Name: KEY

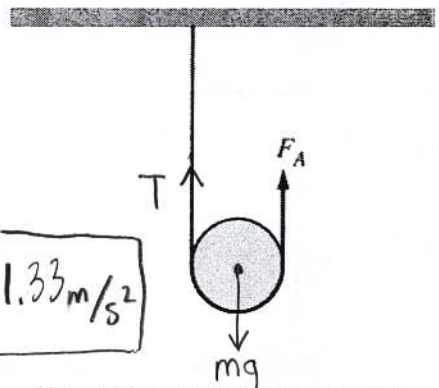
APC Rotational Motion Recitations Part 1

$$2F_A - \tfrac{1}{2}Ma - Mg = Ma$$

$$2F_A - Mg = Ma + \tfrac{1}{2}Ma$$

$$2F_A - Mg = a\,\tfrac{3}{2}M$$

$$a = \frac{2F_A - Mg}{\tfrac{3}{2}M} = \boxed{1.33\ m/s^2}$$



Note: Figure not drawn to scale.

1.
2013
A disk of mass M = 2.0 kg and radius R = 0.10 m is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force F_A . The rotational inertia of the disk around its center is $MR2/ 2$.

(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

$\Sigma F = 0 \qquad T = F_A \qquad 2F_A = mg$

$T + F_A - mg = 0 \qquad F_A = \tfrac{1}{2}mg = \boxed{10\ N}$

At time t = 0, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

$\Sigma F = ma \qquad \Sigma \tau = I\alpha$

(b) Calculate the linear acceleration of the disk.

$T \neq F_A$

$T + F_A - mg = ma \qquad (F_A - T)\cancel{R} = \tfrac{1}{2}MR^{\cancel{2}}\frac{a}{\cancel{R}}$

$(F_A - \tfrac{1}{2}Ma) + F_A - mg = ma \qquad F_A - T = \tfrac{1}{2}Ma$

$T = F_A - \tfrac{1}{2}Ma$

(c) Calculate the angular speed of the disk at t = 3.0 s.

$\omega_0 = 0 \qquad \alpha = \frac{a}{R}$

$\omega = \cancel{\omega_0} + \alpha t \qquad \omega = \frac{a}{R}t = \boxed{40\ rad/s}$

(d) Calculate the increase in total mechanical energy of the disk from t = 0 to t = 3.0 s.

$V = r\omega = 4\ m/s \qquad \Delta x = \cancel{v_0 t} + \tfrac{1}{2}at^2 \qquad E = K + U$

$\Delta x = 6.0\,m = h$

$E = K_{tot} + U$

$E_{tot} = \tfrac{1}{2}I_{cm}\omega^2 + \tfrac{1}{2}Mv^2 + Mgh \qquad E = \tfrac{1}{4}Mv^2 + \tfrac{1}{2}Mv^2 + Mgh$

$E = \tfrac{1}{2}\tfrac{1}{2}M\cancel{R^2}\frac{v^2}{\cancel{R^2}} + \tfrac{1}{2}Mv^2 + Mgh \qquad E = \tfrac{3}{4}Mv^2 + Mgh = 24 + 120 = \boxed{144\ J}$

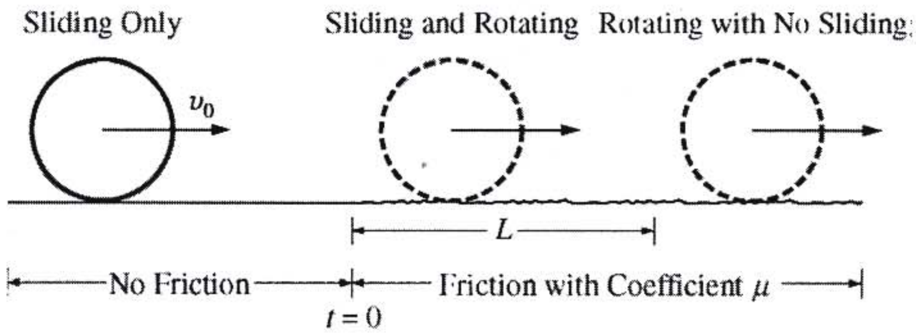
(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

Greater than Less than The same as

Justify your answer. more moment of inertia means less a for the same $\tau \approx F$

$I_{hoop} = MR^2$

$I_{hoop} > I_{cylinder}$



2. 2012

A ring of mass M, radius R, and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L, the ring begins rolling without sliding. Express all answers to the following in terms of M, R, v_0, μ, and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating. $\Sigma F = Ma$ $-F_f = Ma$

i. The linear velocity v of the ring as a function of time t $-\mu Mg = Ma$

$a = \frac{dv}{dt}$ $-\mu Mg = M\frac{dv}{dt}$ $\boxed{\frac{dv}{dt} = -\mu g}$

ii. The angular velocity w of the ring as a function of time t

$\Sigma \tau = I\alpha$

$F_f R = MR^2\alpha$

$\mu \cancel{M} g \cancel{R} = \cancel{M} R^{\cancel{2}} \alpha$

$\mu g = R\frac{d\omega}{dt}$

$\boxed{\frac{d\omega}{dt} = \frac{\mu g}{R}}$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$\frac{dv}{dt} = -\mu g$$

$$dv = -\int \mu g\, dt$$

$$\boxed{V = V_0 - \mu g t}$$

ii. The angular velocity w of the ring as a function of time t

$$\frac{d\omega}{dt} = \frac{\mu g}{R} \qquad d\omega = \int \frac{\mu g}{R}\, dt$$

$$\boxed{\omega = \frac{\mu g t}{R}} + \omega_0 \quad \varnothing$$

(c) Derive an expression for the time it takes the ring to travel the distance L.

$$\omega = \frac{V}{R} \qquad \omega = \frac{\mu g t}{R} \qquad V = \mu g t$$

$$\frac{V}{\cancel{R}} = \frac{\mu g t}{\cancel{R}} \qquad V_0 - \mu g t = \mu g t$$

$$V_0 = 2\mu g t$$

$$\boxed{t = \frac{V_0}{2\mu g}}$$

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L.

$$V = V_0 - \cancel{\mu g}\left(\frac{V_0}{2\cancel{\mu g}}\right) \qquad \boxed{V = \frac{1}{2}V_0}$$

$$V = V_0 - \frac{1}{2}V_0$$

(e) Derive an expression for the distance L.

$$V^2 = V_0^2 + 2a\Delta X$$

$$\left(\frac{V_0}{2}\right)^2 = V_0^2 + 2(-\mu g)L$$

$$\frac{V_0^2}{4} = V_0^2 - 2\mu g L$$

$$\frac{1}{4}V_0^2 - V_0^2 = -2\mu g L$$

$$\cancel{-}\frac{3}{4}V_0^2 = \cancel{-}2\mu g L$$

$$\boxed{\frac{3V_0^2}{8\mu g} = L}$$

$\Sigma F = ma$

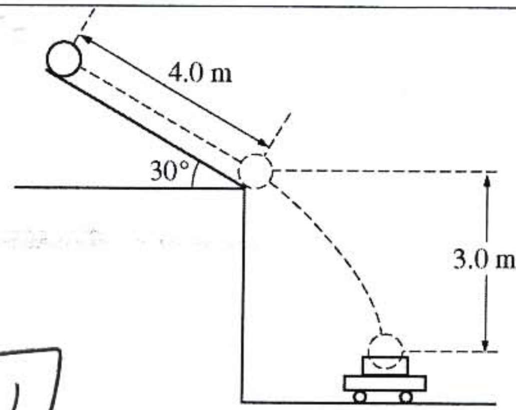
b) $Mg\sin\theta - F_f = Ma$

$\Sigma \tau = I\alpha$

$F_f \cdot R = \frac{2}{5}MR^2\left(\frac{a}{R}\right)$

$F_f = \frac{2}{5}Ma$

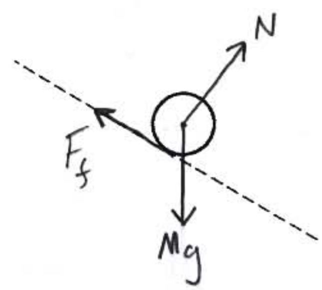
$F_f = \frac{2}{5}M(3.57) = \boxed{8.6N}$



Note: Figure not drawn to scale.

3. $\boxed{2010}$
A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$Mg\sin\theta - \left(\frac{2}{5}Ma\right) = Ma$ $\qquad \cancel{M}g\sin\theta = \frac{7}{5}\cancel{M}a$

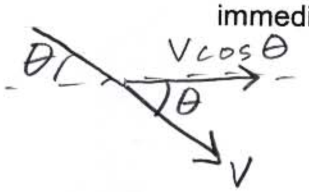
$Mg\sin\theta = Ma + \frac{2}{5}Ma$ $\qquad a = \frac{5}{7}g\sin\theta = 3.57\,m/s^2$

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

$V^2 = V_0^2 + 2a\Delta X$

$V = \sqrt{2a\Delta X} = \boxed{5.34\,m/s}$

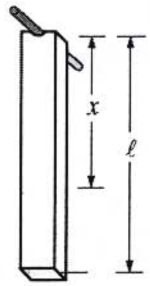
d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.



(COM) $\quad Mv\cos\theta = (M+m)V_f$

$V_f = \frac{Mv\cos\theta}{M+m} = \boxed{1.54\,m/s}$

Always evaluate from CM



treat like pendulum

$F = -kx$

$\tau = -I_b\theta$

Skip 4. 2009

You are given a long, thin, rectangular bar of known mass M and length ℓ with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b, the given quantities, and fundamental constants.

(a)

i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\tau = I\alpha \qquad -Mg\sin\theta\, x = I_b\alpha$$

$$-Mg\sin\theta\, x = I_b\left(\frac{d^2\theta}{dt^2}\right)$$

ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

$$\sin\theta = \theta \qquad -Mgx\theta = I_b\left(\frac{d^2\theta}{dt^2}\right) \qquad T = \frac{2\pi}{\omega}$$

$$\emptyset = \frac{Mgx\theta}{I_b} + \frac{d^2\theta}{dt^2} \qquad T = 2\pi\sqrt{\frac{I_b}{Mgx}}$$

$$\omega^2 = \frac{Mgx}{I_b}$$

(b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b. Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

Find the period by timing the oscillation.

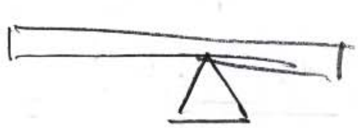
Use $T = 2\pi\sqrt{\frac{I_b}{Mgx}}$

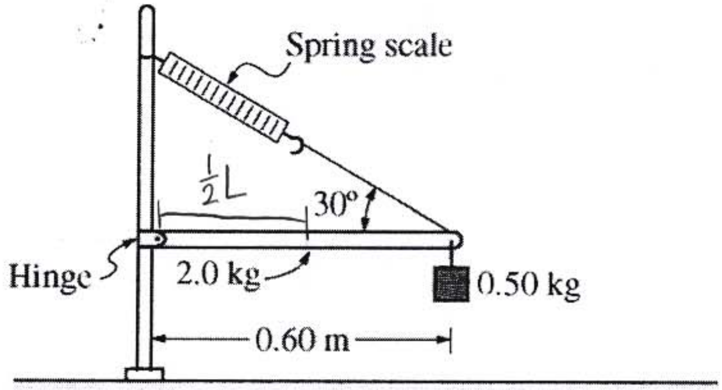
$$\frac{\left(\frac{T}{2\pi}\right)^2}{Mgx} = T$$

(c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

Use a fulcrum and change the position to acheive $\emptyset$ net torque.

$\Sigma\tau = \emptyset$

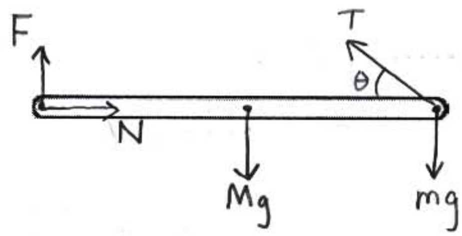




5. 2008

The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30^0 with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.



(b) Calculate the reading on the spring scale.

$\Sigma \tau = \varnothing$

$TL\sin\theta - mgL - Mg\frac{1}{2}L = \varnothing$

$TL\sin\theta = Mg\frac{1}{2}L + mgL$

$TL\sin\theta = \frac{3}{2}gL(M+m)$

$T = \frac{3gL(M+m)}{2L\sin\theta} = \boxed{30N}$

(c) The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length. Calculate the rotational inertia of the rod-block system about the hinge.

$I = I_{cm} + MD^2$

$I = \frac{1}{12}ML^2 + M\frac{L}{2}^2$

$I = \frac{1}{12} + \frac{1}{4}(ML^2)$

$I = \frac{4}{12}ML^2 = \frac{1}{3}ML^2$

point mass

$I_{tot} = \frac{1}{3}ML^2 + mL^2$

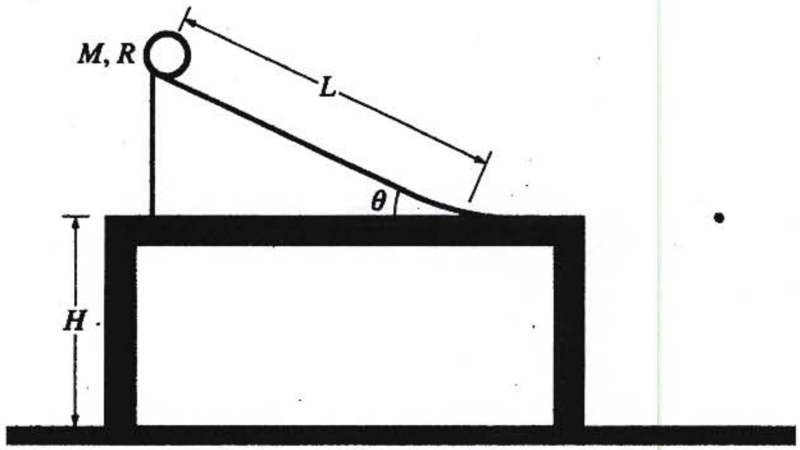
$I_{tot} = 0.24 + 0.18 = \boxed{0.42\ kg \cdot m^2}$

(d) If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

$\Sigma \tau = I\alpha$

$mgL + Mg\frac{1}{2}L = I\alpha$

$\alpha = \frac{mgL + Mg\frac{1}{2}L}{I} = \frac{3+6}{0.42} = \boxed{21.4\ rad/s^2}$



6. 2006

A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

$\Sigma F = Ma$ a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

$Mg\sin\theta - F_f = Ma$ $\qquad \Sigma\tau = I\alpha$ $\qquad \boxed{a = \frac{1}{2}g\sin\theta}$

$F_f R = MR^2\left(\frac{a}{R}\right)$ $\qquad Mg\sin\theta - Ma = Ma$

$F_f = Ma$ $\qquad \cancel{M}g\sin\theta = 2\cancel{M}a$

b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

$E_0 = E$ $\qquad \cancel{M}gh = \cancel{M}v^2$ $\qquad h = L\sin\theta$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ $\qquad \boxed{v = \sqrt{gh}}$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$

c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

$\Delta x = vt$ $\qquad \Delta x = \sqrt{gh}\cdot\sqrt{\frac{2H}{g}} = \sqrt{\cancel{g}h\cdot\frac{2H}{\cancel{g}}} = \boxed{\sqrt{L\sin\theta\cdot 2H}}$

$t = \sqrt{\frac{2H}{g}}$

d. Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

_____ Less than _____ The same as __✓__ Greater than

Briefly justify your response.

A solid disk has a smaller moment of inertia and therefore be traveling faster at the bottom of the ramp.