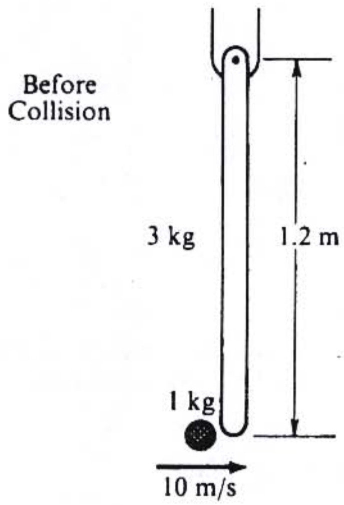
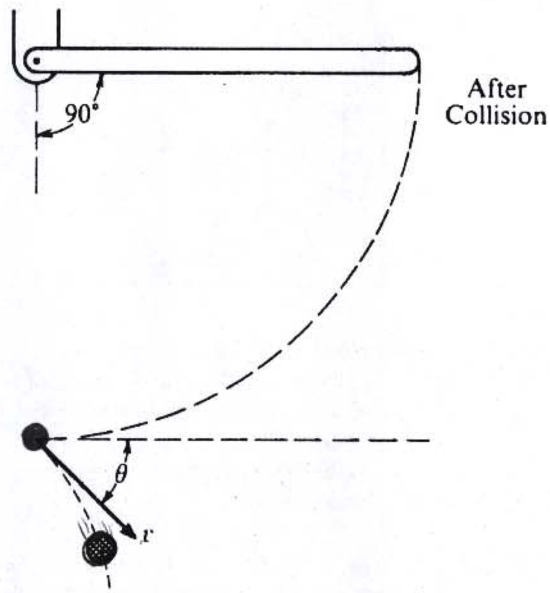




Note: You may use $g = 10\ m/s^2$.

1987M3. A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.

a. Determine the angular velocity of the bar immediately after the collision.

COE

$\frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}I\omega^2 = Mg\frac{1}{2}l$

$\frac{1}{2}\frac{1}{3}\cancel{M}l^{\cancel{2}}\omega^2 = \cancel{M}g\frac{1}{2}\cancel{l}$

$\Delta h_{CM} = \frac{1}{2}l$

$\frac{1}{6}l\omega^2 = \frac{1}{2}g$

$\boxed{\omega = \sqrt{\frac{3g}{l}}}$

$\boxed{\omega = 5\ rad/s}$

b. Determine the speed v of the 1-kilogram object immediately after the collision.

COE

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\cancel{\frac{1}{2}}mv_0^2 = \cancel{\frac{1}{2}}mv^2 + \cancel{\frac{1}{2}}\frac{1}{3}Ml^2\left(\sqrt{\frac{3g}{l}}\right)^2$

$mv_0^2 - \frac{1}{3}Ml^{\cancel{2}}\frac{3g}{\cancel{l}} = mv^2$

$\sqrt{\frac{mv_0^2 - \frac{1}{3}Ml3g}{m}} = v = \boxed{8\ m/s}$

c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.

$L_0 = mv_\perp r$

$L_0 = mv_0 l = \boxed{12\ \frac{kg \cdot m^2}{s}}$

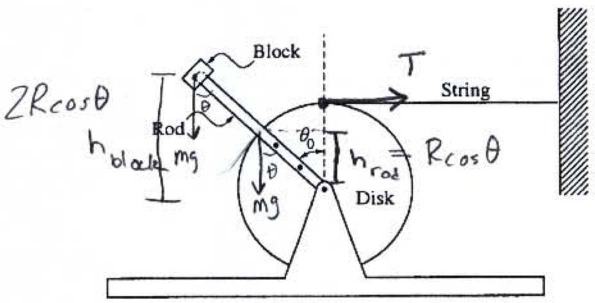
d. Determine the angle θ.

$mv_0 l = mv_\perp r + I\omega$

$mv_0 l = mvl\sin\theta + \frac{1}{3}Ml^2\omega$

$\sin\theta = \frac{mv_0 l - \frac{1}{3}Ml^2\omega}{mvl}$

$\theta = \sin^{-1}\left[\frac{mv_0 l - \frac{1}{3}Ml^2\omega}{mvl}\right] = \boxed{30°}$



a) $\Sigma\tau = \emptyset$

$T\cdot R - mg\frac{1}{2}\ell\sin\theta_0 - 2mg\ell\sin\theta_0 = \emptyset$

$T\cdot R - mgR\sin\theta_0 - 2mg2R\sin\theta_0 = \emptyset$

$T = mg\sin\theta_0 + 4mg\sin\theta_0$

$\boxed{T = 5mg\sin\theta_0}$

1999M3 As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass = 3m, radius = R, moment of inertia about center $I_D = 1.5mR^2$
Rod: mass = m, length = 2R, moment of inertia about one end $I_R = 4/3(mR^2)$
Block: mass = 2m

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

a. Determine the tension in the string.

The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.
   i. The magnitude of the angular acceleration of the disk
   ii. The magnitude of the linear acceleration of the mass at the end of the rod

b) i) $\Sigma\tau = I\alpha$

$5mg\sin\theta_0 R = (I_R + I_D + I_B)\alpha$

$\alpha = \frac{5mg\sin\theta_0 R}{\frac{3}{2}mR^2 + \frac{4}{3}mR^2 + 2m(2R)^2}$

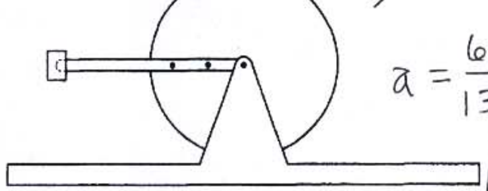
$\alpha = \frac{5mg\sin\theta_0}{\frac{3}{2}mR + \frac{4}{3}mR + 8mR}$

$\boxed{\alpha = \frac{6}{13}\frac{g\sin\theta_0}{R}}$

ii) $a = \alpha\cdot r$

$a = \frac{6g\sin\theta_0}{13\cancel{R}}\cancel{R}$

$\boxed{a = \frac{6}{13}g\sin\theta_0}$



As the disk rotates, the rod passes the horizontal position shown above.

c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.

COE

$\cancel{K_0} + U_0 = K + \cancel{U}$

$U_{g_{rod}} + U_{g_{Block}} = K_R$

$mgh_{rod} + 2mgh_{block} = \frac{1}{2}I\omega^2$

$mgR\cos\theta + 2mg2R\cos\theta = \frac{1}{2}I\omega^2$

$I_{sys} = I_{rod} + I_{block} + I_{Disk}$

$I_{sys} = \frac{3}{2}mR^2 + 2m(2R)^2 + \frac{4}{3}mR^2$

$I_{sys} = \frac{65}{6}mR^2$

$5mgR\cos\theta = \frac{1}{2}\frac{65}{6}mR^2\omega^2$

$\sqrt{\frac{60g\cos\theta}{65R}} = \omega$

$v = \omega\cdot r$

$\boxed{v = \left(\sqrt{\frac{60g\cos\theta}{65R}}\right)2R}$