}{2}gt^2} \qquad \boxed{v_y^{\ 2} = v_{0y}^{\ 2} + 2g\left(y - y_0\right)}$$

The positive and negative signs can cause trouble in these problems. The easiest way to handle the signs is set the direction of initial motion as positive and then to ensure all signs are consistent with this decision. This has one huge benefit. It eliminated the double sign on acceleration. When initial velocity v_0 is used to anchor direction, then a positive acceleration means speeding up and negative acceleration involves slowing down.

Dropped from rest	**Thrown downward**	**Thrown upward**
$v_0 = 0$, but it will move down initially	v_0 is directed downward	v_0 is directed upward
Set downward as positive direction	Set downward as positive direction	Set upward as positive direction
		
Everything is positive and easy	Everything is positive and easy	This is difficult, and depends on where in the flight the problem ends.

Kinematics Problems Involving Changes in the Magnitude of Acceleration

If the magnitude of acceleration changes while solving a kinematics problem then the problem must be solved in separate parts. Unlike displacement x and velocity v, the Kinematic equations do not contain the variables a_0 and a.

Example 1-3: More than one acceleration

A car initially at rest accelerates at 4 m/s^2 while covering a distance of 100 m. Then the car continues at constant velocity for 500 m. Finally it slows to a stop with a deceleration of 3 m/s^2. Determine the total time of this displacement.

Acceleration Phase: $x = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(100)}{(4)}} = 7.07\,s$ and $v = v_0 + at = (0) + (4)(7.07) = 28.3\,m/s$

Constant Velocity Phase: $v = \frac{\Delta x}{t} \qquad t = \frac{\Delta x}{v} = \frac{(500)}{(28.3)} = 17.7\,s \qquad t = \frac{500}{v}$

Deceleration Phase: $v = v_0 + at \qquad t = \frac{v - v_0}{a} = \frac{(0) - (28.3)}{(-3)} = 9.43\,s$

Total Time = 7.07 + 17.7 + 9.43 = 34.2 s

1–2 Vectors and Vectors in Kinematics

Scalar: A physical quantity described by a single number and units. A quantity describing ***magnitude*** only.

Vector: Many of the variables in physics equations are vector quantities. Vectors have ***magnitude*** and ***direction***.

Magnitude: Size or extend. The numerical value.

Direction: Alignment or orientation with respect to set location and system of orientation, such as a coordinate axis.

Notation: $\vec{A}$ or $\xrightarrow{A}$ The length of the arrow represents, and is proportional to, the vectors magnitude. The direction the arrow points indicates the direction of the vector.

Negative Vectors: Have the same magnitude as their positive counterpart, but point in the opposite direction.

If $\xrightarrow{A}$ then $\xleftarrow{-A}$

Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the **<u>result</u>**ant $\vec{R}$.

$\vec{A}+\vec{B}=\vec{R}$ $\qquad \xrightarrow{A} + \dashrightarrow^{B} = \xrightarrow{R}$

When you need to subtract one vector from another think of the one being subtracted as being a negative vector.

$\vec{A}-\vec{B}$ is really $\vec{A}+\left(-\vec{B}\right)=\vec{R}$ $\qquad \xrightarrow{A} + \dashleftarrow^{(-B)} = \xrightarrow{R}$

A negative vector has the same length as its positive counterpart, but its direction is reversed. ***<u>This is very important</u>***. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180^o apart).

Vector Addition: **Parallelogram**

A + ***B***

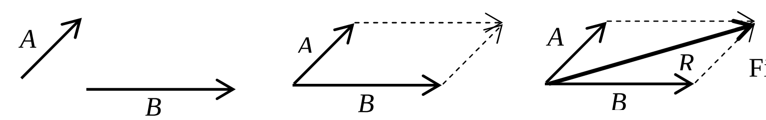


Fig 2.1a

Tip to Tail

A + ***B***

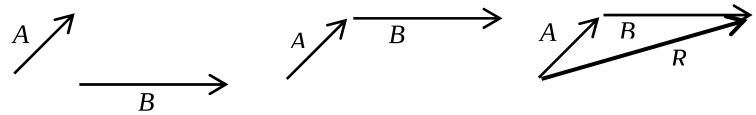


Fig 2.1b

Both methods arrive at the exact same solution since either method is essentially a parallelogram. In some problems one method is advantageous, while in other problems the alternative method is superior.

Reporting Magnitude and Direction:

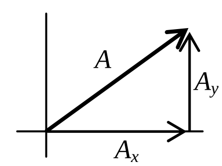


Fig. 2.2a

Component Method: A vector a can be reported by giving the components along the *x*- or *y*-axis. Reporting a vector this way is formally done by employing the unit vectors **i** and **j**. As an example: vector A in fig 2.2a would be $\boxed{A=A_x\mathbf{i}+A_y\mathbf{j}}$, where, $|\mathbf{i}|+|\mathbf{j}|=\mathbf{1}$. There is at third component vector **k** used for three dimensional problems involving the *z*-axis. The vector in Fig. 2.2b shows a numerical application of the component method. In this example you are given the polar coordinates $A=5\ at\ 37^o$. Using trigonometry the components can be established.

$\boxed{A_x=A\cos\theta}=5\cos 37^o=4$ and $\boxed{A_y=A\sin\theta}=5\sin 37^o=3$. Then A can be expressed as follows: $A=4\mathbf{i}+3\mathbf{j}$

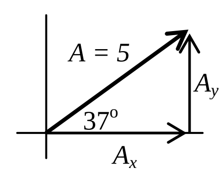


Fig. 2.2b

Polar Coordinates: Vector A in Fig. 2.2b is reported in polar coordinates, $A=5\ at\ 37^o$. This is simply the length of a vector and its angle measured counterclockwise with respect to the positive *x*-axis. (Negative angles are allowed and indicate that direction was measured clockwise from the +*x*-axis. If the component vectors are given, $A=4\mathbf{i}+3\mathbf{j}$, Pythagorean theorem is used to establish the length of the parent vector $\boxed{A=\sqrt{A_x^{\ 2}+A_y^{\ 2}}}=\sqrt{4^2+3^2}=5$.

Arctangent is used to find the direction $\boxed{\theta=\tan^{-1}\dfrac{A_y}{A_x}}=\tan^{-1}\dfrac{3}{4}=37^o$. ***But, watch out! The angle arrived at by the arctangent formula may not be the final answer. The quadrant that the final vector lies in must be established, and an adjustment to the angle may be needed in order to provide an answer that extends from the +x-axis.***

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Component Advantage In Vector Addition:

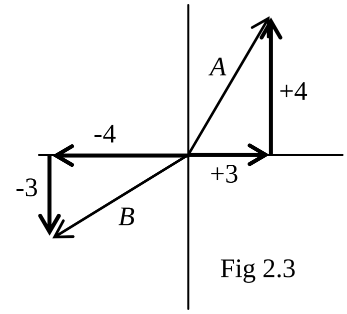


Fig 2.3

1. If vector components along an axis are used, direction can be specified with + and – symbols. Vectors A and B in Fig 2.3 have been converted into components.

   $A = (+3)\,\mathbf{i} + (+4)\,\mathbf{j}$ $\qquad$ $B = (-4)\,\mathbf{i} + (-3)\,\mathbf{j}$

2. This becomes advantageous if vectors A and B need to be added.

   Find the resultant of the x vectors: $\boxed{R_x = A_x + B_x} = (+3) + (-4) = -1$

   Find the resultant of the y vectors: $\boxed{R_y = A_y + B_y} = (+4) + (-3) = +1$

   Then combine them to find R: $\boxed{R = \sqrt{R_x^{\;2} + R_y^{\;2}}} = \sqrt{(-1)^2 + (+1)^2} = 1.41$

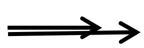
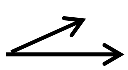
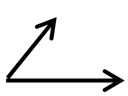
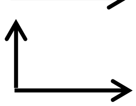
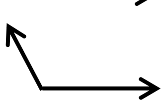
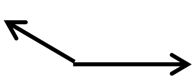
   Find the direction: $\boxed{\theta = \tan^{-1}\dfrac{R_y}{R_x}} = \tan^{-1}\dfrac{(+1)}{(-1)} = -45^o$, but this a 2nd quadrant angle and must be adjusted: $180^o - 45^o = 135^o$. The final vector is has a magnitude of 1.41 and a direction of 135^o.

3. Finding the components simplify problems throughout physics. In Newtonian Mechanics motion, force, and momentum often act at an angle to the *x*- or *y*-axis. Fortunately these vector quantities can be resolved into component vectors along the *x*- or *y*-axis. In addition, equations for motion, force, and momentum can be calculated in the *x*-direction independent of what is happening in the *y*-direction, and vice versa. The first example of this will be projectile motion. In projectile motion there will be a variety of initial and final of velocities at angles.

Scalar (Dot) Product of Two Vectors: The dot product (**A**·**B**) of two vectors **A** and **B** is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Vector (Cross) Product: The cross product (**A**×**B**) of two vectors **A** and **B** is a third vector **C**. The magnitude of this vector is $C = AB\sin\theta$. The direction of this vector is determined by the right-hand-rule. The order that the vectors are multiplied is important (not commutative) If you change the order of multiplication you must change the sign, **A**×**B = – A**×**B**. The magnitude of this vector is largest when $\theta = 90^o$ (perpendicular). The vector is zero when $\theta = 0^o$ (parallel) or when $\theta = 180^o$ (anti-parallel).

Examples: In the following examples vector **A** = 3 and **B** = 2. The direction of vector **A** will vary.

Vectors A & B	Scalar (Dot) Product	Vector (Cross Product)
	$(3)(2)\cos 0^o = \boxed{6}$	$(3)(2)\sin 0^o = \boxed{0}$
	$(3)(2)\cos 30^o = \boxed{5.2}$	$(3)(2)\sin 30^o = \boxed{3.0}$
	$(3)(2)\cos 60^o = \boxed{3}$	$(3)(2)\sin 60^o = \boxed{5.2}$
	$(3)(2)\cos 90^o = \boxed{0}$	$(3)(2)\sin 90^o = \boxed{6}$
	$(3)(2)\cos 120^o = \boxed{-3}$	$(3)(2)\sin 120^o = \boxed{5.2}$
	$(3)(2)\cos 150^o = \boxed{-5.2}$	$(3)(2)\sin 150^o = \boxed{3}$
	$(3)(2)\cos 180^o = \boxed{-6}$	$(3)(2)\sin 180^o = \boxed{0}$

Another way: When cos (parallel) appears in a formula you need two vectors that are parallel (+) or anti-parallel (–). Just use your trig skills to find a component of one of the vectors that points in the same direction as the other vector. In the examples above, find the component of vector **B** that is parallel to vector **A**. When sin (perpendicular) appears in a formula, find the component of vector **B** that is perpendicular to vector **A**.

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1–3 Motion in Two Dimensions

Relative Velocity: Motion in two dimensions, both at constant velocity. A common example is that of a boat crossing a river. In figure 3.1, a boat leaves perpendicular to the shore with a velocity of v_B. The rivers current, v_R, carries it downstream a distance y. In figure 3.2 the boat aims at an angle of θ upstream in order to end up straight across. There is a triangle formed by the solid velocity vectors, and another formed by the dashed displacement vectors. These triangles are similar triangles. The resultant velocity of the boat will be a combination of the boats own velocity and current. This vector is labeled Σv. This is how fast the boat will appear to be moving as seen from a stationary observation point on shore. When calculating the time to cross the stream use the velocity vector and displacement vector that point in the same direction. In figure 3.1, $v_B = \frac{x}{t}$. In figure 3.2, $\sum v = \frac{x}{t}$

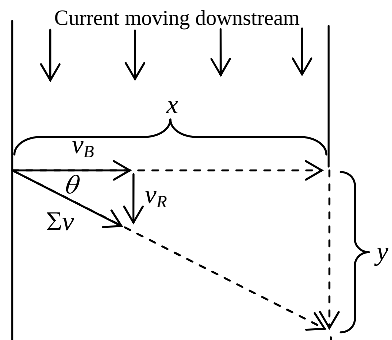


Fig 3.1

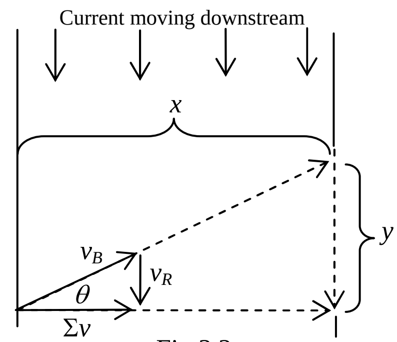


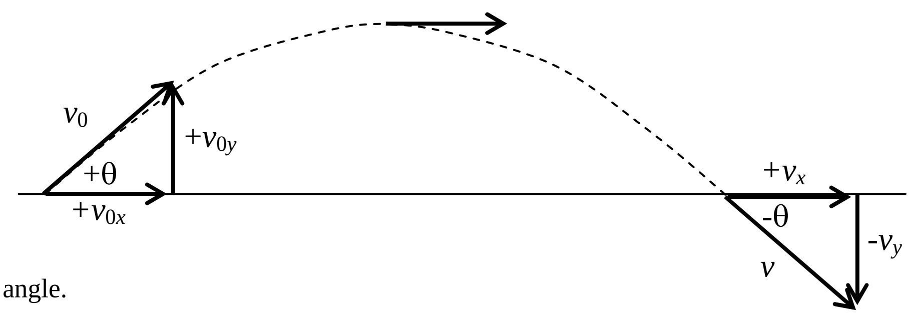
Fig 3.2

Projectile Motion: Motion in one dimension involves acceleration, while the other is at constant velocity.

- In the *x*-direction the velocity is constant, with no acceleration occurring in this dimension.
- In the *y*-direction the acceleration of gravity slows upward motion and enhances downward motion.

Vector Components in Projectile Motion: The *x*-direction and the *y*-direction are independent of each other.

Fig 3.3



Now v_0 is at an angle.

Solve for v_{0x}: $\boxed{v_{0x} = v_0 \cos\theta}$ then use v_{0x} in the kinematic equations to solve for v_x.

Solve for v_{0y}: $\boxed{v_{0y} = v_0 \sin\theta}$ then use v_{0y} in the falling body equations to solve for v_y.

v_x and v_y are component vectors. To find v, use Pythagorean Theorem $\boxed{v = \sqrt{v_x^{\,2} + v_y^{\,2}}}$ and arctangent $\boxed{\theta = \tan^{-1} \frac{v_y}{v_x}}$

The highest point in the flight: $\boxed{v_x = v_{0x}}$ and $\boxed{v_y = 0}$. If the problem ended here these conditions would apply.

Horizontal Launches

The launch angle is $\boxed{\theta = 0^o}$ $\quad v_{0x} = v_0 \cos\theta = v_0 \cos 0^o = v_0 \quad \boxed{v_{0x} = v_0}$

$$v_{0y} = v_0 \sin\theta = v_0 \sin 0^o = 0 \quad \boxed{v_{0y} = 0}$$

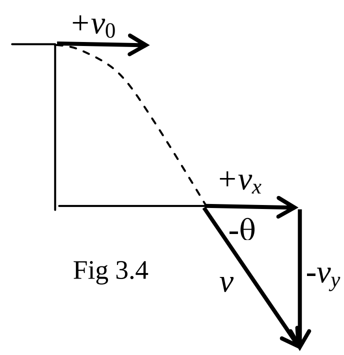


Fig 3.4

The above math is not really necessary. Inspection of the Fig 3.4 shows that v_0 is directed straight down the *x*-axis with no *y*-component vector visible at all. The end of the problem is similar to the problem depicted in Fig 3.3, above.

Coordinate Axis System provides the necessary orientation to handle the following variables and their appropriate signs: launch angle, initial velocities in x & y, final velocities in x & y, final landing height, and final overall velocity. Orientation matters and thus the coordinate axis becomes a powerful tool, as depicted on the next page.

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Initial Launch

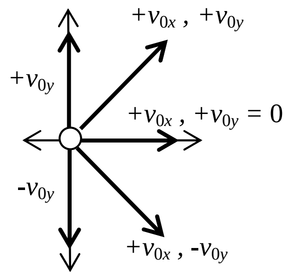

Fig 3.5

Initial displacement

$x_0 = 0$

$y_0 = 0$

Falling bodies: θ = ±90°

$v_{0x} = v_0 \cos 90^o = 0$

$v_{0y} = v_0 \sin 90^o = v_0$

Horizontal launch: θ = 0°

$v_{0x} = v_0 \cos 0^o = v_0$

$v_{0y} = v_0 \sin 0^o = 0$

1st quadrant launch: +θ

$v_{0x} = v_0 \cos\theta$ will be +

$v_{0y} = v_0 \sin\theta$ will be +

4th quadrant launch −θ

$v_{0x} = v_0 \cos\theta$ will be +

$v_{0y} = v_0 \sin\theta$ will be −

During the problem

(At the top $v_x = v_{ox}$ and $v_y = 0$)

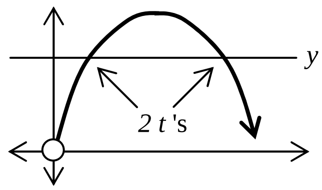

Fig 3.6

If it lands at the same height as it started ($y = y_0$), then $t_{up} = t_{down}$.

There are two t's for every y. The shorter t is for the upward trip. The longer t is for the downward trip.

Solve for maximum height two ways

1. From ground up where $v_y = 0$.

   $v_y^{\,2} = v_{0y}^{\,2} + 2g\left(y - y_0\right)$

2. Or the easy way. Start at the top and pretend it is a falling body. v_x doesn't matter since time is controlled by the y-direction. And at the top v_{oy} is zero. However, this solves for half of the total flight.

   $y = \frac{1}{2} g t^2$ ***Must double time!***

Final Landing

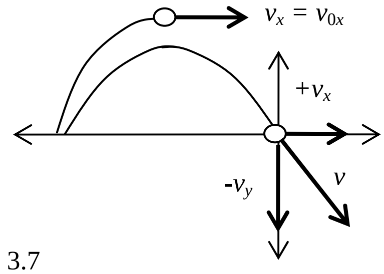

Fig 3.7

$+x$	Always
$-y$	Lands lower than y_0
$y = 0$	Lands same height as y_0
$+y$	Lands higher than y_0

$a_x = 0$ No a in the x-direction

$v_x = v_{0x}$

What is it doing at the end of the problem in the y-direction?

$\boxed{\pm v_y}$

It is usually moving downward at the end of the problem.

So v_y is ***usually negative***

The final $\mathbf{v}$ must be resolved.

$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$

If $y = y_0$

$v_y = -v_{0y}$ & $v = v_0$

Projectile Motion Strategies

1. ***Horizontal Launch:*** Since $v_{0y} = 0$ and $v_{0x} = v_0$, then use $\boxed{y = 1/2\ g t^2}$ and $\boxed{x = v_{0x} t}$.
2. ***When time (t) or range (x) is given:*** Start with $\boxed{x = v_{0x} t}$ and then $\boxed{y = y_0 + v_{0y} t + 1/2\ g t^2}$.
3. ***No x and no t:*** Time is the key to falling body & projectile problems. Two strategies are useful when time is missing.

1st $y = y_0 + v_{0y} t + 1/2\ g t^2$	1st $\boxed{v_y^{\,2} = v_{0y}^{\,2} + 2g\left(y - y_0\right)}$ solve for v_y which is usually ***negative***.
2nd Quadratic Equation	2nd $\boxed{v_y = v_{0y} + g t}$ use $-v_y$ from above to get t.
3rd $\boxed{x = v_{0x} t}$	3rd $\boxed{x = v_{0x} t}$ use t from above to solve for range x.

Other Projectile Motion Facts

Any ***two launch angles that add to 90°*** will arrive at the same landing site if fired on level ground. Examples: 15° and 75°, 30° and 60°, and 40° and 50°. ***Maximum range*** (maximum distance in the x-direction) is achieved by launching at 45° (45° + 45° = 90°). ***Maximum altitude*** (maximum distance in the y-direction) is achieved by firing straight up, at 90°.

Circular Motion

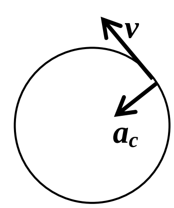

Fig 3.8

Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution, $\boxed{T = \frac{1}{f}}$. Time is in the numerator.

Velocity: In uniform circular motion the magnitude (speed) of the object is not changing. However, the direction is constantly changing, and this means a change in velocity (a vector composed of both magnitude and direction). In circular motion one can describe the rate of motion as either a speed or as a **tangential velocity** $\boxed{v = \frac{2\pi r}{T}}$. This velocity is an instantaneous velocity and it is directed tangent to the curve.

Centripetal Acceleration: The object is continually turning toward the center of the circle, but never gets there due to its tangential velocity. This centripetal (center seeking) change in velocity, is a centripetal acceleration $\boxed{a_c = \frac{v^2}{r}}$

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1–4 Newton's Laws of Motion and Force Vectors

Force: Any push or pull.

Newton's 1st Law: Law of inertia, a restatement of Galileo's principle of inertia. Inertia is controlled by mass, the more mass the more inertia. Essentially an object at rest wants to stay at rest, while an object in motion wants to stay in motion.

Newton's 2nd Law: $\boxed{\sum F = ma}$. The Greek sigma proceeding F is the mathematical notation for taking a sum. In reality there are countless forces interacting with every object in the known universe. When an object is standing still or moving at constant velocity all of these forces are counteracting and neutralizing each other. If a single additional force is applied (***applied force***) it can upset this balance forcing the object to change its inertia and accelerate. But, there may be more than one of these applied forces acting in a physics problem. These applied vector forces need to be added before a determination of the objects acceleration can be made.

Net Force: $\boxed{\sum F = ma}$

$\sum F_x$ is $\sum F$ in the ***x***-direction on the coordinate axis.

$\sum F_y$ is $\sum F$ in the ***y***-direction on the coordinate axis.

$\sum F_{\parallel}$ is the $\sum F$ parallel to a slope.

$\sum F_{\perp}$ is the $\sum F$ perpendicular to a slope.

Newton's 3rd Law: When two entities interact there is an equal and opposite force exerted on each object. Forces come in action-reaction pairs. For every ***action force*** there is an equal & opposite ***reaction force*** (***not*** an equal and opposite reaction). The reaction that is seen also depends on the mass of the object. If a high mass object encounters a low mass object, the one with less mass appears to be effected the most (either moves radically or sustains the most damage). This is due more to its low mass, since the force on both objects is the same.

Common Forces: In addition to the forces below, $F_{\textit{any subscript that makes sense}}$, can be used.

There are many vectors in physics: displacement, velocity, acceleration, force, momentum, gravity fields, electric fields, magnetic fields, etc. The last three mentioned, gravity, electricity, and magnetism are field forces. These are normally associated with invisible force fields. When you push on a box the force is visible and obvious. Gravity pulling down on the box, toward earth, is not visible and not as obvious. These invisible forces are called field forces and motion is along the field lines. ***Often these field forces are not specifically mentioned, but they are implied by the conditions in the problem***.

$\sum F$	$\sum F = ma$	***Sum of Force*** for linear motion, not used in circular motion.
F_1, F_2, …		***Applied Force***, some kind of push or pull.
F_g or W	$F_g = mg$	***Force of Gravity*** which is also known commonly as ***Weight***.
T		***Tension*** is the force that acts along strings, ropes, chains, etc.
N	$N = mg\cos\theta$	***Normal Force***: A contact force that exists when surfaces touch. It is always perpendicular to the surface. The angle θ refers to the angle of a sloping surface. On flat surfaces $\theta = 0^o$ and $\cos 0^o = 1$, so $N = mg$. However, this shortcut is only true as long as there are no applied forces or components of applied forces perpendicular to the surface. Additional y-forces or y-components of force can make the normal force one of the most difficult forces. See Ex. 4.4 on page 8.
f or F_{fr}	$f = \mu N$	***Force Friction***: A combination of the roughness of the surface, μ, and the amount of force pushing the surfaces against each other (the normal force), N.
F_c	$F_c = ma_c$	***Force Centripetal***: Sum of force for circular motion, not used in linear motion.
F_E	$F_E = qE$	***Force of Electricity***
F_B	$F_B = qvB$ $F_B = BI\ell$	***Force of Magnetism*** on a charge moving in a magnetic field. ***Force of Magnetism*** on a current carrying wire in a magnetic field.

From a vector stand point, it does not matter what the exact force is when solving force problems. A force is a force is a force. The variable changes, F_g, T, N, F_c, F_E, F_B, or F_1, F_2, F_3, but it is still force. And when we finish with forces and move to momentum the letter will change to p instead of F, but the vector problem solving technique remains the same.

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Free Body Diagram: Imagine that you are looking down at a box from above, and that four people are pulling on ropes in the direction of the four arrows in Fig. 4.1. A diagram that just shows the object and the forces immediately acting on the object (and nothing else) is known as a free body diagram. As drawn the length of each force vector is an indication of its strength. Obviously persons 1 and 4 are stronger than 2 and 3. In free body diagrams the arrows do not have to be drawn proportionally, they just need to be pointing the correct way and be correctly labeled. The vectors are drawn coming out of the body. This allows an imaginary coordinate axis, shown in Fig. 4.1 by the dashed lines, to be superimposed through the center of the object. This is used to reference the direction of each force vector. Vector direction is simplified as + and – signs can be used to indicate direction along any axis. (In vector math the magnitudes +2 and –2 are equal. They are just opposite in direction.)

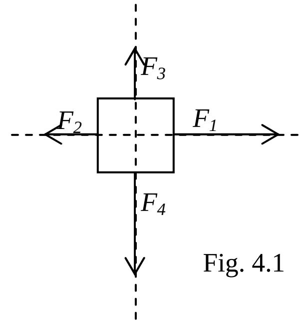

Fig. 4.1

Hints to Complete Successful Free Body Diagrams

1. Identify any invisible force fields like gravity, electricity, and magnetism. Gravity is in every problem and points down.
2. If there is a string or rope, a force of tension exists along the string or rope and it is directed away from the object.
3. If there is a surface or contact point, there is a normal force directed away from and perpendicular to the surface.
4. Applied forces are the most obvious. They are in the text of the problem and may also be in any diagrams if provided.

Finding the Force Resultant (Sum of Force) in Various Situations

Example 4-1: All Vectors Lie Along an Axis.

Look at Fig. 4.1 and suppose that $F_1 = 5$ N, $F_2 = 2$ N, $F_3 = 2$ N, $F_4 = 5$ N.

1. Draw a Free Body Diagram (FBD), as shown in Fig. 4.1.
2. Write vector sum equation in any relevant direction(s). $\sum F_x = F_1 - F_2$ $\quad \sum F_y = F_3 - F_4$
3. Substitute known equations and/or values. $\sum F_x = 5 - 2 = 3$ $\quad \sum F_y = 2 - 5 = -3$
4. Solve. $\sum F_x = 3$ $\quad \sum F_y = -3$
5. If needed: Resolve components into a resultant. $\sum F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(3)^2 + (-3)^2} = \boxed{4.24N}$
6. If needed: Solve for the direction. $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-3)}{(3)} = \boxed{-45^o}$

Example 4-2: Converting Vectors at Angles Into Components on the Coordinate Axis.

In Fig 4.2, suppose that $F_1 = 5$ N at 40°, $F_2 = 2$ N at 230°, $F_3 = 2$ N at 70°, $F_4 = 5$ N at –60°.

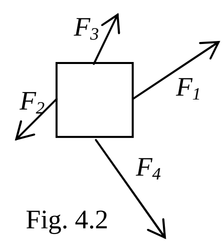

Fig. 4.2

1. Draw a FBD, as shown in Fig. 4.2.
2. State the vector sum equation in direction that matters
   $\sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$ $\quad \sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$
3. Substitute known equations and/or values
   $\sum F_x = F_1\cos\theta + F_2\cos\theta + F_3\cos\theta + F_4\cos\theta$
   $\sum F_y = F_1\sin\theta + F_2\sin\theta + F_3\sin\theta + F_4\sin\theta$
   $\sum F_x = 5\cos 40^o + 2\cos 230^o + 2\cos 70^o + 5\cos -60^o$ $\quad \sum F_y = 5\sin 40^o + 2\sin 230^o + 2\sin 70^o + 5\sin -60^o$
4. Solve $\sum F_x = 5.73N$ $\quad \sum F_y = -0.769N$
5. If needed: Solve for the total with Pythagorean theorem $\sum F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(5.73)^2 + (-0.769)^2} = \boxed{5.78N}$
6. If needed: Solve for the direction $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-0.769)}{(5.73)} = \boxed{-7.64^o}$

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Example 4-3: Using a Flexible Coordinate Axis System.

It will be easier in more complex problems if we set the direction that an object is moving to be positive. Once this direction is declared as positive all vectors pointing that way are positive, and those in the opposite direction are negative.

x-direction:

If object is moving right then,
direction of motion is right.
Right vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

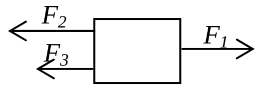

Fig 4.3a

If object is moving left, then
direction of motion is left.
Left vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

y-direction:

If object is moving up, then
direction of motion is up.
Up vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

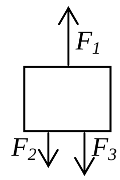

Fig 4.3b

If object is moving down, then
direction of motion is down.
Down vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

Note: The force vectors have already been assigned as positive and negative. So, ***plug in only positive numbers***, such as $g = +9.8$. ***Do not plug in negatives***, like, -9.8, as this will reverse the vector direction decided in the sum of force equation.

Falling Bodies and Projectile Motion Revisited: Try the above method for a falling body that is dropped at rest, or a projectile that is fired horizontally or downward. A vector is a vector, so whether it is force, velocity, or acceleration etc. use the same technique. A falling body is moving downward. So set the direction of motion as positive, thus down becomes positive. Any vector pointing down is now positive.

$y = \frac{1}{2} g t^2$ Both g and y point down and are positive. Everything is now positive. You are in charge of the problem. It does not matter what direction is positive. It matters that there is sign consistency. If down is defined to be positive then any vector pointing upward better have a positive. The beauty of using the direction of motion as positive and assigning plusses and minuses to the vector quantities is that you get to plug in positive values. ***(This is difficult to do in a falling body or projectile motion problem that travels in one direction(up) at the start of the problem and another direction (down) at the end of the problem. For these problems it may be easier to use the rigidly defined coordinate axis system with g as -9.8 that you have previously learned).***

Example 4-4: Deciding On The Relevant Direction.

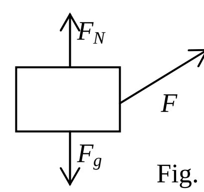

Fig. 4.4

Suppose a 10.0 kg box, in Fig. 4.4, is pulled along a flat surface by a 20.0 N force at a 30.0^o angle with the horizontal. This problem is a little different than the previous ones. The *x*-direction and the *y*-direction are doing different things. The object is accelerating in the *x*-direction, while it is not moving in the *y*-direction. We can solve for the acceleration and for the normal force using the sum of forces in the relevant direction for each of these quantities.

1. Draw a FBD
2. Find the vector sum equation(s). This box will accelerate in the *x*-direction, so if acceleration data is requested, sum the forces in the *x*-direction. If the problem asks for a force in the *y*-direction, such as force normal, sum the forces in the *y*-direction.

   $$\sum F_x = F_x \qquad \sum F_y = N + F_y - F_g$$

3. ***<u>Ask yourself what the object is doing in the relevant direction.</u>*** There are three choices:
   1. Standing still, $a = 0$. 2. Constant velocity, $a = 0$. 3. Accelerating, $a = a$.
   This problem is accelerating in the *x*-direction, while standing still in the *y*-direction.
4. Substitute known equations and values. *The known equations come from the table on page 6.*

   $$ma_x = F\cos\theta \qquad ma_y = N + F\sin\theta - mg$$

   $$10.0a_x = 20.0\cos 30^o \qquad 10.0(0) = N + 10.0\sin 30^o - (10.0)(9.8)$$

5. Solve

   $$a_x = \frac{20.0\cos 30^o}{10.0} \qquad N = (10.0)(9.8) - 10.0\sin 30^o$$

   $$a_x = \boxed{1.73\,m/s^2} \qquad N = \boxed{93.0N}$$

What Is The Object Doing?
It is critical to know how the object is moving.

1. Inertia: The object is following the principle of inertia. It lacks acceleration.
   a. Constant Velocity: $a = 0$, which means that $\sum F = ma = 0$.
   b. Standing Still: $a = 0$, $\sum F = ma = 0$. Standing still, $v = 0$, is just a constant velocity that happens to be zero.
2. Accelerating: $\sum F = ma$. a is solved using this equation, or by other means and then is plugged into this equation.

Example Scenario:

1. Standing still in a car: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = ma = 0$ $\quad$ Inertia.
2. Step on gas peddle: $\Delta v = +$ $\quad a = +$ $\quad \sum F = F_{engine} = ma = +$ $\quad$ Accelerates.

   Initially there is only one applied force acting in this problem, and it is the cars engine (really it is friction between tires and road). This force is in the direction of motion and is a positive force causing positive acceleration. But as soon as the car starts moving it experiences air resistance. It will continue to accelerate as long as the sum of force is positive.

   $\Delta v = +$ $\quad a = +$ $\quad \sum F = F_{engine} - F_{air} = ma = +$ $\quad$ Accelerates.
3. Reach cruising speed: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = F_{engine} - F_{air} = ma = 0$ $\quad$ Inertia.

   Why did it reach a constant velocity with the gas peddle still pressed? The car encounters air resistance, a second force. Air resistance is in the opposite direction and is negative. At first the air resistance is small, but grows until it equals the force of the engine. Adding the positive force of the engine to the now equal negative force of air resistance results in a $\sum F = ma = 0$, and inertia / constant velocity takes over. Note: equal and opposite force does not mean that the car stops. It continues doing whatever it was last doing, i.e., it moves with the final velocity from the acceleration phase.
4. Step on the brake: $\Delta v = -$ $\quad a = -$ $\quad \sum F = -F_{brake} - F_{air} = -ma = -$ $\quad$ Decelerates.

   The brakes (again, really friction) apply a force that opposes the direction of motion. Now there are two forces slowing the car. These negative forces decelerate the car.
5. Stopped: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = ma = 0$ $\quad$ Inertia.

Example 4-5: Balanced Forces

The forces parallel to the relevant direction add to a sum of force of zero.

As an example: All the forces in the Fig. 4.5 are parallel to the x-axis.

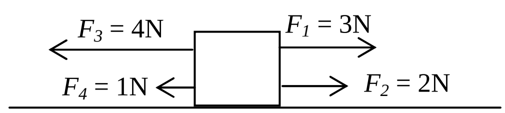

Fig. 4.5

The sum of force is: $\sum F = F_1 + F_2 - F_3 - F_4$

$$\sum F = 3 + 2 - 4 - 1 = 0$$

The object does not accelerate, but rather has constant velocity (which includes $v = 0$).
<u>This implies that constant velocity and standing still are the result of balanced forces, and this can be used as a shortcut.</u>

What if the problem were worded differently?

The box in Fig. 4.6 is moving at constant velocity. What is F_4?

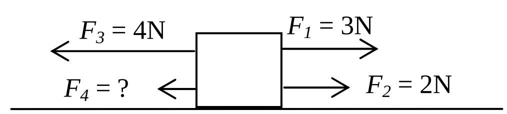

Fig. 4.6

Constant velocity implies that $\sum F = ma = 0$.
Now we know that this implies a balanced force scenario as well.

Solve using the ***sum of forces approach***.

$$\sum F = F_1 + F_2 - F_3 - F_4$$
$$0 = F_1 + F_2 - F_3 - F_4$$
$$F_1 + F_2 = F_3 + F_4$$
$$F_4 = F_1 + F_2 - F_3$$
$$F_4 = 3 + 2 - 4$$
$$F_4 = 1$$

Solving using ***balance force approach***.

All the forces in one direction (added together) must equal all the forces in the opposite direction (added together).

$$F_1 + F_2 = F_3 + F_4$$
$$F_4 = F_1 + F_2 - F_3$$
$$F_4 = 3 + 2 - 4$$
$$F_4 = 1$$

The balanced force approach is faster, as it eliminates the first two steps. ***<u>However, when an object is accelerating balanced forces do not apply, and the problem must start with a sum of forces equation.</u>***

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1–5 Applications of Force

Strategy for Force Problems

1. Draw ***FBD***.
2. Decide direction of motion. Considered this the positive direction. ***If the object does not move, ask which direction it would move if it were free to do so, and set this as the positive direction.***
3. **Which direction matters, the x or the y-direction? What is it doing in the direction that matter?**
4. Construct $\sum F$ equations in the relevant direction, by looking at the ***FBD***. Any force vectors or components of force vectors pointing in the direction of motion are positive while any vectors or components opposing motion are negative.
5. Substitute known equation, $\sum F = ma$, $F_g = mg$, from the table on page 6, into the sum of force equation.
6. Plug in values and solve. All values including 9.8 are positive since the plusses and minuses have already been decided.

Example 5-1: Static Force Problems, and Force Triangles

In beginning physics many force problems contain a force triangle.

A mass is suspended by two strings from the ceiling, as shown in Fig 5.1a. Fig. 5.1b is the FBD. In Fig 5.1c the tensions are separated into x *and* y components. Sum the forces in the x and y-directions separately. The sum of force will be equal to zero.

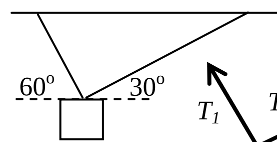

Fig 5.1a

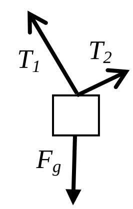

Fig 5.1b

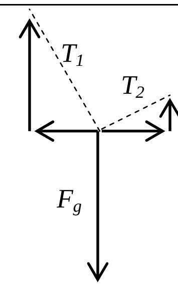

Fig 5.1c

$$\sum F_x = T_{2x} - T_{1x} \qquad \boxed{T_{1x} = T_{2x}}$$

$$\sum F_y = T_{1x} + T_{2y} - F_g \qquad \boxed{T_{1x} + T_{2y} = F_g}$$

Under the right circumstances the three force vectors form a right triangle when added tip to tail. Note the 30° and 60° angles above. The forces complete the 30-60-90 triangle shown in Fig 5.1d. The sum of force is zero. Tensions can be calculated using SOH CAH TOA and Pythagorean Theorem.

$$\boxed{T_1 = F_g \cos 30^o} \quad \boxed{T_2 = F_g \sin 30^o} \qquad \text{or} \qquad \boxed{T_1 = F_g \sin 60^o} \quad \boxed{T_2 = F_g \cos 60^o}$$

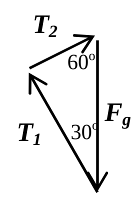

Fig 5.1d

Example 5-2: Lawn Mower

An old fashioned lawnmower is pushed with 90N at a 45° angle against a horizontal retarding force. Fig 5.2a is the FBD, while Figure 5.2b is a diagram of horizontal and vertical component vectors

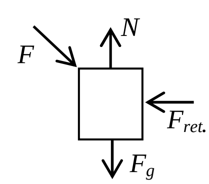

Fig 5.2a

Solve for the Retarding Force

$$\sum F_x = F_x - F_{ret.} \qquad 0 = F_x - F_{ret.} \qquad F_{ret.} = F_x = 90N\cos 45^o = \boxed{63.6N}$$

Solve for the Normal Force

$$\sum F_y = -F_y + N - F_g$$

$$N = \sum F_y + F_y + F_g \qquad N = 0 + \left(90N\sin 45^o\right) + \left[\left(16kg\right)\left(9.8\,m/s^2\right)\right] = \boxed{220N}$$

Solve for F to accelerate from rest to 1.5 m/s in 2.5 s

$$v_x = v_{x_o} + a_x t \qquad a_x = \frac{v_x - v_{x_o}}{t} = \frac{1.5-0}{2.5} = 0.6\,m/s^2 \qquad \sum F_x = ma_x = \left(16kg\right)\left(0.6\,m/s^2\right) = \boxed{9.6N}$$

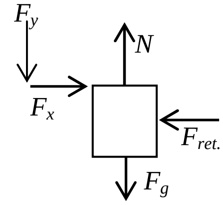

Fig 5.2b

You need this force to accelerate, but you still need to overcome the retarding force.

$$\sum F_x = F_x - F_{ret.} \qquad F_x = \sum F_x + F_{ret.} = 9.6N + 63.6N = 73.2N$$

But you aren't pushing in the x-direction. You need the push at 45° to generate 73.2 N in the x-direction.

$$F_x = F\cos 45^o \qquad F = \frac{F_x}{\cos 45^o} = \frac{73.2N}{0.707} = \boxed{104N}$$

Apparent Weight: When you ride in an elevator upward you will feel heavier when the elevator accelerates and lighter as it slows to a stop. When riding downward you will feel lighter when it accelerates and heavier when it stops. When you ride a roller coaster you experience the same sensations when moving up and down. On "Superman the Ride" you can actually feel weightless. Astronauts and pilots experience these same sensations when moving away or toward the center of the earth (or the moon). Astronauts also feel weightlessness as well. This effect is not limited to the vertical or y direction.

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When you “step on it” in a car you feel yourself pressed into the seat, and when you panic stop you feel yourself thrown forward. These sensations have the same characteristics as gravity or weight. Up to now we have analyzed the motion of the car, the plane, the rocket, etc. using straight forward force and kinematics. <u>With apparent weight we are dealing with a false force that the passenger feels</u>. The effect is really created by the passenger’s inertia. When you “step on it” you don’t sink into the car seat. You actually follow inertia and stay at rest, while the car hits you from behind. The real acting forces are the opposite of what our brain thinks. ***To analyze this feeling of force for a passenger we need apparent weight***. Apparent weight is the weight that would show on a bathroom scale if you were between you and the surface causing the force. Contact with any surfaces is a normal force and bathroom scales measure normal force. So weight apparent is also F_N.

$\boxed{F_{g_{apparent}} = mg \pm ma}$ This equation adds the acceleration of a passenger’s vehicle to the real weight.

x* direction:** $F_{g_{apparent\ x}} = mg_x \pm ma_x$, but with no ***g in the ***x*** direction $\boxed{F_{g_{apparent\ x}} = \pm ma_x}$

Positive: accelerating, you feel heavier. Negative: decelerating, you feel lighter.

***y* direction:** $\boxed{F_{g_{apparent\ y}} = mg_y \pm ma_y}$, so acceleration adds / subtracts from the real / actual weight.

Positive: moving away from the center of gravity (up or away from Earth). Negative: moving toward the center of gravity.

Why do you feel weightless on “Superman the Ride”? Because, the acceleration is downward and matches gravity.

$$F_{g_{apparent\ y}} = mg_y \pm ma_y = m(9.8) - m(9.8) = 0\,m/s^2$$

g’s: The acceleration of gravity can also be expressed in g’s. $\boxed{1g = 9.8\,m/s^2}$ This is commonly used in flight terminology.

Example 5-3: Compound Bodies, One Dimension

These problems have two or more masses connected by a string or pressed against each other.

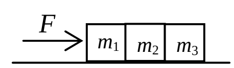


Fig 5.3a

In Fig 5.3a a force F presses against block 1 which presses with a normal force against block 2 which then presses with a normal force on block 3.

If all the blocks are the same mass what is the acceleration of the blocks? Simply treat all the masses as one larger block, and remember to use the sum of force to find acceleration.

$$\sum F_{system} = F \qquad m_{system}a = F \qquad (m_1 + m_2 + m_3)a = F \qquad \boxed{a = \frac{F}{(m_1 + m_2 + m_3)}}$$

How much force F is acting on each block? The force will distribute proportionally based on mass. If the blocks are of equal mass then the force on each of the three blocks will be one third. $\boxed{F_1 = \frac{F}{3}}$ $\boxed{F_2 = \frac{F}{3}}$ $\boxed{F_3 = \frac{F}{3}}$

If the blocks do not have the same mass then you must distribute the force using the mass ratio.

How much force is acting at the boundary between the blocks? This force is a normal force as it is created by surface contact. Block 1 requires one third of F to move. So two thirds of F remain to push blocks 3 and 4. The force at the boundary between blocks 1 and 2 is two thirds F, and this is the force needed to push the blocks behind the boundary. The last block C only requires one third of force F and thus the force at the boundary between blocks B and C is one third F. This may make more sense when examined vertically. ***If the same blocks are stacked on a table one could imagine that they are gymnasts standing on each other’s shoulders.*** How much of the total force F is felt on the bottom gymnasts feet? All of the force. How much is at the boundary? The boundary can be analyzed from either surface of contact. If we look at the bottom gymnasts shoulders, he must push up with 2/3 F to hold 2/3 m. The middle gymnasts feet must support his own weight and that of the gymnast above, so his feet must push with 2/3 F. No matter how you look at the boundary you arrive at the same answer. The boundary between the middle and top gymnast is 1/3 F, as the middle gymnast shoulder and top gymnasts feet only need to support the top gymnasts 1/3 m.

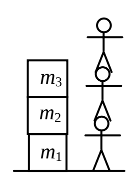


Fig 5.3b

The problem is exactly the same if the blocks are tied by strings and hanging from the ceiling. Only now we s[illegible]e for tension instead of force normal, and the gymnasts are hanging from a cliff. If the top rope (top arm) in Fig 5.3c has a tension of F, what is the tension between blocks 1 and 2? 2/3 F. The top gymnast lower arm and the middle gymnasts upper arm both have to support 2/3 m. How much tension is in the rope between blocks 2 and 3? 1/3 F

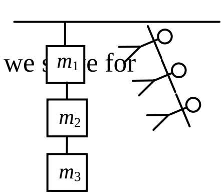
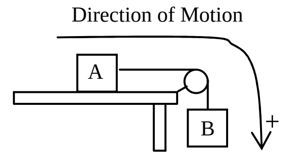
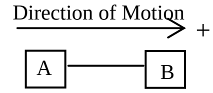


Fig 5.3c

Compound Bodies in Two Dimensions: These problems have two or more masses connected by a strings and involve pulleys. Pulleys are devices that change the direction of force. The pulleys in the following examples will be considered massless and frictionless, and as a result they do not change the magnitude of force. As two dimensions are involved simultaneously the assignment of positive and negatives on the various forces can be tricky. The easiest method to achieve sign consistency throughout a complex problem is to identify the direction the masses are moving, or the direction they are most likely to move. ***Set the direction of motion as positive***, as shown in the top figure to the right. Every vector in the direction of motion will be considered positive. Those opposed to the direction of motion are negative. If you are not sure what the direction of motion will be, take a guess. If you calculate a negative value for acceleration, you were wrong, the masses actually moved in the direction opposite your prediction. ***A very useful shortcut is to reorient the problem into one dimension.*** Pulleys are devices that change the direction of force. So pretend the pulley is not there, as shown in the figure at the bottom right.





Example 5-4: Compound Body Moving in Two Dimensions

Solve for acceleration: Fig 5.4a shows the scenario. As there are two masses there are two FBD's shown in Fig 5.4b. Fig 5.4c is an informal sketch of connected boxes. Use this sketch and the combined mass method to solve for overall acceleration. ***<u>Remember, when you use the combined method you must total all the masses for the sum of force</u>.*** F_g and F_N acting on mass A are perpendicular to motion. They cancel each other. Fig 5.4c shows that the ***<u>tension in the rope also cancels</u>***. It is the same rope so the value at both ends is the same and the direction of tension is opposite. So tension cancels in the shortcut to find acceleration.

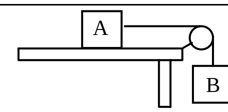


Fig 5.4a

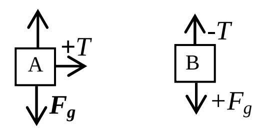


Fig 5.4b

$$\sum F_{AB} = F_{gB} \qquad (m_A + m_B)a = m_B g \qquad \boxed{a = \frac{m_B g}{(m_A + m_B)}}$$

Solve for the tension in the rope: In order to solve for tension you need a formula with tension in it. You must solve for one of the masses by itself. Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{frA} \quad \text{or} \quad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + \mu m_A g} \qquad \boxed{T = m_B g - m_B a}$$

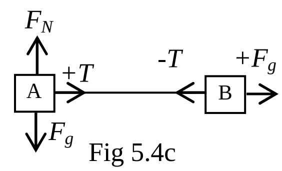


Fig 5.4c

Plug the acceleration from part one into either equation above and you will get the same final answer.

Example 5-5: Atwood Machine:

Atwood created a device to artificially slow the acceleration of gravity. In Fig 5.4a it doesn't say which mass is greater. I picked the two masses B and C as the more massive side and used this to set the direction of motion.

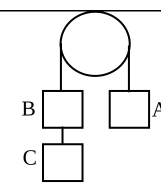


Fig 5.5a

Solve for acceleration: The FBD's for all blocks are shown in Fig 5.5b. Use the combined mass method to solve for overall acceleration. *<u>Remember, when you use the combined method you must total all the masses for the sum of force</u>*. In addition it is easier if you treat blocks B and C as though they are one larger block having a single mass. Fig 5.5c is a sketch of the masses as a linear problem, with the left masses combined.

$$\sum F_{ABC} = F_{gBC} - F_{gA}$$

$$(m_A + m_B + m_C)a = (m_B + m_C)g - m_A g \qquad a = \frac{(m_B + m_C)g - m_A g}{(m_A + m_B + m_C)}$$

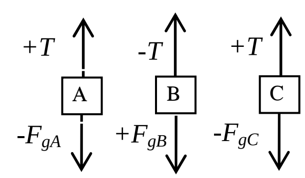


Fig 5.5b

If asked for the tension in the rope connecting mass A and B you must sum the forces for any block connected to the rope. A problem might only give information for one of the two blocks, or one of the blocks will be much simpler to solve. Learn to identify the easy block. If more than one mass is suspended by a rope, then add the masses suspended by the rope. This is the case for blocks B and C. Both of the possible solutions are detailed, one using block A on the left, and the other using blocks B and C to the right.

$$\sum F_A = T - F_{gA} \quad \text{or} \quad \sum F_{BC} = F_{gBC} - T$$

$$T = \sum F_A + F_{gA} \qquad T = F_{gBC} - \sum F_{BC}$$

$$\boxed{T = m_A a + m_A g} \qquad \boxed{T = (m_B + m_C)g - (m_B + m_C)a}$$

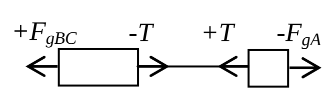
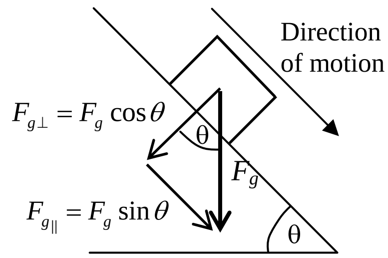


Fig 5.5c

Plug in the acceleration from above to solve for F_T. To solve for the tension in the rope between block B and C, just use the mass of block C. Refer to the right most FBD in Fig 5.5b.

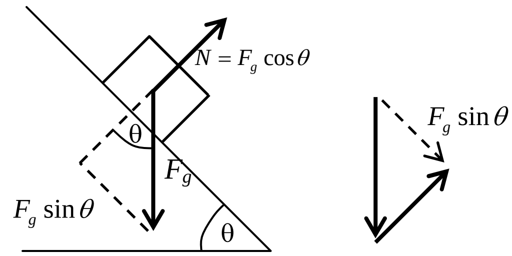
$$\sum F_C = F_{gC} - T \qquad T = F_{gC} - \sum F_C \qquad \boxed{T = m_C g - m_C a}$$

Force Gravity on Slopes: Motion on a slope is parallel to the slope. F_g is then at an angle to this motion. Any vector at an angle to motion should be split into components parallel and perpendicular to the chosen orientation (direction of motion). In this case F_g should split into $F_{g\,||}$ and $F_{g\perp}$ to the slope. F_g is actually pulling the block into the slope $F_{g\perp} = F_g \cos\theta$ and down the slope $F_{g_{||}} = F_g \sin\theta$.



But the block is not moving perpendicular to the slope, so the sum of forces in the perpendicular direction is zero. If gravity is pulling the block into the slope then the slope must push back with an equal and opposite force. Any force involving contact with a surface is call a **Normal Force, *N***. In slope problems $\boxed{N = F_g \cos\theta}$.

When a block moves on a flat surface the slope angle is 0^o. Plug 0^o into the N equation and it reduces to $\boxed{N = F_g}$. This is an abbreviation for N that works only on flat surfaces. The component of gravity parallel to the slope $\boxed{F_{g||} = F_g \sin\theta}$ is then left over. $F_{g\,||}$ is the resultant when F_g and N are added by vector addition. This is the force of gravity that can accelerate objects down slopes.



Example 5-6: Combined Bodies and Slopes

Solve for acceleration: This is just like Example 5-4, only this time block A is on a slope. Just use the same problem solving approach and recognize that gravity is different on slopes. Fig 5.7a diagrams the problem. Fig 5.7b shows the FBD's for the two masses. Make a note of the orientation of the vectors for block A. Fig 5.6c depicts the problem as a linear problem. As usual gravity is pulling block B, and as it is felt entirely in the y direction, it is acting with full strength. However, gravity is acting at an angle to the motion of block A on the slope. ***We need the component of gravity acting in the direction of motion, $F_g\sin\theta$, in order to solve the problem.***

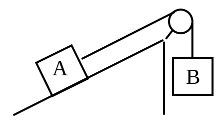


Fig 5.6a

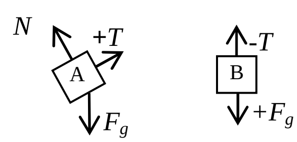


Fig 5.6b

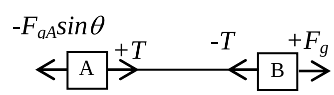


Fig 5.6c

$$\sum F_{AB} = F_{g\,B} - F_{gA}\sin\theta$$

$$(m_A + m_B)a = m_B g - m_A g \sin\theta \qquad \boxed{a = \frac{m_B g - m_A g \sin\theta}{(m_A + m_B)}}$$

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{gA}\sin\theta \quad \text{or} \quad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + m_A g \sin\theta} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Friction: Opposes motion and is always negative. Motion is always parallel to a surface, so friction always acts parallel.

Static Friction: Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction). ***Oddly enough the maximum value for static friction is measured just as the object breaks loose and begins to move.*** *Static friction is the strongest friction since the surfaces have a stronger adherence when stationary*.

Kinetic Friction: Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still opposes motion.

Coefficient of friction: μ is a value of the adherence or strength of friction. μ_k for kinetic and μ_s for static.

$\boxed{f = \mu N}$ so $\boxed{f = \mu mg\cos\theta}$ On flat surfaces $\theta = 0^o$, $\boxed{f = \mu mg}$

Example 5-7: Two Dimension Compound Body with Friction

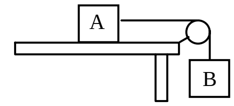


Fig 5.7a

Solve for acceleration: This is a repeat of Example 5-4, only this time friction appears in the FBD for mass A. Friction opposes motion and is therefore negative.

$\sum F_{AB} = F_{gB} - F_{frA}$ $\qquad (m_A + m_B)a = m_B g - \mu m_A g$ $\qquad \boxed{a = \dfrac{m_B g - \mu m_A g}{(m_A + m_B)}}$

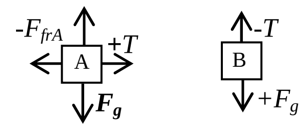


Fig 5.7b

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$\sum F_A = T - F_{frA}$ $\qquad$ or $\qquad \sum F_B = F_{gB} - T$

$\boxed{T = m_A a + \mu m_A g}$ $\qquad \boxed{T = m_B g - m_B a}$

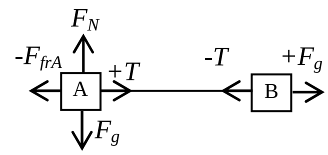


Fig 5.7c

Plug the acceleration from part one into either equation above and you will get the same final answer.

Example 5-8: A Complex Slope Problem

Objects can move up or down a slope. $\boxed{F_g \sin\theta}$ is simply the component of gravity pulling on an object down the slope in a direction parallel to the slope. In Fig 5.8a a man is pushing mass B up a slope. In this case gravity is opposite the direction of motion, as is friction. The FBD's for both blocks are shown in Fig 5.8b, with the main difference being the presence of the true forces, F_g and F_N instead of $F_g \sin\theta$. Remember, F_g and F_N are summed to create $F_g \sin\theta$. The direction of motion (positive direction) is up the slope.

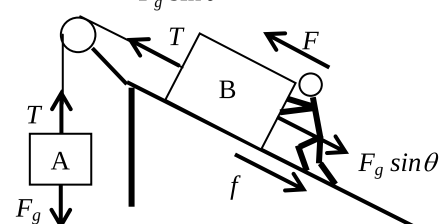


Fig 5.8a

$\sum F_A = F_{gA} - T$ $\qquad \sum F_B = F + T - F_{gB}\sin\theta - f_B$

$T = F_{gA} - \sum F_A$ $\qquad T = \sum F_B - F + F_{gB}\sin\theta + f_B$

$$F_{gA} - \sum F_A = \sum F_B - F + F_{gB}\sin\theta + f_B$$

$$m_A g - m_A a = m_B a - F + m_B g\sin\theta + \mu m_B g\cos\theta$$

$$m_A a + m_B a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

$$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

If constant velocity $\qquad \boxed{0 = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}$

If accelerating $\qquad \boxed{a = \dfrac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

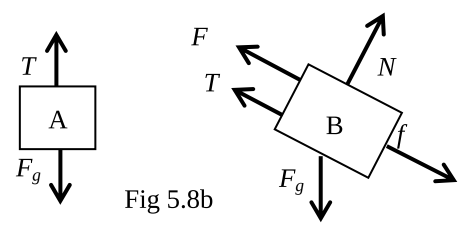
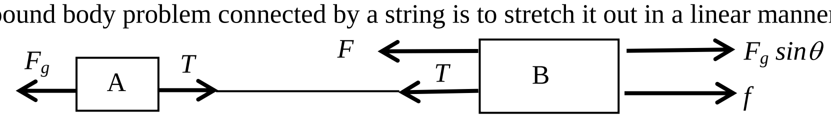


Fig 5.8b

An easier way to solve any compound body problem connected by a string is to stretch it out in a linear manner as shown in Fig 5.8c.

Fig 5.8c


The direction of motion is to the left, so all left arrows are positive, and all right arrows are negative. The string attaching the two boxes is the same and the two equal and opposite tensions cancel each other out. This allows you to solve for an overall Force Net in fewer steps. ***<u>The main difference here is that the mass in the sum of force substitution is the total mass of the whole problem</u>***.

$$\sum F = F_{gA} + F - F_{gB}\sin\theta - f_B$$

$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$, which is identical to the sixth line in the longer version above.

Constant velocity $\boxed{0 = m_A g + F_P - m_B g\sin\theta - \mu m_B g\cos\theta}$ or accelerating $\boxed{a = \dfrac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

However, this is not going to solve for the force of tension in the string. You must select one of the masses and work with its FBD. Block A obviously has the simpler of the FBD, so choose it.

$\sum F_A = F_{gA} - T$ $\qquad T = F_{gA} - \sum F_A$ $\qquad \boxed{T = m_A g - m_A a}$

If you are lucky the blocks are standing still or moving at constant velocity, in which case a = 0. If not use the a from above.

1–6 Work, Energy, and Power

Work: Force applied to an object that moves a distance.

$\boxed{W = \mathbf{F} \bullet \Delta \mathbf{r} = F \Delta r \cos\theta}$ θ is the angle between direction of motion and applied force.

Work is a Scalar (Dot) Product: The dot product (**A·B**) of two vectors **A** and **B** is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Work can be solved with either version of the formula. We will use both in example 6–1 below. The formula $W = \mathbf{F} \bullet \Delta \mathbf{r}$ involves vectors, which are annotated in bold print. Vectors have both magnitude and direction. The positive and negative values of force and displacement are important when using this version of the formula. The other version $W = F \Delta r \cos\theta$ involves italicized print indicating that only the magnitude of each vector is needed. Only positive numbers are used. The angle in the formula is the angle measured between the two vectors. In this version of the formula $\cos\theta$ solves the directional aspect. Compare both methods in the example below.

Example 6-1: Various Orientations of Force and Displacement.

In the first three scenarios a force $F = 5$ N acts on a mass which is displaced $r = 2$ m.

1. Force vector is parallel to the displacement vector and points in the same direction:

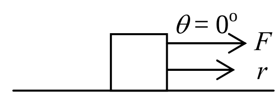


Fig 6.1a

$W = \mathbf{F} \bullet \Delta \mathbf{r}$

$W = (+5)(2)$

We decided F was positive

$W = (+5)(2) = \boxed{10\,\text{J}}$

$W = F \Delta r \cos\theta$

$W = (5)(2)\cos 0^o$

Here the angle solves for the positive

$W = (5)(2)(+1) = \boxed{10\,\text{J}}$

2. Force vector is parallel to the displacement vector and points in the opposite direction:

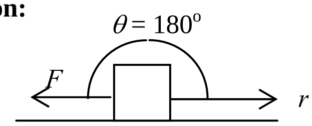


Fig 6.1b

$W = \mathbf{F} \bullet \Delta \mathbf{r}$

$W = (-5)(2)$

We decided F is negative

$W = (-5)(2) = \boxed{-10\,\text{J}}$

$W = F \Delta r \cos\theta$

$W = (5)(2)\cos 180^o$

Here the angle solves for the negative

$W = (5)(2)(-1) = \boxed{-10\,\text{J}}$

Forces opposing motion are negative, and are associated with negative acceleration and negative work.

3. Force and displacement vectors are perpendicular:

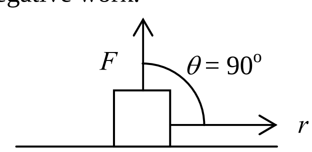


Fig 6.1c

$W = \mathbf{F} \bullet \Delta \mathbf{r}$

$W = (0)(2)$

We decided F has no affect on motion

$W = (0)(2) = \boxed{0\,\text{J}}$

$W = F \Delta r \cos\theta$

$W = (5)(2)\cos 90^o$

Here the angle solves for the zero

$W = (5)(2)(0) = \boxed{0\,\text{J}}$

Forces acting perpendicular to motion have no affect in the direction of the original motion. They do not speed up or slow the object in the direction being investigated. The object experiences inertia (stays at rest or continues at constant velocity) in the direction it was originally moving. No work is done in the original direction of motion. The force may accelerate the object in the perpendicular direction. However, this will no affect the direction of motion.

4. Force and displacement vectors are at angles other than parallel or perpendicular:

In this scenario $F = 5$ N at 37^o and $r = 2$ m

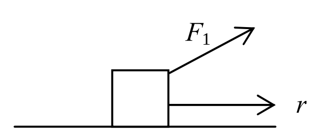


Fig 6.1d

$W = \mathbf{F} \bullet \Delta \mathbf{r}$

To use this formula you need a component of F parallel to r

$W = F_x r$

$W = (5\cos 37^o)(2) = \boxed{8\,\text{J}}$

$W = F \Delta r \cos\theta$

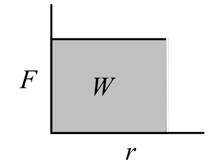
$W = (5)(2)\cos 37^o$

$W = (5)(2)(0.8) = \boxed{8\,\text{J}}$

Both methods result in the same calculations. They are just derived with slight differences in problem solving logic. I prefer using the formula on the left. I am in the habit of searching out my own components anyway. I know that any components perpendicular do not matter, since W = 0 in these cases. So I look for components of force that match the displacement. You can also look for components of displacement that match force. I prefer finding the correct component, since the formula on the right requires a specific angle that is not always the given angle in the problem. By solving for components I take control of the problem and avoid plugging in wrong angles.

Work is the Area Under the Force Displacement Curve

This is the integral of the force distance function in the calculus based course. In the non-calculus course these areas will be simple enough (squares, rectangles, and triangles) to allow us to use geometry to find the area. AP Physics C students need to be ready to deal with complex curves using the calculus expressions below.



Calculus: $\boxed{W = \int F \ dr}$ If the integral of force over an interval of displacement is work, then it should follow that the derivative of work with respect to the same interval of displacement is force. $F = \frac{dW}{dr}$

Energy: A capacity an object has to do work. There are many forms of energy including, mechanical, chemical, electrical, thermal, nuclear, etc. An object may have many types and amounts of energy at the start of a problem. Calculating the total energy an object has is impossible, and it is also unnecessary. During the course of a problem the type and amount of energy, or both, may change, and usually this change is restricted to only a few forms of energy. Energy can also be passed from object to object. Instead of trying to find the total energy we will focus on the forms of energy that change or that transfer from object to object. Tracking the change or movement of energy is very manageable, flexible, and extremely useful.

Mechanical Energy: The sum of Kinetic and Potential (gravitational) Energies

Kinetic Energy: Energy of motion. Depends on mass (inertia of the object) and velocity. Velocity has a greater effect on kinetic energy as it is squared in the formula $\boxed{K = \frac{1}{2}mv^2}$. Double a cars mass and you double kinetic energy. Double a cars velocity and you quadruple kinetic energy.

Potential Energy (gravitational): Energy of position. Depends on an objects mass, the gravity pulling the object, and on the height that the object is located at $\boxed{U_g = mgh}$. For mathematical simplicity and convenience the lowest possible point that the object can reach is designated to have zero height, and h is measured from this point. This is arbitrary and any point can be chosen, but choosing the lowest point as zero avoids dealing with negative heights.

Work Revisited: Think of work as the energy that is added (+***W***) to the system or subtracted (-***W***) from the system.

System: The object that the problem is focusing on.

Environment: The surrounding. The entire universe, except the object in the problem.

Work Energy Theorem: $\boxed{W = \Delta Energy}$ Work put into a system equals the change in energy of the system.

Example 6-2: Work and Potential Energy

Lifting a Mass: In order to lift a mass at constant velocity a force must be directed upward and be equal to the force of gravity. Use $W = F\Delta r \cos\theta$. Substitute F_g for F and height h for r. $W = F_g h \cos\theta$. The force and displacement are in the same direction, so $\theta = 0^o$ and thus $W = F_g h$. Substitution leads to $W = mgh$. Strangely this is the formula for gravitational potential energy? $U_g = mgh$. So does $W = U_g$?

Not really. Work-Energy Theorem states that work is a change in energy.

$$W = \Delta Energy \qquad W = \Delta(mgh) \qquad W = mgh_f - mgh_i$$

But if we set the lowest height in the problem (we lifted the mass so h_i is the lowest point) to be zero, then $W = mgh_f$.

So, if we arbitrarily identify the lowest height as zero, then the work done to raise an object does equal the energy it has at its new position, relative to its starting point. We are doing positive work, and this adds to the energy of the object.

If we lift an object by doing 20 J of work, then the object will have 20 J of additional energy. ***If we pretend that the object had 0 J at the start of the problem***, then it now has 20 J at the end. Caution: No object ever has zero energy. This is just a mathematical trick or simplification. We are not concerned with the amount of total energy the object has. Instead we are focusing on the change in energy. Many forms of energy are present (chemical, electrical, nuclear, etc.), but they did not change in this problem. The only energy that changed was the energy of height or position, known as potential energy. For convenience we set this energy to zero at the start of the problem (we simply moved the number line). Besides avoiding negative heights it allows easier analysis of useable energy and energy flow. The work done to lift the mass is added to the mass and becomes the final energy of the mass. This is really just the added energy, and since the object cannot fall any further than the lowest height (zero potential energy), it can only loose this much potential energy. ***We are really just tracking changes in usable energy.*** This is why Work and Work-Energy Theorem is such a useful entity.

20 J of work were added to the system, doing positive work and raising the systems energy. Where did it come from? It came from the environment. We will soon learn that energy is conserved and just either moves (transfers) between locations (system and environment) or it changes form (potential to kinetic, etc.). If you drop the object it will loose 20 J of potential energy. Since it is loosing energy, 20 J of negative work is done by gravity. Where does this energy go? It turns into 20 J of kinetic energy. Then the object hits the table and stops. Now it has lost all of its height (potential energy) and all of its velocity (kinetic energy). Where did the 20 J go? There was a sound, so molecules of air were pushed aside. The kinetic energy of the molecules increased. The table vibrated. The kinetic energy of the molecules in the table increased. This vibration is felt as heat. Where are the air molecules and table molecules? They are in the environment. So the 20 J returned to the environment. What do you mean by returned to the environment? Well, when I picked up the mass in the first place I was part of the environment, so the original 20 J used to lift the mass came from the environment in the very beginning.

Conservation of Energy:
Energy cannot be created or destroyed, but it can change forms.

There are many forms of energy. Those that will be used in this course are shown in the energy wheel below (Note: AP Physics C students will not cover Thermal or Modern). All the forms of energy are included here as this serves as a review of the entire year. If any energy is unfamiliar to you at this point in the year don't worry, we will cover them soon enough.

If energy can change form, then any energy can be equal to any other energy. If you drop an object the object looses height, but gains velocity. In this case potential energy is turning into kinetic energy. If the object looses all of its height, then there is a 100% transfer of energy to kinetic energy. $\boxed{mgh_i = \frac{1}{2}mv_f^2}$. But, what if the energy is an incomplete transfer. What if the problem ends before reaching zero height (zero potential energy). The object will then have two energies at the end. $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^2}$. Energy is a scalar. It is directionless and simply adds without worrying about direction.

Basically, conservation of energy means that the total energy at the beginning of the problem must equal the total energy at the end of the problem. What if the object has height and is moving at the beginning of the problem, and still has height and is moving at the end of the problem? $\boxed{mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2}$. The total energy in the problem is either $\boxed{E_{total} = mgh_i + \frac{1}{2}mv_i^2}$ or $\boxed{E_{total} = mgh_f + \frac{1}{2}mv_f^2}$. The total energy is conserved, so the total energy is present at the beginning, and it is still present at the end. What happens if the object is thrown straight upward, from the lowest point, and then reaches the highest point of flight where the velocity is zero? $\boxed{\frac{1}{2}mv_i^2 = mgh_f}$. What if it is thrown upward at an angle so that when it reaches its highest point it still has some velocity in the x- direction? $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^2}$. We can see that conservation of energy is a very flexible and powerful tool.

Remember: Energy is directionless. Simply ask yourself:

1. What energy/energies are present initially and add them up on the left.
2. What energy/energies are present finally and add them up on the right.

Can energy be lost?
No! Lost energy goes to the environment. A car (system) looses energy due to air resistance, so air molecules (environment) gain energy and move faster. Energy is conserved in the universe.

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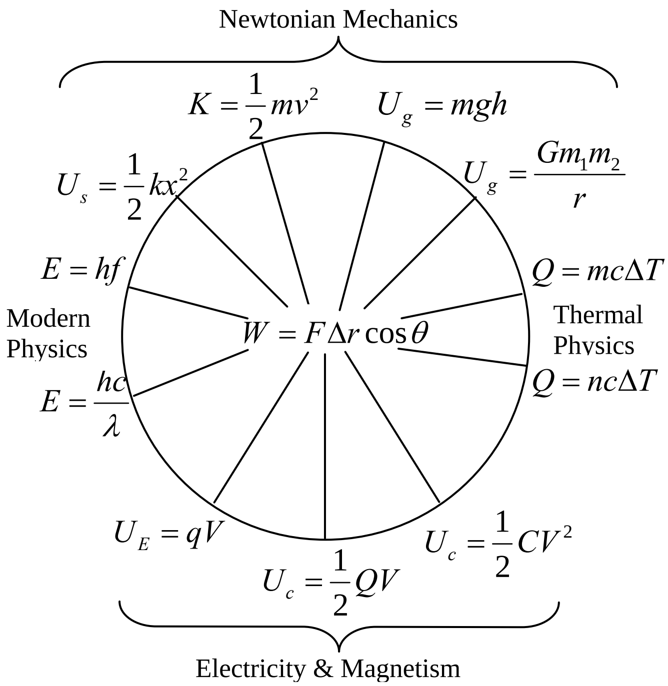

The wheel to the left shows all the energies learned in this class around the outside. They are grouped into the five major strands of physics. In the center is work.

Conservation of Energy: Energy cannot be created or destroyed it can change forms.

Changing forms means that any energy/energies initial around the outside of the circle can change into any other energy/energies final. $\boxed{\sum Energy_i = \sum Energy_f}$

In this class we often assume 100% energy transfer. One energy may be present initially, which disappears, and another energy is present at the end of the problem. In these 100% energy transfer problems any energy around the wheel can equal any other energy. If more than two energies are mentioned then you simply add up all energies present initially and set them equal to the total of all the final energies.

Work Energy Theorem: Work is a change in any energy. $\boxed{W = Any\ \ Energy_f - Any\ \ Energy_i}$

Most of the time one of the energies, either final or initial is equal to zero. So then Work can equal any energy around the outside. And since $W = Fr$, then Fr can equal any energy around the outside.

When do we use Conservation of Energy as Opposed to Work Energy Theorem? Conservation of energy is used when energy stays in the system and simply changes form. Work Energy Theorem is used when energy moves from the environment to the system and vice versa. You will see me use a hybrid of the two, in the examples below, when energy is lost to the environment due to friction.

The following is an incomplete list of examples. Many of these are common problems, and have been represented in AP Exams. In addition many are based on 100% efficiency of energy transfer. This is not really the case (see thermodynamics, soon), but it makes the problems easier in the same way that an airless and frictionless world helped us to start kinematics.

Example 6-3: Height Turning Into Velocity

The path does not matter, only the change in height.

For a 100% transfer: $\boxed{mgh_i = \frac{1}{2}mv_f^{\ 2}}$

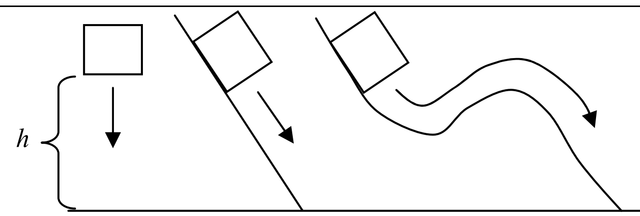

Fig 6.3

Example 6-4: Roller Coaster

A roller coaster is a good example of incomplete transfer of energy. At different points along the track there are various amounts of both kinetic and potential energy. One key to all these height problems, measured from a planetary surface, is to declare the lowest point in a problem to be $h = 0$. This gives a reference that is easy to add or subtract from. Then if you are given the height at any point on the track you can find the carts velocity:

$$\boxed{mgh_i + \frac{1}{2}mv_i^{\ 2} = mgh_f + \frac{1}{2}mv_f^{\ 2}}$$

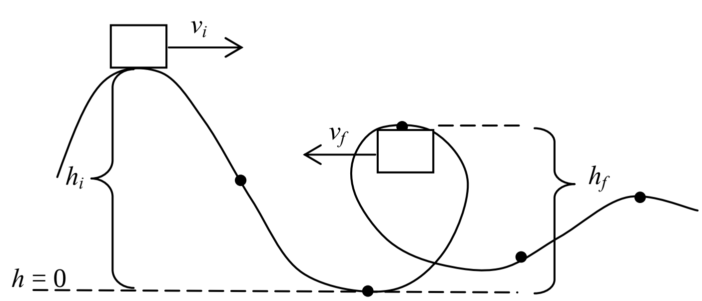

Fig 6.4

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Example 6-5: Friction Stops an Object

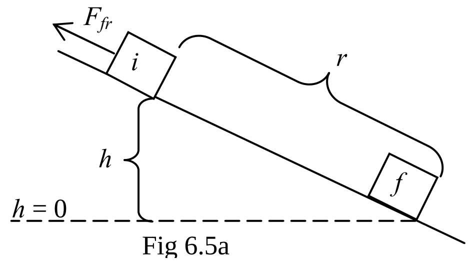

Fig 6.5a

Fig 6.3a shows an object that had height and stopped due to friction. Friction is a force and the only way to use a force in an energy equation is to multiply the force by the displacement, in other words use work. In this case it is the work of friction.

$$mgh_i = W_{fr} \qquad \boxed{mgh_i = F_{fr}r}$$

If the object in Fig 6.3a did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$mgh_i = W_{fr} + \frac{1}{2}mv_f^{\ 2} \qquad \boxed{mgh_i = F_{fr}r + \frac{1}{2}mv_f^{\ 2}}$$

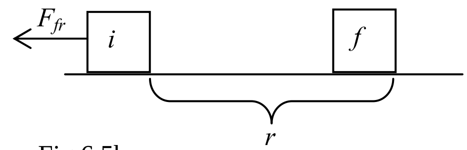

Fig 6.5b

In figure 6.3b the object is sliding along a horizontal surface. It initially has kinetic energy and is stopped by friction.

$$\frac{1}{2}mv_i^{\ 2} = W_{fr} \qquad \boxed{\frac{1}{2}mv_i^{\ 2} = F_{fr}r}$$

If the object in Fig 6.3b did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$\frac{1}{2}mv_i^{\ 2} = W_{fr} + \frac{1}{2}mv_f^{\ 2} \qquad \boxed{\frac{1}{2}mv_i^{\ 2} = F_{fr}r + \frac{1}{2}mv_f^{\ 2}}$$

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Power: Power is the rate of work, or rate of energy change. In other words it is the rate that energy is used, transferred, or generated during a one second interval. Since it involves energy, power is by extension as important.

$\boxed{P = \frac{W}{t}}$ you can substitute for work $P = \frac{F\Delta r}{t}$, and if you note that displacement over time is velocity then,

$\boxed{P = Fv}$. The first boxed equation is useful when you have work or energy, the second is useful when you have force. Even though the first equation contains the expression for work, you must be flexible. You must realize that work is a change in any energy $\boxed{P = \frac{\Delta Any\ \ Energy}{t}}$. You can plug in any energy from the preceding page.

$\boxed{P = \frac{mgh}{t}}$, $\boxed{P = \frac{1/2mv^2}{t}}$, etc. ***<u>If a problem contains any form of energy or work or the units of joules, and any quantity of time or any units of time, then it will involve power.</u>***

If energy is flexible then so is power. Occasionally a question gives power as a variable, but you need energy to solve the problem. Simply set the time equal to one second, then $P = \frac{W}{1s}$ and work/energy will have the same numerical value (but different units) as power for that one second. Use this value for Work/Energy to solve the problem. Just remember that all answers obtained in the problem are based on one second. If time is given later on, just multiple the energy of "one second" by the number of seconds and you've got your answer.

Powerful machines do more work in the same time, or the same work in less time.

Calculus: $\boxed{P = \frac{dW}{dt}}$ Power is another rate (function of time) and is therefore a derivative expression. Integrating power during a time interval will return work or energy values. $\boxed{W = \int P\ dt}$

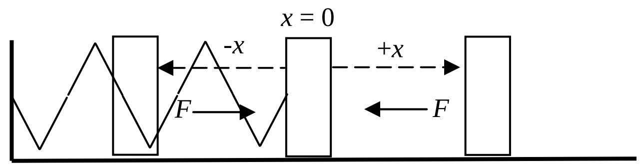
<u>Solve problems by looking for energy, work, and power first, then force, last of all kinematics.</u>

1–7 Oscillations

Period: $\boxed{T = \frac{1}{f}}$ Time for one revolution.

Frequency: The number of revolution, vibrations, oscillations, or rotations per second.

Springs: A Simple Harmonic Oscillator (SHO)



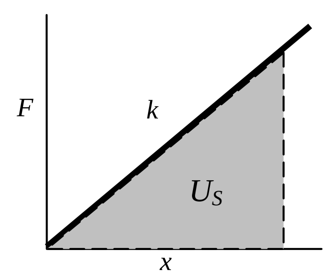
ΣF is max	$\Sigma F = 0$	ΣF is max
a is max	$a = 0$	a is max
U is max	$U = 0$	U is max
$K = 0$	K is max	$K = 0$
$v = 0$	v is max	$v = 0$

Restoring force: A spring attached to a block is shown at the right. Equilibrium ($x = 0$) is the rest position when no external forces are applied. If the spring is compressed so the block moves to $-x$ or the spring is stretched so the block moves to $+x$ the spring will have a force directed toward the equilibrium position. This is the restoring force $\boxed{F = -kx}$, and is known as Hooke's Law. ***k*** is the spring constant reflecting the quality or strength of the spring. The minus sign shows that the restoring force is opposite the displacement. Move a spring right and it wants to restore to the left. Restoring force is highest at maximum displacement (amplitude), located at $+x$ and $-x$. At these positions the spring has the highest force and acceleration, but has an instantaneous velocity of zero. It also has the highest amount of spring potential energy.

Potential Energy: The energy of position. This time we are working with a spring's position, $\boxed{U_s = \frac{1}{2}kx^2}$. When a spring is at equilibrium it is at rest and has zero displacement. This position is thus has zero potential energy. This is just like gravitational potential energy. When an object is a rest on the ground it is said to have zero potential energy. When it is displaced against the force of gravity it gains potential energy. It wants to return to the ground. The mass is displaced against the force of the spring, which wants to return it to equilibrium. So a displaced spring has stored energy, also known as potential energy. When the spring is fully displaced at either $-x$ or $+x$ it comes to rest.

Amplitude: The distance from equilibrium to the point of maximum displacement. This is the distance form $x = 0$ to $-x$, or from $x = 0$ to $+x$. It is not the distance from $-x$ to $+x$. That distance is two amplitudes. ***How many amplitudes does a spring go through in one complete cycle, from -x to x = 0 to +x, and then back to x = 0 and finally back to -x? Four.***

When the spring reaches $-x$ or $+x$ there it comes to an instantaneous stop, so it has no kinetic energy, but it has potential energy and a very large restoring force. When it moves through the equilibrium position the force on the spring and acceleration are both zero, but the velocity and kinetic energy is now at a maximum. Here any potential energy stored by displacing the spring from rest is turned into kinetic energy. Conservation of energy dictates that if one form of energy disappears and another appears, then the two energies must be equal. So $U_S = K$, or $\boxed{\frac{1}{2}kx^2{}_{\text{at either end}} = \frac{1}{2}mv^2{}_{\text{at the middle}}}$



Springs are a variable force. $F_s = -kx$ The more a spring is displaced, x, the larger the force, F, that is needed to displace it. At first no force is needed to move the spring. At the end of the problem a force F_S is needed. So the average force needed during the entire displacement is $\bar{F}_s = \frac{F_s - 0}{2} = \frac{1}{2}F_s = \frac{1}{2}kx$. Work is $W = F \cdot d = \frac{1}{2}kx \cdot x = \boxed{\frac{1}{2}kx^2}$. Force and displacement can be graphed as shown at the right. The equation $F_s = kx$ is the equation for a line $y = mx + b$, where k is the slope.

U_S is the area under the curve, $\boxed{\frac{1}{2}bh} = \frac{1}{2}(x)(F_s) = \frac{1}{2}(x)(kx) = \boxed{\frac{1}{2}kx^2 = U_s}$

Period of a spring: Time required for one oscillation, $\boxed{T_s = 2\pi\sqrt{\frac{m}{k}}}$. Depends on mass of object attached to spring and k.

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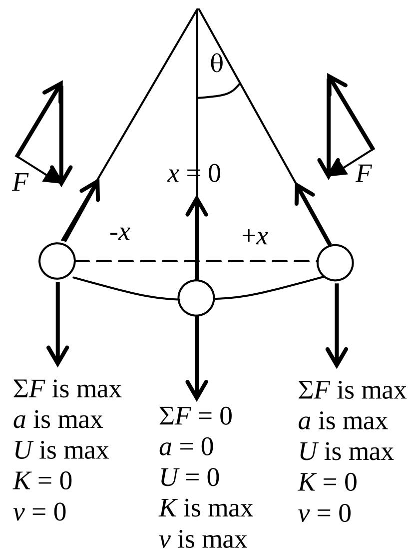
Pendulums: Approximates a Simple Harmonic Oscillator (SHO)



Restoring force: The restoring force is the sum of the tension and gravity vectors diagramed to the right. When the object is at its extreme displacement ($+x$ or $-x$) the restoring force is greatest. When the object returns to the equilibrium position ($x = 0$) the sum of the vectors is zero, and there is no restoring force. This is just like the spring above. There is never any net force at the equilibrium position ($x = 0$), so there is no acceleration at this point. Both the pendulum and the spring move through the equilibrium by inertia. It is interesting to note that when the restoring force is highest (at maximum displacement, either $+x$ or $-x$), the velocity is zero. When the net force is zero (equilibrium position, $x = 0$), the velocity is at its highest. All of the proceeding is true for any harmonic oscillators. Note: the dashed displacement line is not the same length as the arc that a pendulum actually moves through. The difference is very small at angles less than or equal to 15^o. Beyond this angle pendulums are not good harmonic oscillators. So at angles under 15^o pendulums behave almost like a true SHO.

Energy: The pendulum has potential energy at $+x$ and $-x$. It has kinetic energy at $x = 0$. The potential energy at one either end equals the kinetic energy in the middle. Potential energy in a pendulum has to do with height, not the spring constant $\boxed{mgh_{\text{at either end}} = \frac{1}{2}mv^2{}_{\text{at the middle}}}$

Period: Time required for one oscillation, $\boxed{T_p = 2\pi\sqrt{\frac{\ell}{g}}}$. Depends on length of the pendulum and $\boldsymbol{g}$.

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1–8 Linear Momentum and Collisions

Momentum: $\boxed{\mathbf{p} = m\mathbf{v}}$ Quantity of motion (inertia in motion). Measure of how difficult it is to stop an object.

Impulse: $\boxed{\mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}}$ Trade off between time taken to stop and force needed to stop. Velocity, acceleration, and momentum were understood early on in the developing days of physics. These concepts were used by Newton to establish his 2nd Law: $\mathbf{F}\Delta t = \Delta\mathbf{p}$ $\quad \mathbf{F}\Delta t = m\Delta\mathbf{v}$ $\quad \mathbf{F} = m\dfrac{\Delta\mathbf{v}}{\Delta t}$ $\quad \mathbf{F} = m\mathbf{a}$

Calculus: Force is the derivative of momentum $\boxed{\mathbf{F} = \dfrac{d\mathbf{p}}{dt}}$. Analogous to acceleration being the derivative of velocity $\boxed{\mathbf{a} = \dfrac{d\mathbf{v}}{dt}}$. If you divide the force and momentum vectors by the scalar mass you get the acceleration and velocity vectors.

Physics is full of meaningful patterns. Impulse is the integral of force during a time interval $\boxed{\mathbf{J} = \int \mathbf{F}\ dt = \Delta\mathbf{p}}$. This means that if you take the derivative of impulse with respect to time you will calculate force $\boxed{\mathbf{F}=\dfrac{d\mathbf{J}}{dt}}$.

Collisions and Conservation of Momentum: Momentum in any situation must ***always*** be conserved. When two objects, each having momentum, collide the total momentum during the problem remains the same. If mass 1 and mass 2 are moving then they both a momentum. Their momentums add together to calculate a total momentum $\boxed{p_{total} = m_1v_{1i} + m_2v_{2i}}$. If they collide they might bounce off each other with different velocities than before, but the total momentum must remain the same $\boxed{p_{total} = m_1v_{1f} + m_2v_{2f}}$. If the initial momentums and final momentums are both equal to the same total momentum then the sum of the initial momentums must equal the sum of the final momentums $\boxed{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$. If there are more than two masses involved, just keep adding them to both sides.

Momentum is a Vector: Unlike energy (a scalar) which simply adds, momentum is a vector. Vectors have direction, and this means that the vector directions (like force vectors) must be accounted for in the math. You must decide on a positive direction in a conservation of momentum problem. Once decided any mass traveling in that direction has a positive momentum. Any mass traveling opposite the chosen direction has a negative momentum. Any mass traveling at an angle to the chosen direction must be split into components, with the x and y directions analyzed separately. Use the same strategies that were learned in forces.

Elastic Collision: Collisions in which kinetic energy is conserved. This can only happen when two objects do not touch each other. One example that may make sense and will be used latter in electricity is the collision between two protons. Protons have positive charges and in electricity like charges repel. If two protons approach each other head on the repulsion for each other will slow them to a stop before they touch one another. Then the repulsion will repel them away from each other. In effect they bounce off each other without touching

Inelastic Collisions: Collisions in which kinetic energy is lost. Since energy is never really lost, it must go somewhere. Lost energy is just energy that was lost by the system to the environment or it is energy that changed into a form that is not very recognizable. When masses collide and touch each other the masses vibrate. This vibration is heat. In collisions where objects touch each other some of the original kinetic energy is lost as heat.

Perfectly Inelastic Collision: The objects collide and stick together (one mass, one velocity) $\boxed{m_1v_{1i} + m_2v_{2i} = v_f\left(m_1 + m_2\right)}$.

Explosion: The opposite of a perfectly inelastic collision. A single object fractures and sends fragments in many directions. If we look at the simpliest case where it fractures into two bodies moving in opposite directions then $v_i\left(m_1 + m_2\right) = m_1v_{1f} + m_2v_{2f}$. Usually the original object is considered stationary in beginners examples, but it does not have to be $0 = m_1v_{1f} + m_2v_{2f}$. So this means that $\boxed{m_1v_{1f} = -m_2v_{2f}}$. The negative sign implies that one object must move in the opposite direction of the other.

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Example 8-1: Conservation of Linear Momentum

Elastic Collision: Mass 1, m_1 = 2.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 4.00 m/s. The collision is diagrammed before and after in Fig 8.1a. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(2)(4) + (2)(0) = (2)v_{1f} + (2)(4) \qquad \boxed{v_{1f} = 0 m/s}$$

All the momentum from mass 1 was transferred to mass 2.

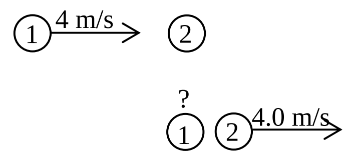


Fig 8.1a

Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 5.30 m/s. The collision is diagrammed before and after in Fig 8.1b. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(4)(4.00) + (2)(0) = (4)v_{1f} + (1)(5.30) \qquad \boxed{v_{1f} = 1.35 m/s}$$

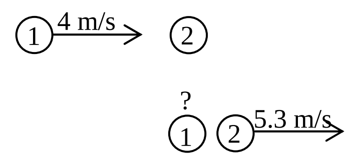


Fig 8.1b

Perfectly Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision the mass stick together. The collision is diagrammed before and after in Fig 8.1c. What is the velocity of the combined mass after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(4)(4.00) + (2)(0) = (4+2) v_f \qquad \boxed{v_{1f} = 2.67 m/s}$$

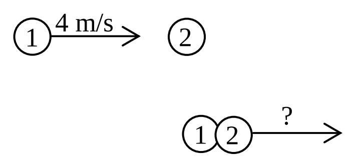


Fig 8.1c

Explosion: A large mass fractures into two smaller masses, m_1 = 4.00 kg and m_2 = 2.00 kg. Mass 2 moves to the right at 2.0 m/s. How fast is mass 1 going after the explosion?

$$(m_1 + m_2) v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$(4+2)(0) = (4)v_{1f} + (2)(2) \qquad \boxed{v_{1f} = -1 m/s}$$

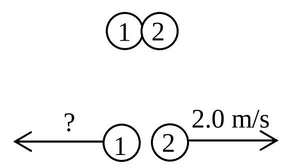


Fig 8.1d

<u>The minus sign in the last example means that the mass is going to the left. Remember to watch the minus signs. Harder problems will have masses moving in different directions. Missing the sign convention will destroy the problem.</u>

Collisions and Energy: **Total energy is always conserved**, $E_{1i} + E_{2i} = E_{1f} + E_{2f}$.

However, kinetic energy is only one form of energy. If we look only at kinetic energy it sometime seems to disappear.

Kinetic energy is conserved only in <u>elastic collisions</u>. $\qquad K_{1i} + K_{2i} = K_{1f} + K_{2f}$

Kinetic energy is lost (dissipated) in <u>inelastic collisions</u>. $\qquad (K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$

But where does the lost kinetic energy go? When two or more bodies collide the molecules that they are composed of are set into faster vibration. The speed of molecules is directly proportional to temperature, and temperature is one component of ***<u>internal energy</u>***. So while kinetic energy is lost, total energy is not. The kinetic energy lost is turning into another form of energy, which is not one of the ***<u>mechanical energies</u>*** (kinetic and potential). It is now in the form of vibrating atoms and molecules: ***<u>internal energy</u>*** (a thermal energy).

Example 8-2: Two dimensional Collision

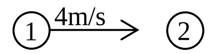


Fig 8.2a

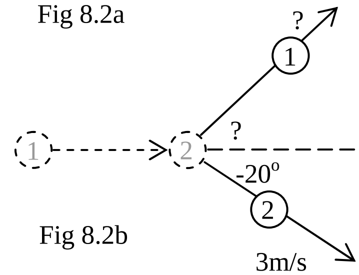


Fig 8.2b

Mass 1, m_1 = 2kg, is moving at 4m/s to the right. Mass 2, m_2 = 1kg, is stationary and is hit by mass 1 just a little off center. This causes mass 2 to move a 3m/s at an angle of 20° below the x-axis. The collision is diagrammed in Fig 8.2a (initial conditions) and in Fig. 8.2b (final conditions). What is the speed and direction of mass 1 after the collision?

The total momentum before the collision must equal the total momentum after. But after the collision the masses are moving at angles to the initial motion of m_1. We have to work in components. The total momentum in the x-direction must be conserved and the total momentum in the y-direction must be conserved.

***x*-direction**: Two bodies before and two after.

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$(2)(4) + (1)(0) = (2) v_{1fx} + (1)\left(3\cos\left(-20^o\right)\right) \qquad v_{1fx} = 2.59 m/s$$

***y*-direction**: This dimension is an explosion. Initially there is no motion in the y-direction at all. Then there are two bodies moving in opposite directions.

$$(m_1 + m_2) v_{iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

$$0 = (2) v_{1fx} + (1)\left(3\sin\left(-20^o\right)\right) \qquad v_{1fy} = 0.513 m/s$$

When you have two component vectors Pythagorean Theorem them back together and use arctangent to find the angle.

$$v_f = \sqrt{v_{1fx}^{\ 2} + v_{1fy}^{\ 2}} = \sqrt{(2.59)^2 + (0.513)^2} = \boxed{2.64 m/s} \qquad \theta = \tan^{-1}\frac{v_{1fy}}{v_{1fx}} = \tan^{-1}\frac{(0.513)}{(2.59)} = \boxed{11.2^0}$$

What amount of kinetic energy is lost? Remember energy is a directionless scalar, so x and y are meaningless.

$$(K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$$

$$\left(\frac{1}{2} m v_{1i}^{\ 2} + \frac{1}{2} m v_{2i}^{\ 2}\right) - K_{lost} = \left(\frac{1}{2} m v_{1f}^{\ 2} + \frac{1}{2} m v_{2f}^{\ 2}\right)$$

$$\left(\frac{1}{2}(2)(4)^2 + \frac{1}{2}(1)(0)^2\right) - K_{lost} = \left(\frac{1}{2}(2)(2.64)^2 + \frac{1}{2}(1)(3)^2\right) \qquad \boxed{K_{lost} = 0.0304 J}$$

Example 8-3: Ballistic Pendulum

The ballistic pendulum is used to determine projectile speed. The sequence of events is as follows. First a projectile, like a bullet (b), is fired into a block (B). This collision is perfectly inelastic, so $\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B) v_f}$ is used to solve for the v_f of the bullet block combination. v_f for this first phase becomes the v_0 for the second phase. In the second phase the bullet block combination swings as a pendulum to a new height, as shown in the diagram to the right. Conservation of energy applies. $\boxed{\frac{1}{2} m v_0^{\ 2} = mgh}$ is used to determine the height of the swing.

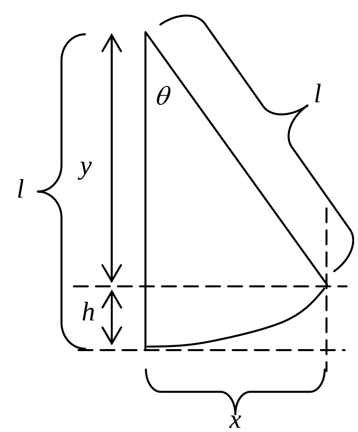
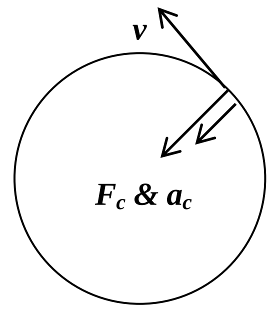


Fig 8.3

But, the point is to find the velocity of the bullet, so this problem is actually done backwards. The length of the rope holding the pendulum is known (l) and the distance the pendulum moves in the x direction (x) is measured (or the angle of swing, θ, is measured). You must use the geometry of a pendulum swing diagramed in Fig 8.2 to find the height (h) that the pendulum rises to $\boxed{y = \sqrt{l^2 - x^2}}$ and $\boxed{l = y + h}$, so $\boxed{h = l - y}$. Plug this final h into $\boxed{\frac{1}{2} m v_0^{\ 2} = mgh}$ and solve for the initial v_0 of the energy phase. Then recognize that this is the same as v_f for the collision $\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B) v_f}$ Solve for v_b.

1–9 Uniform Circular Motion and Gravity

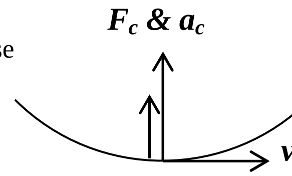


Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution. $\boxed{T = \frac{1}{f}}$ Time is in the numerator.

Velocity: Direction and thus velocity are continuously changing in circular motion. The magnitude of velocity and speed are not. You can measure an instantaneous velocity, which is tangential to the curve. Tangential Velocity or speed $\boxed{v = \frac{2\pi r}{T}}$

Centripetal Acceleration: Inertia would make a mass leave the circle following the tangential velocity. Instead the direction of the mass is being changed toward the center. In other words the mass is accelerated toward the center. Centripetal means center seeking. $\boxed{a_c = \frac{v^2}{r}}$



Centripetal Force: If an object is changing direction (accelerating) it must be doing so because a force is acting. Remember objects follow inertia (in this case the tangential velocity) unless acted upon by an external force. If the object is changing direction to the center of the circle it must be forced that way. $\boxed{F_c = ma_c}$ $\boxed{F_c = m\frac{v^2}{r}}$

Problem Solving Strategy

1. **As always, ask what the object is doing.** If it is moving in a circle, or even part of a circle, shown above right.
2. **Draw a FBD.** Remember ***F_c is the sum of force*** for circular motion. ***The sum of force is not shown in the FBD***.
3. **Set the direction of motion as positive.** Toward the center is positive, since this is the desired outcome.
4. **Identify the sum of force equation.** In circular motion F_c is the sum of force. F_c can be any of the previous forces.
5. **Substitute the relevant force equations and solve.**

Example 9-1: Vertical Circular Motion

A ball at the end of a string is swung in a vertical circle. Any forces pointing to the center are positive, while force vectors pointing away from the center are negative. Sum the forces. In circular motion F_c is the sum of force. **Find the tension in the string when the ball is at the top and at the bottom.**

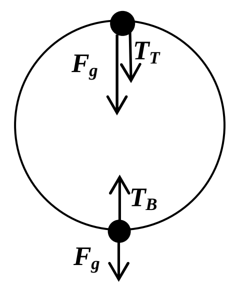


Fig 9.1

$$F_c = F_g + T_T \qquad F_c = -F_g + T_B$$

$$T_T = F_c - F_g \qquad T_B = F_c + F_g$$

$$\boxed{T_T = m\frac{v^2}{r} - mg} \qquad \boxed{T_B = m\frac{v^2}{r} + mg}$$

Example 9-2: Horizontal Circular Motion

A penny on a circular disk rotating horizontally (or a car turning a corner). Something must be keeping it going in a circle. Friction keeps it in place. If friction let go the penny would move due to inertia in a direction tangent to the disk. Force centripetal is the sum of forces for circular motion. **Find a formula for the maximum velocity if the coefficient of friction is known.**

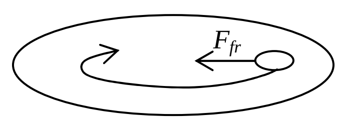


Fig 9.2

$$F_c = F_{fr} \qquad \cancel{m}\frac{v^2}{r} = \mu \cancel{m}g \qquad \boxed{v = \sqrt{\mu g r}}$$

Example 9-3: Top of a Loop and Apparent Weightlessness

Apparently weightless means that you are in freefall. The only force acting on you is F_g. To feel weightless at the top of the loop the roller coaster car can have no F_N (no pressure from the track). So for an instant at the top the car is not touching the track. **What at the top of the loop makes this possible?**

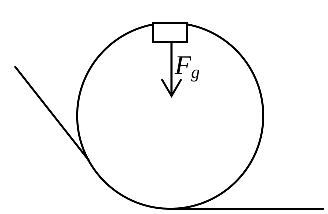


Fig 9.3

$$F_c = F_g \qquad \cancel{m}\frac{v^2}{r} = \cancel{m}g$$

 © R H Jansen

Example 9-4: Conical Pendulum

m_1 is suspended by a string that passes through a tube. At the other end of the tube m_2 is hanging from the same string. m_1 is spun at a velocity that keeps m_2 stationary.

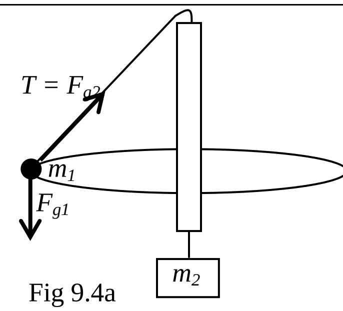


Fig 9.4a

Solve for the force centripetal. Force centripetal is the sum of force that points to the center of the circular motion. The two acting forces on m_1 are causing the circular motion, and they must sum together as force centripetal. If you add the two acting force vectors tip to tail they form a force vector triangle, shown in Fig 9.4b.

$$T^2 = F_c{}^2 + F_{g1}{}^2 \qquad \boxed{F_c = \sqrt{T^2 - F_{g1}{}^2}}$$

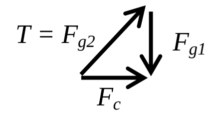
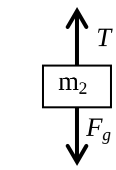


Fig 9.4b

Solve for the tension in the rope. Both masses hang from the rope, so either one can be used. Pick the easiest, in this case the vertically hanging mass. It's FBD is shown in Fig 9.4c. $\sum F = T - F_{g2}$ $\quad 0 = T - F_{g2}$ $\quad \boxed{T = F_{g2}}$

Fig 9.4c



$$F_c = \sqrt{T^2 - F_{g1}{}^2} \qquad m\frac{v^2}{r} = \sqrt{F_{g2}{}^2 - F_{g1}{}^2} \qquad \boxed{v = \sqrt{\frac{r\sqrt{F_{g2}{}^2 - F_{g1}{}^2}}{m}}}$$

Example 9-5: Gravitron

This is the ride at amusement parks where it spins and the floor drops down, leaving the occupants stuck to the wall.

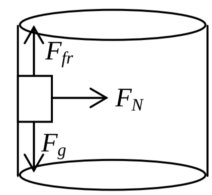


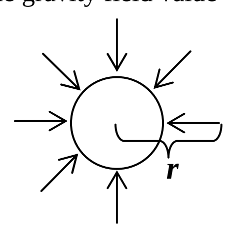
Fig 9.5

Solve for the tangential velocity. You feel pressed against the wall because the wall exerts a normal force toward the center. In other words the normal force is force centripetal, $F_c = F_N$. In the vertical dimension you are prevented from sliding down the wall by an upward and equal friction force, $F_{fr} = F_g$. Friction depends on force normal. $F_{fr} = \mu F_N$. Put all these equations together, and substituting for F_c and F_g.

$$F_c = F_N \qquad F_c = \frac{F_{fr}}{\mu} \qquad F_c = \frac{F_g}{\mu} \qquad m\frac{v^2}{r} = \frac{F_g}{\mu} \qquad \cancel{m}\frac{v^2}{r} = \frac{\cancel{m}g}{\mu} \qquad \boxed{v = \sqrt{\frac{rg}{\mu}}}$$

Gravity: One of the fundamental forces. This force is a field force, and the field is g, the acceleration of gravity. Every mass in the universe generates a gravity field. The gravity field is directed toward the center of mass. While the nature of the force is not understood the mathematics are detailed in Newton's Law of Universal Gravitation $\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ where $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$. This equation is the force between two masses. Remember the force between objects is equal and opposite. Combine this with the equation for weight $\boxed{F_g = mg}$ to get $mg = G\frac{m_1 m_2}{r^2}$, which simplifies as $\boxed{g = G\frac{m}{r^2}}$.

Each equation has its usefulness depending on the situation. The last equation is important for finding the gravity field value g around any mass at a distance r. To find the gravity at a point in space near Earth, use the mass of earth (which creates the gravity) and the distance from Earth's center. r is not a radius, but is the distance measured from center of a mass. r is used since gravity radiates in rays from the center of mass in a spoke like manner. Viewed in this way every distance in gravity is a radius. If you calculate g at a point in space near a mass you also know g for all points on a sphere of that radius (equipotential, since all points have the same potential energy).



Inverse Square Law: This can be used for both formulas with r^2 in the denominator. If r doubles (x2), invert to get ½ and then square it to get ¼. Gravity is ¼ its original value so F_g is ¼ of what it was and g is ¼ of what it was. Multiply the old F_g by ¼ to get the new weight, or multiply g by ¼ to get the new acceleration of gravity.

Example 9-6: Gravity on an Unknown Planet

Mars has roughly half the radius of Earth and has one-tenth the mass.

What is the gravity on the surface of Mars? Many students want to look up the radius and mass of Mars and plug into this equation $\boxed{g = G\dfrac{m}{r^2}}$. But there is another way. The problem gives us the relationship to Earth for a reason. We also already know the gravity on Earth $9.8 = G\dfrac{m_{Earth}}{r_{Earth}^{\ 2}}$. What if we just use some logic and the Inverse Square Law? The gravity on Mars is going to be Earth's gravity adjusted by pretending Earth shrinks to half its radius and one-tenth its mass.

$g_{Mars} = G\dfrac{(m_{Earth}\times 0.1)}{(r_{Earth}\times 0.5)^2}$ $\quad g_{Mars} = G\dfrac{m_{Earth}}{r_{Earth}^{\ 2}}\times\dfrac{(0.1)}{(0.5)^2}$ and we already know that $9.8 = G\dfrac{m_{Earth}}{r_{Earth}^{\ 2}}$, so

$g_{Mars} = 9.8\times\dfrac{(0.1)}{(0.5)^2}$ $\quad \boxed{g_{Mars} = 3.92\, m/s^2}$. Again, just pretend Earth shrinks to become Mars. This last line is all the work you need to show.

Example 9-7: Superposition of Gravity Fields

Superposition is a term referring to the addition, or superimposing, of two or more force fields. In Fig 9.7a mass A and mass B both create gravity at all points in space to infinity. If an object is positioned at point P it will feel the gravity of both masses. The two gravities must be added together using vector addition. $m_A = 2.00\times10^{20}$ kg, $m_B = 4\times10^{20}$ kg, The masses are 2.00×10^{8} m apart. Point P is located at a point half way between the masses.

What gravity is felt at point P? First solve for the gravity of each planet, at point P, separately. Use $g = G\dfrac{m}{r^2}$

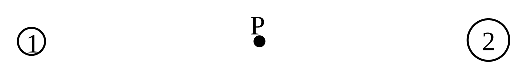


Fig 9.7a

$$g = \left(6.67\times10^{-11}\right)\frac{\left(2.00\times10^{20}\right)}{\left(1.00\times10^{8}\right)^2} = \boxed{1.33\times10^{-6}\, m/s^2}$$ toward m_1 (left)

$$g = \left(6.67\times10^{-11}\right)\frac{\left(4.00\times10^{20}\right)}{\left(1.00\times10^{8}\right)^2} = \boxed{2.67\times10^{-6}\, m/s^2}$$ toward m_2 (right)

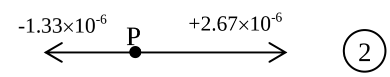


Fig 9.7b

g is a vector, and vectors have direction. Assign a positive sign to the vector pointing right and a negative sign to the vector pointing left, as shown in Fig 9.7b. Then add the two vectors together $\left(-1.33\times10^{-6}\right)+\left(+2.67\times10^{-6}\right) = \boxed{+1.33\times10^{-6}}$.

The positive answer implies that the gravity at point P is 1.33×10^{-6} m/s^2 directed toward the right.

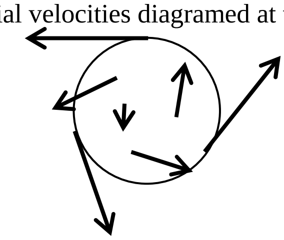
If a 100 kg mass were to be positioned at point P, what would the force of gravity be? The beauty of finding g is that you can easily apply it to any mass at that location to find the force of gravity $F_g = (100)\left(1.33\times10^{-6}\right) = \boxed{1.33\times10^{-4}\, N}$

Potential Energy Revisited:

There is another equation to find potential energy using the universal gravity constant. Use the work formula and work energy theorem, $W_g = \Delta U_g = F_g\Delta r$. Set the initial displacement as zero and it simplifies to $U_g = F_g r$. Use this with $\boxed{F_g = -G\dfrac{m_1m_2}{r^2}}$ to get $\dfrac{U_g}{r} = -G\dfrac{m_1m_2}{r^2}$. This simplifies to $\boxed{U_g = -G\dfrac{m_1m_2}{r}}$.

Where did the minus sign come from? Suddenly it is added to Newton's Law of Universal Gravitation. This is the formal version of the law. It can be used with either a positive sign (simplified and common version) or a negative sign (formal version) and is situational dependent. In formal physics a point at infinity is said to have zero potential energy. Since a central point of zero potential energy cannot be located in the universe, it makes sense to pick infinity to be zero potential energy. All points in the universe are the same infinite distance from infinity. However, this means that close to Earth's surface potential energy is negative. It is common practice when viewing planets from a great distance to set infinity as $U_g = 0$, and when on a planets surface to set the lowest height as $U_g = 0$. These are just conventions used to make specific problems easier to solve. Remember, the exact energy that an object has is not really important. What matters is how much of that energy is usable to do work. And work is a change in energy. Therefore, we can really declare any point as zero energy and measure changes from that point.

1–10 Introduction to Rotation and Torque



Rotation: In rotation the entire object spins around its center of mass. Looking at the tangential velocities diagramed at the right, we see that they are all in different directions and all vary in magnitude. Points near the outer edge have to move through a larger circumference in the same period than those closer to the center. The outer edge must be moving faster to cover the longer distance in the same period or time. All of these points have one thing in common, they all travel through the same number of degree or radians during a period. Rotational velocity measured in radian per second is called **angular velocity**. However, **All the equations for an object in circular motion hold true if we are looking at a single point and only a specific point on a rotating object.** Rotating objects have **rotational inertia** and an accompanying **angular momentum**, meaning that a rotating object will continue to rotate (or not rotate) unless acted upon by an **<u>unbalanced</u> torque**, discussed below.

(Note: Planets and satellites follow circular motion, as they are not attached. Inner planets move faster as they are closer to the sun and must have larger tangential velocities. They also travel a shorter circumference. Thus they have shorter periods.)

Angular momentum: Depends on mass (like regular momentum) and it also depends on mass distribution. As an ice skater brings their arms closer to the body they begin to spin faster, since the mass has a shorter distance to travel.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

Torque: In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$\boxed{\tau = rF\sin\theta}$ Strongest when the force is **<u>perpendicular</u>** to the lever arm (since sin 90° equals one).

Balanced Torque: The sum of torque is zero. No rotation.

Unbalance Torque: Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** The convention when in doubt is that counterclockwise is positive. This corresponds to projectile motion where angles measured from the horizon counterclockwise were positive. But, just like in forces if you know the direction of motion call it positive. It will either rotate clockwise or counterclockwise. If you pick the wrong direction your final answer will be negative. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so the opposing torques cancel.
3. **Identify the sum of torque equation.**

$\sum\tau = \sum\tau_{cw} - \sum\tau_{ccw}$ **or** $\sum\tau = \sum\tau_{ccw} - \sum\tau_{cw}$

4. **Substitute the relevant force equations and solve** (examples assume clockwise was positive direction)

Rotating: $\sum\tau = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$

Not Rotating: $0 = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$ $\sum(rF\sin\theta)_{cw} = \sum(rF\sin\theta)_{ccw}$

Example 10-1: Torque and a Seesaw

Three masses are positioned on a seesaw as shown in Fig 10.1. m_A = 4.0 kg, m_B = 2.0 kg, and m_3 = 3.0 kg. Distances are shown in the diagram.

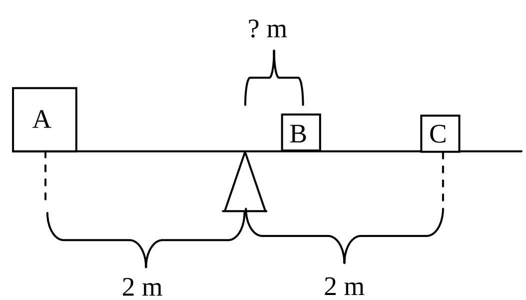


Fig 10.1

How far from the fulcrum must m_B be positioned in order for the system to balance? Keep in mind that measurements are made from the center of mass. It is as though all the mass is mathematically located at a point at the center of the object. It is not rotating, so the clockwise torques must equal the counterclockwise torques.

$$\sum\tau_{cw} = \sum\tau_{ccw}$$

$$\tau_A = \tau_B + \tau_C$$

$$r_A \cdot F_A = r_B \cdot F_B + r_C \cdot F_C$$

$$r_A \cdot m_A \cancel{g} = r_B \cdot m_B \cancel{g} + r_C \cdot m_C \cancel{g}$$

$$(2)(4) = r_B(2) + (2)(3) \qquad r_B = \boxed{1.0m}$$

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1−11 Rotation Detailed, Rolling, and Angular Momentum

Note: This detail on rotation is needed for AP Physics C: physical science majors, calculus based.

Center of Mass: Objects rotate around a central axis and around a center of mass. It is therefore important to be able to locate the center of mass. The center of mass is each for shapes like squares, rectangles, circles, spheres, or equilateral triangles. It is in the middle. The following equation will find the center of mass of a system of point masses or for a system of geometric shapes (those just mentioned, that you can find the center of by inspection) $\boxed{\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m}$. Unusual shapes can be found experimentally by hanging the object from two or more positions, drawing vertical lines from the point of attachment of the string, and looking for an intersection. Or integral calculus can be used. For this course these last two methods will not be discussed.

Example 11-1: Center of Mass

Find the center of mass for the object in Fig 11.1a. It is a thin flat object composed of a rectangle (2m by 4m in length, mass 5 kg) and a square (2m long sides, mass 3 kg). Set up a coordinate axis system. For convenience place the coordinate axis at one corner of the object and divide the object into a rectangle and a square, as shown in Fig 11.1b. Find the center of mass the rectangle, relative to the coordinate axis, by inspection, $x = 1,\ y = 2$. Find the center of mass the rectangle, relative to the coordinate axis, by inspection, $x = 3,\ y = 1$. **Now you can pretend that the rectangle and square are point masses at these locations. The remainder of the problem is the method for solving for point masses in two dimensions.** You must work in each dimension separately.

$$x_{cm} = \frac{\sum mx}{\sum m} \qquad x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \qquad x_{cm} = \frac{(5)(1)+(3)(3)}{(5)+(3)} = 1.75m$$

$$y_{cm} = \frac{\sum my}{\sum m} \qquad y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \qquad x_{cm} = \frac{(5)(2)+(3)(1)}{(5)+(3)} = 1.63m$$

The center of mass is located at $\boxed{\mathbf{r}_{cm} = 1.75\mathbf{i}m + 1.63\mathbf{j}m}$

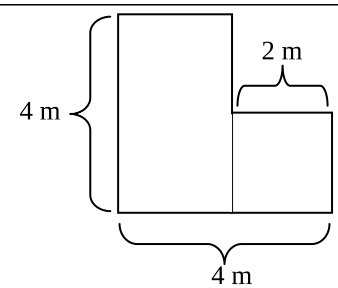

Fig 11.1a

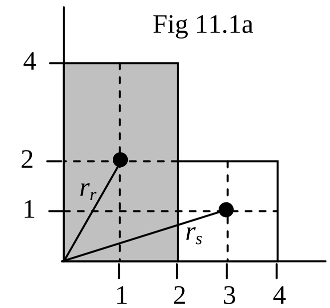

Fig 11.1b

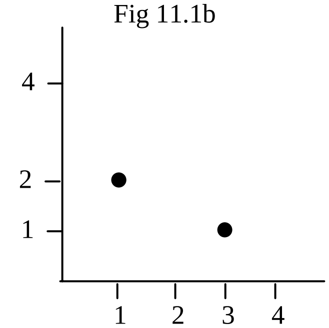

Fig 11.1c

Rotation: Since every point on a rotating object experiences a different tangential velocity displacement, velocity, and acceleration cannot be expressed in terms of meters. A particle on the outside edge of a rotating object covers a greater distance in the same time interval than a particle closer to the center. The only quantity that both points share in any given time interval is the angle through which they move, as shown to the right. In rotation we have to work in radians instead of degrees. This means that for every variable in linear (***translational***) motion there is a corresponding variable for rotation. And every equation in linear motion has a rotational counterpart. Displacement x is replaced by ***radians*** θ (radians). Velocity v is replaced by ***angular velocity*** ω (radians per second). Acceleration a is replaced by ***angular acceleration*** α (radians per second squared) The following three equations form a bridge between linear motion and rotation and should be memorized. $\boxed{x = r\theta}$ $\boxed{v = r\omega}$ $\boxed{a = r\alpha}$. The chart below, and on the following pages, compares rotation to linear motion. There is an analogous quantity and an analogous equation for rotation that parallels those learned in linear translational motion. Keep the three equations listed above in mind and become familiar with the new quantities.

	Angular	Linear
Position	$\theta = \dfrac{arc\ length}{r}$	x
Displacement	$\Delta\theta = \theta - \theta_0$	$\Delta x = x - x_0$
Average Speed	$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$	$\bar{v} = \dfrac{\Delta x}{\Delta t}$ $\bar{v} = \dfrac{v_0 + v}{2}$
Instantaneous Speed	$\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t}$ $\omega = \dfrac{d\theta}{dt}$	$v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$ $v = \dfrac{dx}{dt}$ **Slope of displacement - time graph**
Average Acceleration	$\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$	$\bar{a} = \dfrac{\Delta v}{\Delta t}$
Instantaneous Acceleration	**Tangential Acceleration** $\alpha = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t}$ $\alpha = \dfrac{d\omega}{dt}$	$a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$ $a = \dfrac{dv}{dt}$ **Slope of velocity - time graph**
Kinematic Equations	$\boxed{\omega = \omega_0 + \alpha t}$ $\boxed{\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2}$ $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$	$\boxed{v = v_0 + at}$ $\boxed{x = x_0 + v_0 t + \dfrac{1}{2}at^2}$ $\boxed{v^2 = v_0{}^2 + 2a(x - x_0)}$
Tangential Speed	$\boxed{v = r\omega}$ $\omega = \dfrac{2\pi}{T}$	$v = \dfrac{2\pi r}{T}$
Centripetal Acceleration	**Radial Acceleration** $\boxed{a_c = \dfrac{v^2}{r} = \omega^2 r}$ Radial Acceleration is the acceleration directed along a radial (spoke) line. It is directed toward the center.	$a_c = \dfrac{v^2}{r}$
Inertia	**Moment of Inertia:** Depends on mass and distribution and thus varies for each object $\boxed{I = \int r^2 dm = \sum mr^2}$ Since these vary from object to object they are usually given. The three shown here are commonly used. The first one is the common shape for pulley, which are the most used. Cylinder: $\boxed{I = \frac{1}{2}MR^2}$ Cylindrical hoop: $I = MR^2$ Sphere: $I = \frac{2}{5}MR^2$	m
Force and Torque	**Torque:** Unbalance torques cause rotation. $\boxed{\tau = \mathbf{r} \times \mathbf{F}}$ $\boxed{\sum \tau = \tau_{net} = I\alpha}$	**Force:** Unbalanced forces cause translation. $\mathbf{F}$ $\boxed{\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}}$
Kinetic Energy	$\boxed{K = \dfrac{1}{2}I\omega^2}$	$\boxed{K = \dfrac{1}{2}mv^2}$

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	Angular	Linear
Work	$W = \int \tau d\theta$ $W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0{}^2$	$\boxed{W = \int \mathbf{F} \cdot d\mathbf{r}}$ **Area under force – distance curve** $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$
Power	$\boxed{P = \frac{dW}{dt}}$ $P = \tau\omega$	$\boxed{P = \frac{dW}{dt}}$ $\boxed{P = Fv}$

Vector Product and Torque: Torque is a cross product of vectors. The **magnitude of a cross product is the area of the parallelogram formed by the contributing vectors**. The **direction of a cross product vector is determined by using the right hand rule**. So the direction of torque is out of the page for counterclockwise rotation, and into the page for clockwise rotations.

Translation vs. Rotation: Hit an object with a force directed into or out of the center of mass and it will translate (linear motion). Hit an object with a force perpendicular to a radial line extending from the center of mass and at the very edge of the object, and the object will rotate. Hit and object with a force between the center of mass and the edge and it will translate and rotate.

(Note: Planets and satellites follow circular motion, as they are not attached. Inner planets move faster as they are closer to the sun and must have larger tangential velocities. They also travel a shorter circumference. Thus they have shorter periods.)

Angular momentum: Masses that experience linear motion (translation) have velocity and thus have linear momentum. Rotating masses have angular velocity and thus have angular momentum. While linear momentum depends on mass and velocity, angular momentum depends on mass, mass distribution, and angular velocity. Think about it. In rotating objects the points of mass farther from the center are moving faster and thus have higher instantaneous momentum values than those closer to the center. Lots of mass, far from the center of mass, means higher angular momentum than the same mass, near the center of mass.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

	Angular	Linear
Momentum	$\boxed{\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}}$	$\boxed{p = mv}$
Conservation of Momentum	$\mathbf{L}_i = \mathbf{L}_f$ $I\boldsymbol{\omega}_i = I\boldsymbol{\omega}_f$	$p_i = p_f$ $mv_i = mv_f$

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Example 11-2: Compound Bodies and Pulleys with Mass

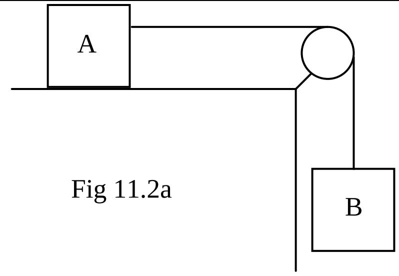

Fig 11.2a

A compound body consisting of, m_A = 6.0 kg, m_B = 8.0 kg, M_{pulley} = 1 kg, R_{pulley} = 0.10 m, is shown in Fig. 11.1a.

What is the acceleration of the system?

There are three masses, so there are three FBD's, shown in Fig 11.2b. Make note of the interesting new FBD for a pulley. Gravity acts through the center and down, as usual. The normal force is created by the support pushing the pulley away from the table, and it follows the direction of the support through the center of the pulley. ***The tensions are tangent to the pulley. These tensions are a distance R (radius of pulley) from the center and they are perpendicular to the R. This provides the torque that rotates the pulley. Also note that there are two tensions. When we work with real pulleys that have mass the rope connecting the masses has different tensions in every separate segment.***

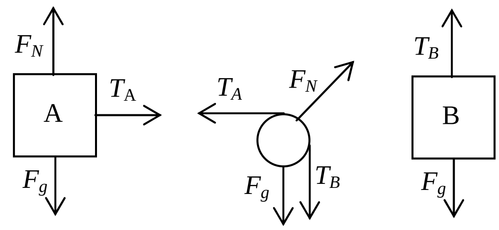

Fig 11.2b

Set up sum of force and sum of torque equations for relevant masses. As before use the direction of motion to assign positives and negatives. The direction of motion is to the right, clockwise, and then down.

$$\sum F_A = T_A \qquad \sum \tau = \tau_{cw} - \tau_{ccw} \qquad \sum F_B = F_g - T_B$$

$$m_A a = T_A \qquad I\alpha = R \cdot T_B - R \cdot T_A \qquad m_B a = m_B g - T_B$$

$$T_A = m_A a \qquad I\frac{a}{R} = R \cdot T_B - R \cdot T_A \qquad T_B = m_B g - m_B a$$

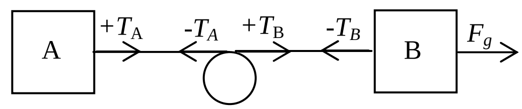

Fig 11.2c

Combine the three equations above to get

$I\frac{a}{R} = R\left(m_B g - m_B a\right) - R\left(m_A a\right)$ Substitute in the moment of inertia of a cylindrical disk (pulley) $I = \frac{1}{2}MR^2$

$\left(\frac{1}{2}MR^2\right)\frac{a}{R} = R\left(m_B g - m_B a\right) - R\left(m_A a\right)$ Cancel out the pulleys radius, group all expression with a, and simplify.

$$\frac{1}{2}Ma = m_B g - m_B a - m_A a \qquad m_A a + m_B a + \frac{1}{2}Ma = m_B g \qquad \boxed{a = \frac{m_B g}{\left(m_A + m_B + \frac{1}{2}M\right)}}$$

This looks familiar. If we did the problem the old way, with a massless pulley, we would look at it as linear, like Fig 11.2c

It would be $\sum F_{total} = F_{gB}$ $\qquad \left(m_A + m_B\right)a = m_B g \qquad a = \frac{m_B g}{\left(m_A + m_B\right)}$

This is identical except for the expression for half the pulley's mass. Can we just do all pulley problems the old way and just add a $\frac{1}{2}M$ to all the regular masses in the denominator. It seems to work, but you might loose points for not showing work.

And the $\frac{1}{2}M$ only works with pulleys that have a moment of inertia of $I = \frac{1}{2}MR^2$. If it were a spherical pulley, would we add $\frac{2}{5}M$ to the denominator, since its moment of inertia is $I = \frac{2}{5}MR^2$. Verify it on your own and see.

Fig 11.2c shows the problem sketched linear. Any forces that are perpendicular to the direction of motion were removed from this sketch. Vectors pointing in the direction of motion are noted with positive signs and those opposing motion are negative. It is apparent that tension cancels as before. However, unlike previous work in forces the pulley is not erased as it now has mass. It must be accounted for. Is there a shortcut method using this linear sketch that would show adequate supporting work?

Example 11-3: Rolling Down an Incline

A spherical mass rolls 2 m down an incline shown in Fig 11.3a. The FBD for the sphere is shown in Fig 11.3b. In figure 11.3c the gravity and normal force vectors have been summed and the component of force down the slope is shown. $F_g\sin\theta$ pulls the sphere down the incline in the usual manner. However, the friction force vector is a distance R from the center of the sphere. These two vectors are perpendicular. This creates an unbalanced torque on the sphere, which causes it to rotate. The combined motion of rotating and moving down the slope is rolling.

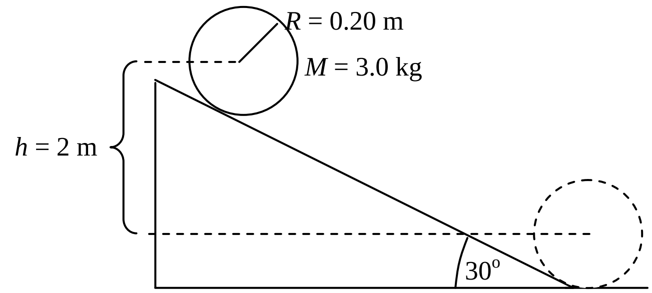

Fig 11.3a

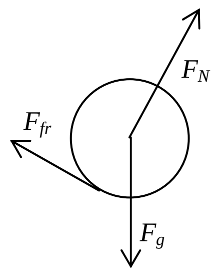

Fig 11.3b

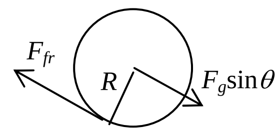

Fig 11.3c

What is the kinetic energy of the sphere at the bottom of the incline? The sphere is translating and rotating at the same time. The total kinetic energy is the addition of the translational and rotational kinetic energies.

$$K_{total} = K_{translation} + K_{rotation}$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

It is also equal to the potential energy that converted in kinetic energy. Given the quantities in the problem, this is easiest to solve

$$K_{total} = mgh \qquad K_{total} = (3.0)(9.8)(2.0) = \boxed{58.8J}.$$

How fast is the sphere going at the bottom of the incline?

Now the first kinetic energy equation has relevance, $K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. Combine this with the moment of inertia equation of a sphere $\frac{2}{5}MR^2$, and the equation $\omega = \frac{v}{R}$ which converts angular values into linear values.

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 \qquad K_{total} = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 \qquad K_{total} = \frac{7}{10}Mv^2$$

Rearrange for velocity, plug in values, and solve.

$$v = \sqrt{\frac{10}{7}\frac{K_{total}}{M}} \qquad v = \sqrt{\frac{10}{7}\left(\frac{58.8}{3.0}\right)} = \boxed{5.29\,m/s}$$

Another expression can be derive here also $v = \sqrt{\frac{10}{7}gh}$

What is the linear acceleration of the sphere down the inline?

The length down the ramp is $hyp = \frac{h}{\sin\theta}$

$$v^2 = v_0^{\ 2} + 2a(x - x_0)$$

$$\left(\sqrt{\frac{10}{7}\frac{K_{total}}{M}}\right)^2 = (0)^2 + 2a((x) - 0)$$

$$\left(\sqrt{\frac{10}{7}\frac{Mgh}{M}}\right)^2 = (0)^2 + 2a\left(\frac{h}{\sin\theta}\right)$$

$$a = \frac{5}{7}g\sin\theta \qquad a = \frac{5}{7}(9.8)\sin 30^o = \boxed{3.5\,m/s^2}$$

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