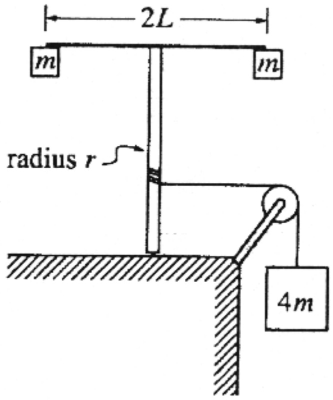
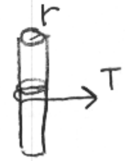
KEY





Experiment A

200[illegible]M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

$\alpha = \frac{a}{r}$

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

$I = \Sigma m r^2$ $\quad I = mL^2 + mL^2$ $\quad \boxed{I = 2mL^2}$

$T = \frac{I\alpha}{r}$ $\quad \rightarrow T = 4mg - 4ma$

$\uparrow T$

$\downarrow 4mg$

b. Determine the downward acceleration of the large block.

$\Sigma F = ma \quad 4mg - T = 4ma$

$\Sigma \tau = I\alpha$

$I\alpha = Tr$

$4mg - 4ma = \frac{I\alpha}{r}$

$4mg - 4ma = \frac{2mL^2\alpha}{r}$

$\frac{2\cancel{m}L^2 a}{r^2} = 4\cancel{m}g - 4\cancel{m}a$

$\frac{2L^2 a}{r^2} + 4a = 4g$

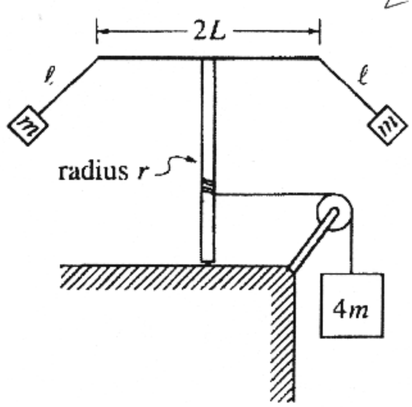
$a\left(\frac{2L^2}{r^2} + 4\right) = 4g$

$\boxed{a = \frac{2gr^2}{L^2 + 2r^2}}$

c. When the large block has descended a distance *D*, how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD* ? Check the appropriate space below and justify your answer.

Greater than 4mgD ______ Equal to 4mgD __✓__ Less than 4mgD ________

Energy is conserved



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

$I \uparrow$

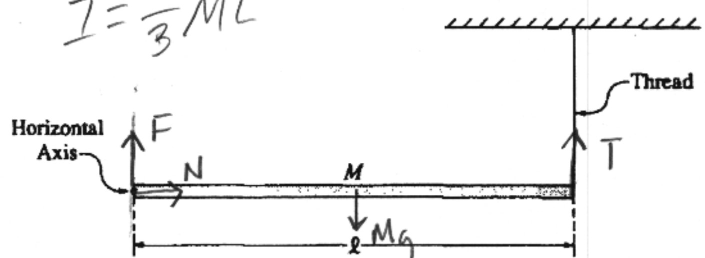
d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? Check the appropriate space below and justify your answer.

Greater before ________ Equal to before ________ Less than before __✓__

less a = less v = less K

15

$I = \frac{1}{3}ML^2$



1993M3. A long, uniform rod of mass M and length l is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of M, l, and g.

4 a. Determine the magnitude and direction of the force exerted on the rod by the axis.

$1 \quad \Sigma\tau = \emptyset \qquad \tau = rF_\perp$

$Mg\frac{1}{2}L - FL = \emptyset$

$Mg\frac{1}{2}\cancel{L} = F\cancel{L}$

$1 \quad \boxed{F = \frac{1}{2}Mg} \quad \boxed{\text{upward}} \quad 1$

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

2 b. The angular acceleration of the rod about the axis

$1 \quad \Sigma\tau = I\alpha$

$\cancel{M}g\frac{1}{2}L = \frac{1}{3}\cancel{M}L^2\alpha$

$\alpha = \frac{3g\cancel{L}}{2L^{\cancel{2}}} = \boxed{\frac{3g}{2L}} \quad 1$

2 c. The translational acceleration of the center of mass of the rod

$1 \quad a = r\alpha$

$a = \frac{1}{2}\cancel{L}\left(\frac{3g}{2\cancel{L}}\right) = \boxed{\frac{3}{4}g} \quad 1$

3 d. The force exerted on the end of the rod by the axis

$1 \quad \Sigma F = Ma$

$Mg - F = Ma \quad 1$

$F = Mg - Ma$

$F = Mg - M\left(\frac{3}{4}g\right)$

$F = \left(\frac{4}{4} - \frac{3}{4}\right)Mg$

$\boxed{F = \frac{1}{4}Mg} \quad 1$

4 The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod has swung.

$\Delta K = \Delta U$

$1 \quad \frac{1}{2}I\omega^2 = mgh \quad 1$

$\cancel{\frac{1}{2}}I\omega^2 = mg\cancel{\frac{1}{2}}L\sin\theta \quad 1$

$\omega^2 = \frac{mgL\sin\theta}{I} = \frac{3\cancel{m}g\cancel{L}\sin\theta}{\cancel{m}L^{\cancel{2}}}$

$\omega^2 = \frac{3g\sin\theta}{L} \quad 1 \quad \boxed{\sqrt{\frac{3g\sin\theta}{L}}}$

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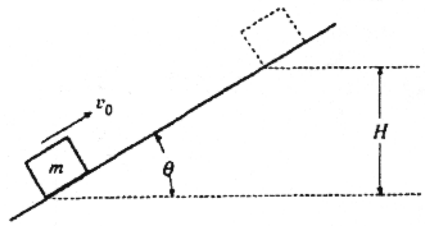


4 1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.

1 $K_o + U_o = K + U$

$\frac{1}{2}mv_o^2 = mgH$

$\boxed{H = \frac{v_o^2}{2g}}$ 1

5 b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.

1 $K_o = W_f + U$

1 $W_f = F_f d$

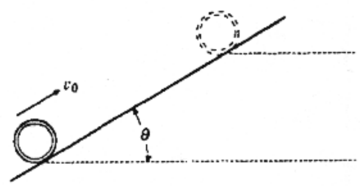
1 $F_f = \mu mg\cos\theta$

$d = \frac{H}{\sin\theta}$ 1

$\frac{1}{2}mv_o^2 = (\mu mg\cos\theta)\left(\frac{H}{\sin\theta}\right) + mgH$

$\frac{1}{2}mv_o^2 = mgH(\mu\cot\theta + 1)$

$\boxed{\frac{v_o^2}{2g(\mu\cot\theta + 1)} = H}$ 1



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_o in the position shown.

4 c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities.

$K_o + U_o = K + U$

1 $(K + K) = U$

$\frac{1}{2}mv_o^2 + \frac{1}{2}I\omega^2 = mgH$ 1

$\frac{1}{2}mv_o^2 + \frac{1}{2}mR^2\frac{v_o^2}{R^2} = mgH$ 1

$\frac{1}{2}v_o^2 + \frac{1}{2}v_o^2 = gH$

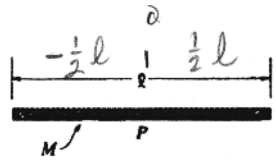
$\boxed{\frac{v_o^2}{g} = H}$

2 d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.

$K_o + U_o = K + U$

$\frac{1}{2}mv_o^2 = mgH$ 1

$\boxed{H = \frac{v_o^2}{2g}}$ 1



1996M3. Consider a thin uniform rod of mass M and length l, as shown above.

a. Show that the rotational inertia of the rod about an axis through its center and perpendicular to its length is $Ml^2/12$.

$dm = \lambda dr$

$\lambda = \frac{M}{l}$

$I = \int r^2 dm$

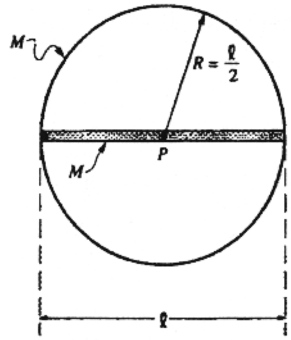
$I = \int r^2 \lambda dr$

$I = \lambda \int_{-\frac{1}{2}l}^{\frac{1}{2}l} r^2 dr$

$I = \frac{M}{l}\left[\frac{r^3}{3}\right]_{-\frac{1}{2}l}^{\frac{1}{2}l}$

$I = \frac{M}{l}\left[\frac{\frac{1}{8}l^3}{3} + \frac{\frac{1}{8}l^3}{3}\right]$ $\frac{M}{l}\left[\frac{\frac{2}{8}l^3}{3}\right]$

$\boxed{I = \frac{1}{12}Ml^2}$



The rod is now glued to a thin hoop of mass M and radius R/2 to form a rigid assembly, as shown above. The centers of the rod and the hoop coincide at point P. The assembly is mounted on a horizontal axle through point P and perpendicular to the page.

b. What is the rotational inertia of the rod-hoop assembly about the axle?

$I_{hoop} = MR^2$

$\Sigma I = M\left(\frac{l}{2}\right)^2 + \frac{1}{12}Ml^2 = \frac{1}{4}Ml^2 + \frac{1}{12}Ml^2 = \frac{4}{12}Ml^2 = \boxed{\frac{1}{3}Ml^2}$

Several turns of string are wrapped tightly around the circumference of the hoop. The system is at rest when a cat, also of mass M, grabs the free end of the string and hangs vertically from it without swinging as it unwinds, causing the rod-hoop assembly to rotate. Neglect friction and the mass of the string.

c. Determine the tension T in the string.

T

$\Sigma F = Ma$

Mg

$Mg - T = Ma$

$T = Mg - Ma$

$\Sigma \tau = I\alpha$

$TR = \frac{1}{3}Ml^2\alpha$

$T\frac{1}{2}l = \frac{2}{3}Mla$

$\alpha = \frac{a}{\left(\frac{l}{2}\right)}$

$\boxed{T = \frac{4}{7}Mg}$

$T = Mg - M\frac{3}{7}g$

d. Determine the angular acceleration a of the rod-hoop assembly.

$T = \frac{2}{3}Ml\alpha$

$\cancel{M}g - \cancel{M}a = \frac{2}{3}\cancel{M}l\alpha$

$g - \frac{3}{7}g = \frac{2}{3}l\alpha$

$g\frac{4}{7} = \frac{2}{3}l\alpha$

$\frac{12g}{14l} = \alpha$

$\boxed{\frac{6g}{7l} = \alpha}$

e. Determine the linear acceleration of the cat.

$T = \frac{4}{3}Ma$

$\cancel{M}g - \cancel{M}a = \frac{4}{3}\cancel{M}a$

$g = \frac{4}{3}a + a$

$g = \frac{7}{3}a$

$\boxed{a = \frac{3}{7}g}$