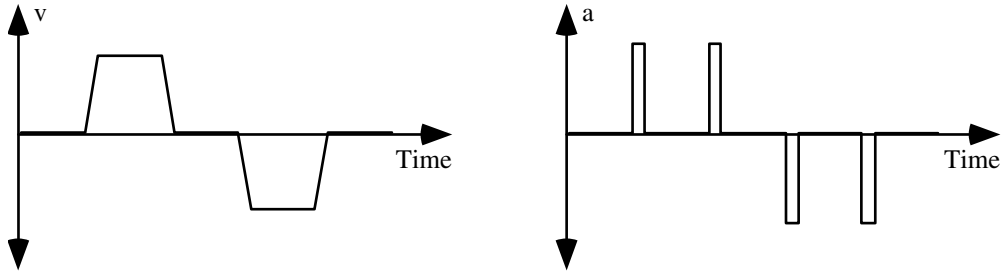
CONTENTS

NT3 MOTION IN ONE DIMENSION

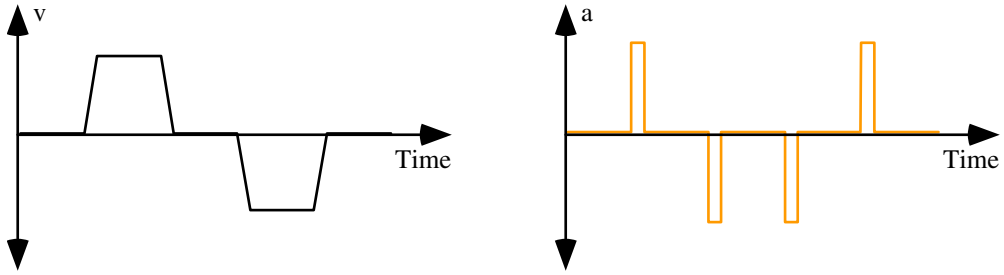
NT3A-WWT1: VELOCITY VS. TIME GRAPH II—ACCELERATION VS. TIME GRAPH

A student obtains a graph of an object's velocity versus time and then draws the graph of the acceleration versus time for the same time interval.



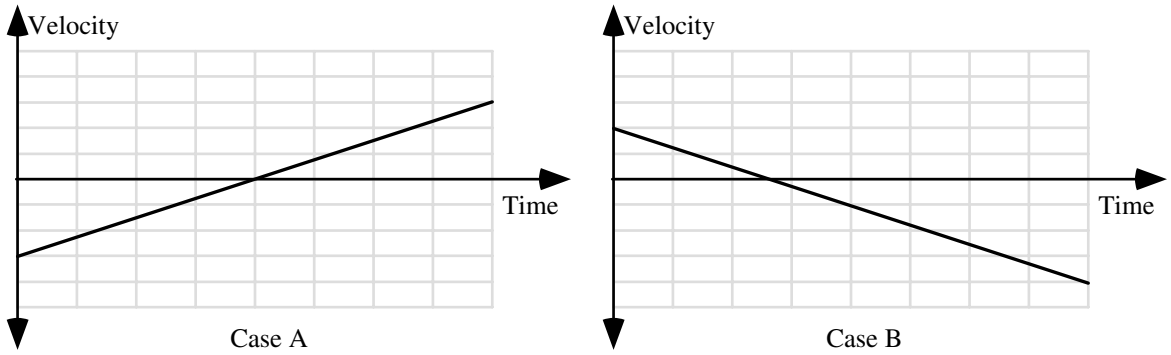
What, if anything, is wrong with the graph of the acceleration versus time? If something is wrong, identify it and explain how to correct it. If the graph is correct, explain why.

Answer: The acceleration is the slope of the velocity graph. Thus, the third and fourth peaks should be reversed as shown below because the sign of the acceleration is the same as the sign of the slope of the velocity-time graph.



NT3A-CT2: VELOCITY VS. TIME GRAPHS—DISPLACEMENT

The graphs represent the velocity of two toy robots moving in one dimension for a particular time interval. Both graphs have the same time and velocity scales.



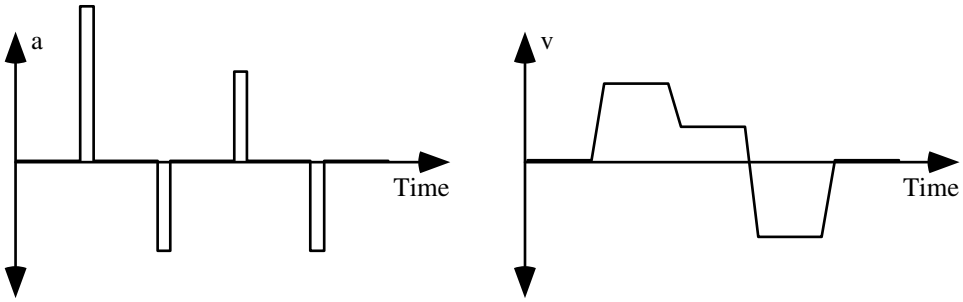
Is the magnitude of the displacement of the robot in Case A *greater than, less than,* or *equal to* the magnitude of the displacement of the robot in Case B?

Please explain your reasoning.

The displacement is the area between the velocity-time curve and the time axis. The displacement in Case A is zero while the displacement for case B is negative. The magnitude of the displacement is greater in case B.

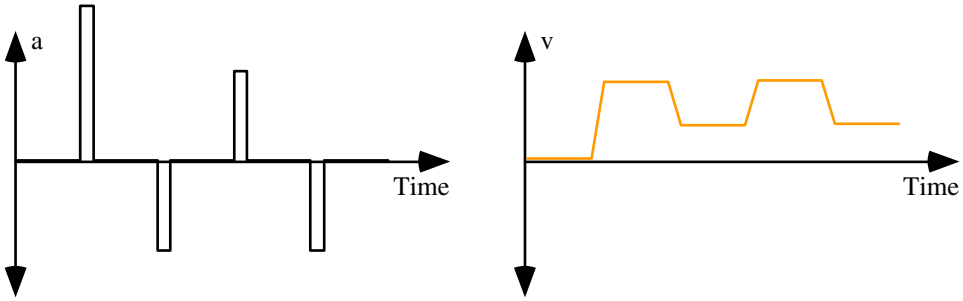
NT3A-WWT3: ACCELERATION VS. TIME GRAPH—VELOCITY VS. TIME GRAPH

A student obtains a graph of an object's acceleration versus time and then draws the graph of the velocity versus time for the same time interval. The object starts from rest.



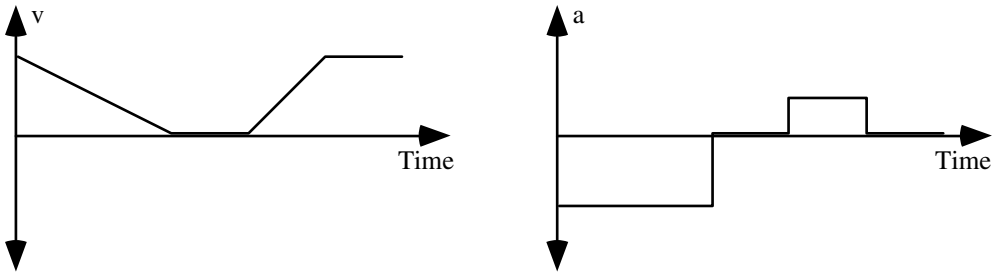
What, if anything, is wrong with the graph of velocity versus time? If something is wrong, identify it and explain how to correct it. If the graph is correct, explain why.

Answer-the change in velocity is related to the area under the acceleration vs time graph. Thus the velocity graph should be like the graph below. The second, third and fourth accelerations are all the same magnitude and are smaller than the initial acceleration, so the velocity does not return to zero at the end of the interval.



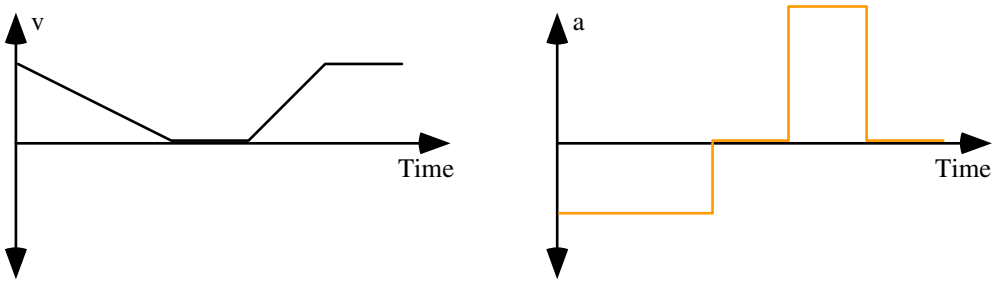
NT3A-WWT4: VELOCITY VS. TIME GRAPH—ACCELERATION VS. TIME GRAPH

A student obtains a graph of an object's velocity versus time and then draws the graph of the acceleration versus time for the same time interval.



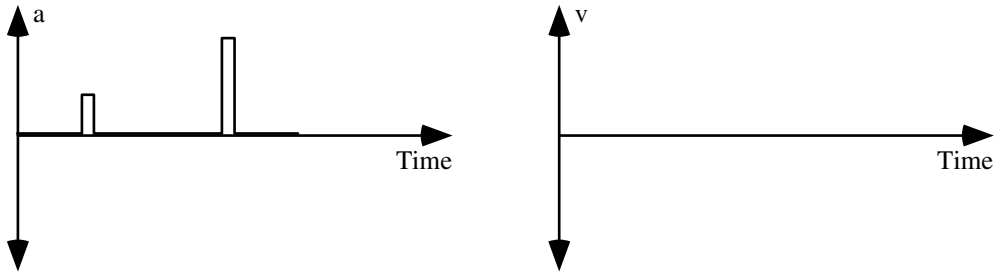
What, if anything, is wrong with the graph of the acceleration versus time? If something is wrong, identify it and explain how to correct it. If the graph is correct, explain why.

Answer-the change in velocity is related to the area under the graph of acceleration vs. time or the acceleration is related to the slope of the velocity graph, so we need to adjust the acceleration graph to bring the velocity back to the original value. Changing the acceleration graph so that both rectangles have the same area gives this:



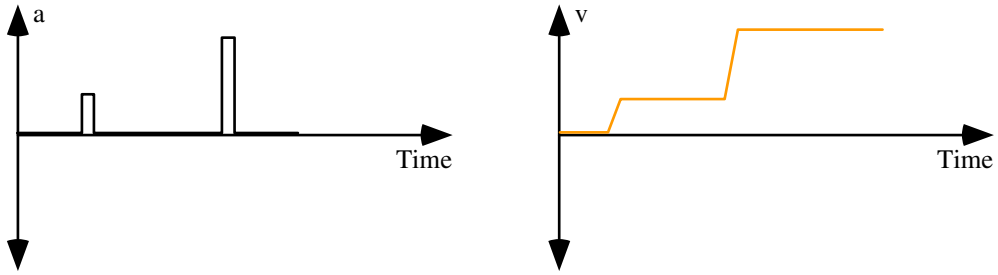
NT3A-CRT5: ACCELERATION VS. TIME GRAPH—VELOCITY VS. TIME GRAPH

Sketch a possible velocity versus time graph given the acceleration graph for the same time interval.



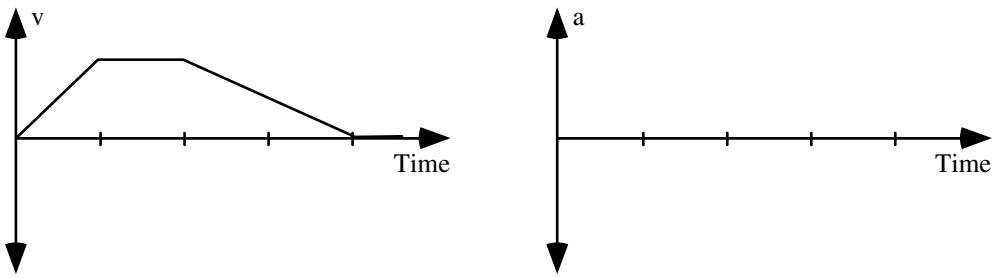
Explain.

Answer-the change in the velocity is related to the area under the acceleration graph. The second blip is taller than the first so it contains more area. Consequently it represents a greater change in velocity. Since there is no particular initial velocity given, any graph of the same shape but starting at some other initial velocity would also be a correct choice.



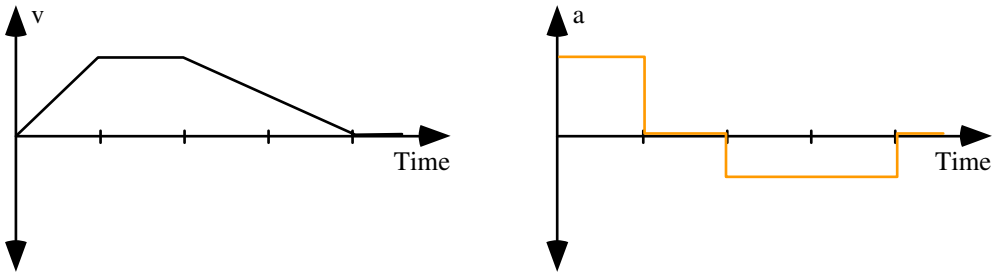
NT3A-CRT6: VELOCITY VS. TIME GRAPH—ACCELERATION VS. TIME GRAPH

Sketch the acceleration versus time graph given the velocity versus time graph for the same time interval.



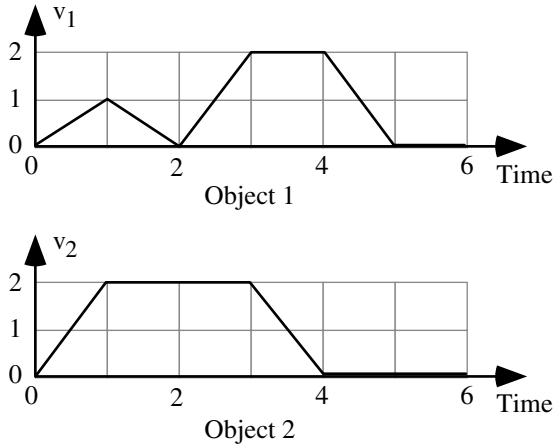
Explain.

Answer-the acceleration is the slope of the velocity graph thus we have a positive acceleration for the first interval, then zero since the velocity doesn't change and finally a smaller—lower slope—negative acceleration during the last two intervals.



NT3A-CT7: VELOCITY VS. TIME GRAPHS OF TWO OBJECTS I—DISPLACEMENT

The graphs below show the velocity of two objects during the same time interval.



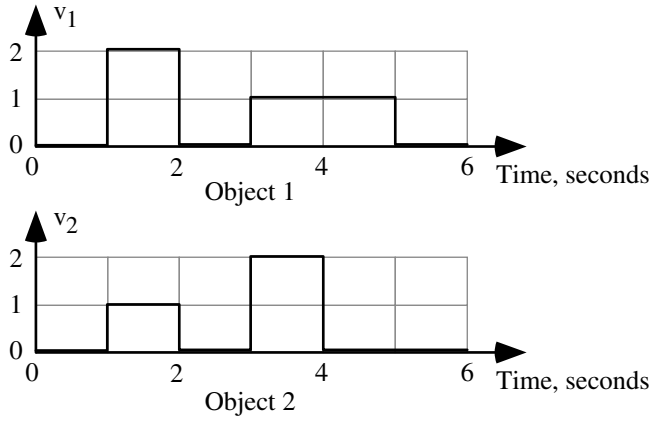
After 5 seconds, is the displacement of Object 1 in the top graph *greater than, equal to,* or *less than* the displacement of Object 2 in the bottom graph?

Please explain.

Answer- The displacement of Object 1 is less than Object 2. Object 2 has a larger displacement since the displacement is equal to the area under the graph of velocity vs. time and the area is larger for Object 2.

NT3A-CT8: VELOCITY VS. TIME GRAPHS OF TWO OBJECTS II—DISPLACEMENT

The graphs below show the velocity of two objects during the same time interval.



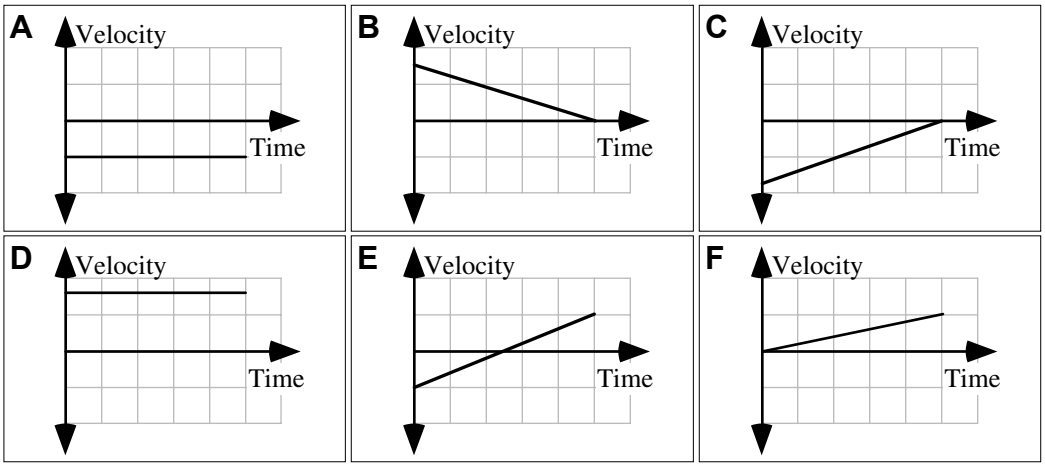
After 5 seconds, is the displacement of Object 1 in the upper graph *greater than, equal to,* or *less than* the displacement of Object 2 in the lower graph?

Please explain.

Answer: The displacement is greater for Object 1 since the displacement is equal to the area under the line or curve representing the motion in a graph of velocity vs. time.

NT3A-RT9: VELOCITY VS. TIME GRAPHS—DISPLACEMENT

Shown below are six velocity-time graphs for toy robots that are traveling along a straight course. All the robots are initially facing the same way. All graphs have the same time and velocity scales.



Rank these situations on the basis of the displacement during these intervals.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The displacement during these intervals is the same but not zero for all these robots. ___

OR, The displacement during these intervals is zero for all these robots. ___

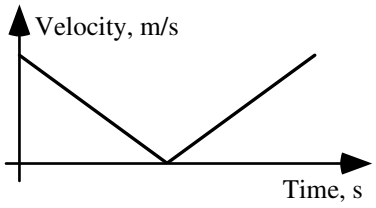
OR, We cannot determine the ranking for the displacements of these robots. ___

Please explain your reasoning.

Answer: D > B > F > E > C > A; since these are velocity versus time graphs the displacements are given by the areas "under" (between the lines and the time axis) the lines in the graphs.

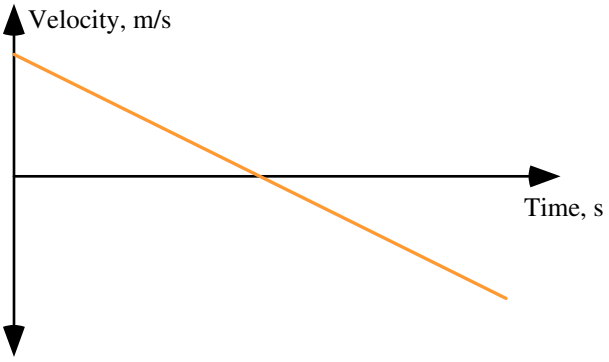
NT3A-WWT10: BALL THROWN UPWARD AND COMES BACK DOWN—VELOCITY VS. TIME GRAPH

A ball is thrown straight upward and falls back to the same height. A student makes this graph of the velocity of the ball as a function of time.



What, if anything, is wrong with the student's graph? If something is wrong, explain the error and how to correct it. If the graph is correct, explain why.

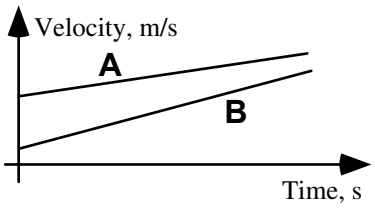
The line should go across the axis and continue in a straight line into the negative region since the velocity is negative on the way back down. The slope should be constant since the acceleration is constant. Note the graph would be correct if it were a graph of the speed of the object.



NT3A-WWT11: VELOCITY VS. TIME GRAPH OF TWO OBJECTS—FASTEST OBJECT

A student is shown the velocity-time graphs for two objects and is asked to decide which object is moving faster. The student responds:

"B is faster because it has the steeper slope."



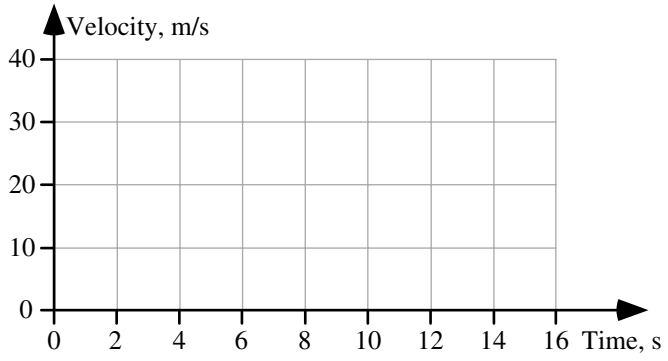
What, if anything, is wrong with the student's statement? If something is wrong, explain the error and how to correct it. If the statement is correct, explain why.

Answer: A is faster because its velocity value is above that of B at any time shown on the graph. At any time, then, it is moving faster than B is. The slope of the velocity-time graph is the acceleration. Therefore, B has the larger acceleration during the entire time because it has the larger slope.

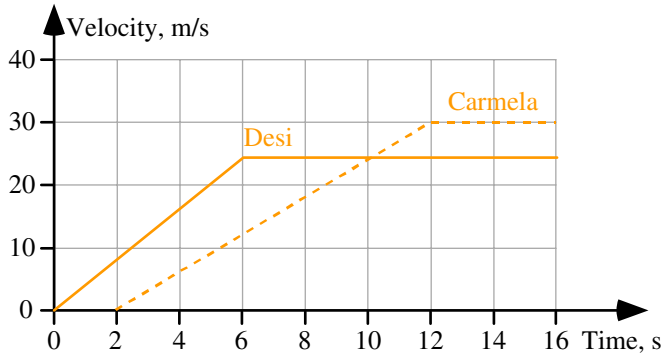
NT3A-CRT12: TRAVELING STUDENTS—VELOCITY VS. TIME GRAPH

Carmela and Desi leave their physics classroom separately and travel west. They both start from rest. Desi left first, traveling with an acceleration of 4 m/s^2 west for the first 6 seconds, and then he traveled at a constant velocity. Two seconds after Desi started, Carmela began with an acceleration of 3 m/s^2 west for 10 seconds, and after that she traveled at a constant velocity.

Graph the velocity of both travelers as a function of time up to *t* = 16 seconds starting at time *t* = 0 when Desi leaves the classroom. Use a solid line for Desi's velocity and a dashed line for Carmela's velocity.

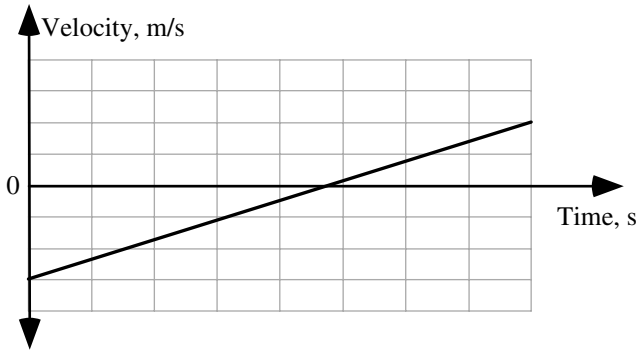


Answer-



NT3A-CCT13: VELOCITY VS. TIME GRAPH—DISPLACEMENT

The graph shown represents the velocity of a toy robot moving in one dimension for a particular time interval. Three students studying this graph make the following statements:



Arnold: *"The robot's displacement is positive because the slope of the graph is positive."*

Betty: *"No, the robot's displacement will be zero since it moves in both the positive and negative directions during this time."*

Cindy: *"I think the displacement is negative since the robot has a negative velocity for a longer time."*

Which, if any, of these three students do you agree with and think is correct?

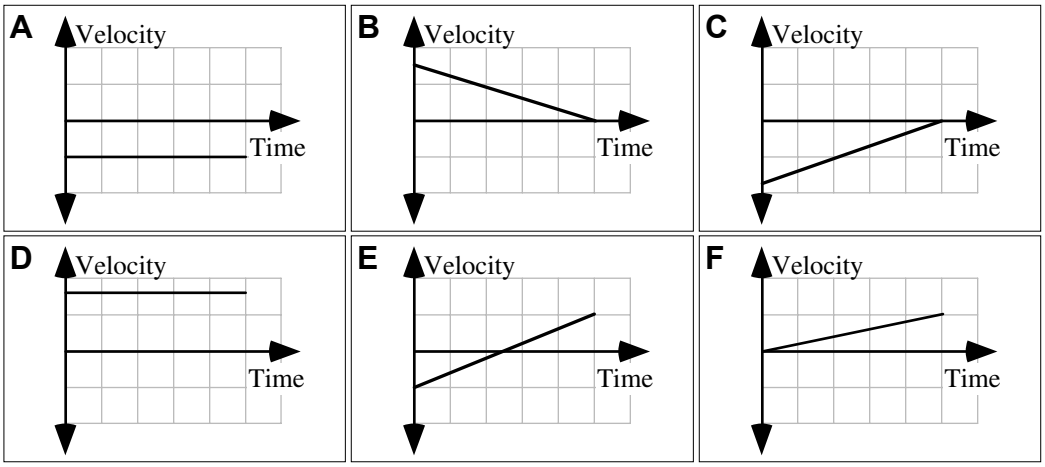
Arnold _____ Betty _____ Cindy _____ None of them_____

Please explain your reasoning.

Answer: Cindy is correct but her reasoning could be more complete. The displacement can be found from the area between the time axis and the velocity-time graph. For the time interval shown, the robot moves in the negative direction until the graph crosses the axis, and the displacement in this direction is equal to the area of the triangle bounded by the two axes and the line representing the robot's velocity. This area is larger than the area of the triangle above the time axis representing the robot's displacement in the positive direction up to the end of this time interval, so there is an overall displacement in the negative direction.

NT3A-RT14: VELOCITY VS. TIME GRAPHS—DISTANCE TRAVELED

Velocity-time graphs for six toy robots that are traveling along a straight path are shown. All graphs have the same time and velocity scales.



Rank these situations on the basis of the distance traveled during these intervals.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The distance during the intervals indicated is the same but not zero for all these robots. ___

OR, The distance during the intervals indicated is zero for all these robots. ___

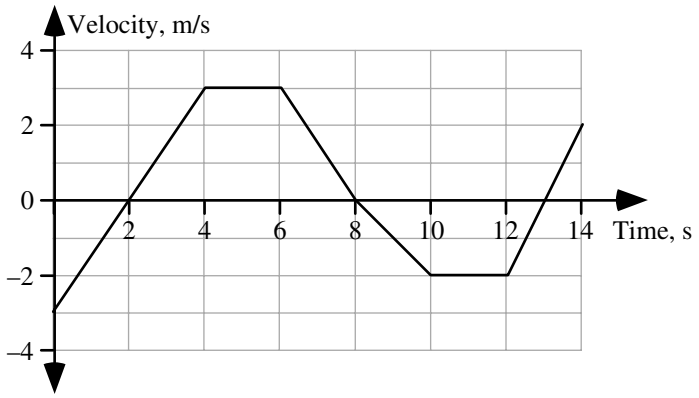
OR, We cannot determine the ranking for the distances traveled of these robots. ___

Please explain your reasoning.

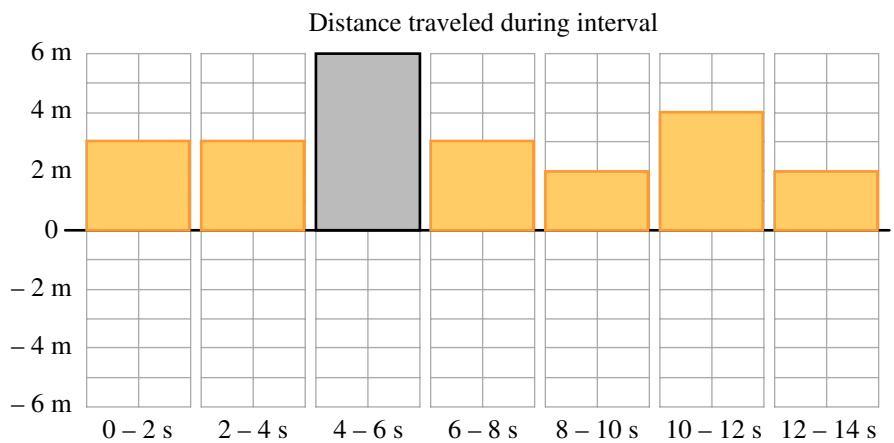
Answer: D > A > B = C > E = F. The distance traveled is the sum of the absolute values of the areas between the horizontal axis and the graph. For graphs of motion that don't change direction (all cases except E), the magnitude of the displacement is the same as the distance traveled. For case, E, the distance traveled is the sum of the distance traveled backward and the distance traveled forward. The absolute value of the two areas in case E is the same as the area in case F.

NT3A-BCT15: VELOCITY VS. TIME GRAPH—DISTANCE TRAVELED

The graph represents the motion of a toy robot moving in one dimension during a 14-second interval.



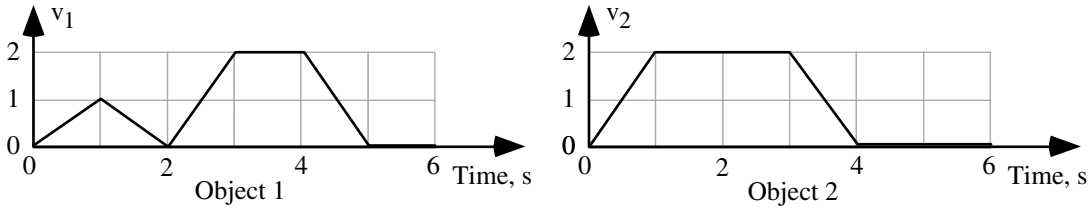
In the histogram below, the bar represents the distance the robot travels during the 2-second interval from 4 s to 6 s. **Draw additional bars to represent the distance traveled during the other 2-second intervals.**



Since we are looking for a distance traveled rather than a displacement, we use the absolute value of the area between the line and the time axis.

NT3A-CCT16: VELOCITY VS. TIME GRAPHS OF TWO OBJECTS I—DISPLACEMENT

The graphs below show the velocity of two objects during the same time interval.



Three students are discussing the displacements of these objects for this interval.

Andy: *"I think Object 2 will have the greater displacement because it gets to a higher speed faster than Object 1."*

Badu: *"No, Object 1 will have the greater displacement because it travels for longer than Object 2."*

Connor: *"I think Andy has the right answer, but for the wrong reason. We can see that Object 2 has the larger displacement because the area under the graph is greater."*

Which, if any, of these three students do you agree with?

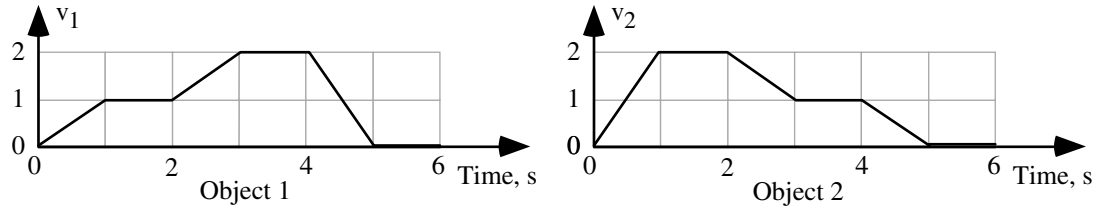
Andy_____ Badu _____ Connor _____ None of them_____

Please explain your reasoning.

Answer: Coen is correct, because the displacement is determined by the area under a velocity-time graph.

NT3A-CCT17: VELOCITY VS. TIME GRAPHS OF TWO OBJECTS II—DISPLACEMENT

The graphs below show the velocity of two objects during the same time interval.



Three students are discussing the displacements of these objects for this interval.

Apriel: *"I think Object 2 will have the greater displacement because it gets to a higher speed faster than Object 1."*

Brody: *"Object 1 spends most of its time speeding up, but object 2 spends most of its time slowing down. Object 1 will go farther."*

Cyril: *"The displacement is found from the integral or area of the velocity graphs. But in this case we don't know what the integration constant or the initial position is that we need to add to the integral or area. We don't have enough information to find the displacement."*

Which, if any, of these three students do you agree with?

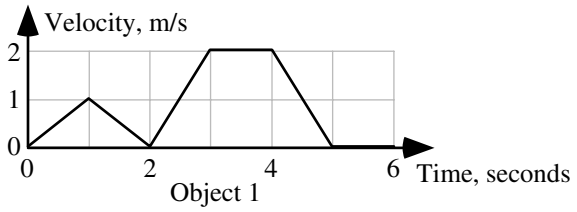
Apriel_____ Brody _____ Cyril _____ None of them_____

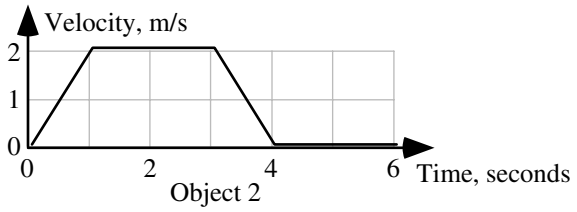
Please explain your reasoning.

Answer: None of these students is correct. The displacement is determined by the area under the velocity-time graph, which is the same in both cases.

NT3A-CRT18: VELOCITY VS. TIME GRAPHS OF TWO OBJECTS I—VELOCITY EQUATIONS

The graphs below show the velocity of two objects during the same time interval.





Write the equations for the velocity in m/s as a function of time in seconds for these two motions for the first 5 seconds.

Time	**Velocity Equation for Object 1**	**Velocity Equation for Object 2**	
0 s—1 s		$v(t) =$	$v(t) =$
1 s—2 s		$v(t) =$	$v(t) =$
2 s—3 s		$v(t) =$	$v(t) =$
3 s—4 s		$v(t) =$	$v(t) =$
4 s—5 s		$v(t) =$	$v(t) =$

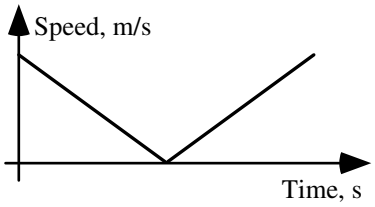
Answer: one solution is a fit to line mt +b where m is the slope which can be read from graph and then determine b by fitting it to one end or the other of the segment (check by testing it against the other end of the segment)

Interval	***Object 1***	***Object 2***
0 s—1 s	***v (t) = 1(m/s2) t***	***v (t) = 2(m/s2) t***
1 s—2 s	***v (t) = 2(m/s) - 1(m/s2) t***	***v (t) = 2(m/s)***
2 s—3 s	***v (t) = -4 (m/s)+ 2(m/s2) t***	***v (t) = 2(m/s)***
3 s—4 s	***v (t) = 2 (m/s)***	***v (t) = 8(m/s) - 2(m/s2) t***
4 s—5 s	***v (t) = 10 (m/s) - 2(m/s2) t***	***v (t) = 0***

Another but more physical equation/approach would use the velocity at the beginning of the interval and use the time elapsed during that interval. So for object 2 in the interval between 3 and 4 seconds this would give v (t) = 2(m/s) - 2(m/s^2) (t- 3 s) for example.

NT3A-CCT19: BALL THROWN UPWARD AND COMES BACK DOWN—ACCELERATION

A ball is thrown straight upward and falls back to the same height. A student makes the graph of the speed of the ball as a function of time. Three students who are discussing this graph make the following contentions:



Akira: *"I don't think this can be correct because the sign of the acceleration changes on this graph, but the acceleration on the ball will be constant."*

Burt: *"No, I think this is right because it is only showing what happens to the speed, which will decrease to zero at the top and then increase as the ball falls. Since the slopes for both segments are the same except for sign that means the acceleration is constant."*

Catalina: *"This graph makes sense to me because it shows the speed decreasing. I disagree with Burt, because I think this means the acceleration is also decreasing until the ball gets to the top and stops. Then both the speed and acceleration increase as the ball falls down again."*

Which, if any, of these three students do you agree with and think is correct?

Akira _____ Burt _____ Catalina _____ None of them_____

Please explain your reasoning.

Answer: Burt is correct because a speed versus time graph will have a negative slope as the ball moves upward, since it is slowing down, and then a positive slope coming down since it is speeding up. But both lines have to have the same slope value since the acceleration is constant throughout.

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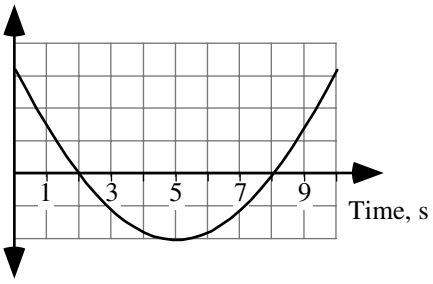
NT3A-QRT20: KINEMATICS GRAPHS—CHANGE DIRECTION

The graph at right is for an object in one-dimensional motion. The vertical axis is not labeled.



a) If the vertical axis is position, does the object ever change direction? If so, at what time or times does this change in direction occur?

Explain.

Answer: The velocity of the object is given by the slope of the graph, and the object has a negative velocity from time zero to 5 seconds. After 5 seconds, the object has a positive velocity. So the object changed direction at 5 seconds.

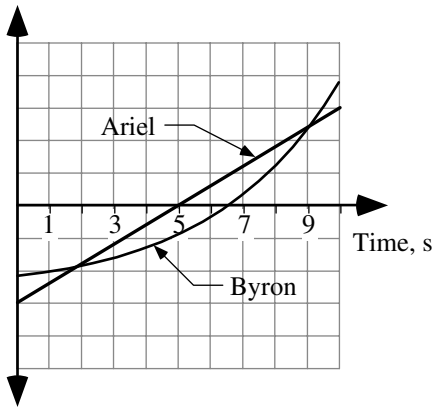
b) If the vertical axis is velocity, does the object ever change direction? If so, at what time or times does this change in direction occur?

Explain.

Answer: The velocity of the object is given by the value of the graph, and the object has a positive velocity from time zero to 2 seconds. From 2 seconds to 8 seconds, the object has a negative velocity. After 8 seconds, the object has a positive velocity. So the object changed direction at 2 seconds, and again at 8 seconds.

NT3A-QRT21: POSITION TIME GRAPHS OF TWO CHILDREN—KINEMATICS

The graph at right is of the motion of two children, Ariel and Byron, who are moving along a straight hallway. The vertical axis is not labeled.



a) If the vertical axis is position, does either child ever change direction? If so, at what time or times does this change in direction occur?

Explain.

Answer: The velocity of the object is given by the slope of the graph, and the slope of the graph is positive for both children at all times shown. So both children are moving in the same direction along the hallway at all times and do not change direction.

b) If the vertical axis is position, are the two children ever at the same position along the hallway? If so, at what time or times?

Explain.

Answer: The two children will be in the same position along the hallway for times that the graphs of their motions have the same position value. So when the lines cross at 2 seconds and at 9 seconds they are at the same position.

c) If the vertical axis is position, do the two children ever have the same speed? If so, at what time or times?

Explain.

Answer: The velocity of the object is given by the slope of the graph, and the speed is the absolute value of the velocity. The two children have the same speed when the slopes of the graphs have the same absolute value. A tangent line drawn to a point on the curve representing Byron's position gives his speed, and at around 6 seconds this tangent will be parallel to the line representing Ariel's position. The two children have the same speed at about 6 seconds.

d) If the vertical axis is position, do the two children ever have the same acceleration? If so, at what time or times?

Explain.

Answer: The velocity of the object is given by the slope of the graph, and for Ariel this slope is constant in time. So Ariel's velocity does not change with time, and Ariel's acceleration is zero. That is, Ariel is moving down the hallway at a constant speed. On the other hand, the slope of Byron's position-time graph is always different, meaning that Byron's velocity is always changing with time. So Byron has a nonzero acceleration at all times. The two children never have the same acceleration.

e) If the vertical axis is velocity, do either of the children ever change direction? If so, at what time or times does this change in direction occur?

Explain.

Answer: The velocity of each child is given by the value of the graph, so Ariel's velocity changes from negative to positive at time 5 seconds, and Byron's velocity changes from negative to positive approximately at time 6.5 seconds. So Ariel changes direction at 5 seconds and Byron changes direction at 6.5 seconds. Between 5 and 6.5 seconds, the children are moving in opposite directions along the hallway.

f) If the vertical axis is velocity, do the two children ever have the same velocity? If so, at what time or times?

Explain.

Answer: The velocity of each child is given by the value of the graph, so Ariel's velocity is the same as Byron's velocity at times 2 seconds and 9 seconds.

g) If the vertical axis is velocity, do the two children ever have the same acceleration? If so, at what time or times?

Explain.

Answer: The acceleration of each child is given by the slope of the graph, so Ariel's velocity is the same as Byron's velocity when the graphs have the same slope, at around time 6 seconds.

NT3B-WWT22: MOTION EQUATIONS IN ONE DIMENSION—AVERAGE VELOCITY

An object moves along the x-axis according to the following (with x in meters and t in seconds):

$$x = 10\text{m} - 4(\text{m/s})\,t + 2(\text{m/s}^3)\,t^3$$

A student is planning to calculate the object's average velocity between $t = 0$ and $t = 2$ seconds using the equation

$$v_{average} = \frac{v_{initial} + v_{final}}{2}$$

What, if anything, is wrong with this? If something is wrong, identify it and explain how to correct it. If this is correct, explain why.

This equation is only valid for constant acceleration, and in this case the acceleration is changing with time (note the t^3 term or the a=12t).

NT3B-WBT23: POSITION EQUATION—PHYSICAL SITUATION

Describe the motion of an object that is represented by the equation below:

$$x = 33.6 \text{ m} - (2.8 \text{ m/s})t$$

Answer:
This object began 33.6 m away from the origin and traveled at a constant velocity of 2.8 m/s in the negative direction towards the origin, reaching it after 12 seconds. It continues to move at 2.8 m/s in the negative direction away from the origin at 12 seconds.

NT3B-CCT24: BICYCLIST ON A STRAIGHT ROAD—AVERAGE SPEED

Three students are discussing a situation where a bicyclist travels at a steady 18.0 m/s for 10 minutes, then at 6.0 m/s for 20 minutes and finally at 12.0 m/s for 15 minutes along a straight level road. Students make the following contentions about the bicyclist's average speed for the overall trip:

Aaron: *"I think the average speed for the entire period is 18 m/s because to find an average you sum the three values and divide by two."*

Bessie: *"I disagree. The average speed is 12 m/s because you add the three velocities, but then you have to divide by three."*

Cesar: *"No, you are both wrong. The average speed is 10.7 m/s because that is what you get when you divide 28,800 m, the total distance traveled on the straight road, by 2700 seconds, the total time it took."*

Which, if any, of these three students do you agree with?

Aaron_____ Bessie _____ Cesar _____ None of them_____

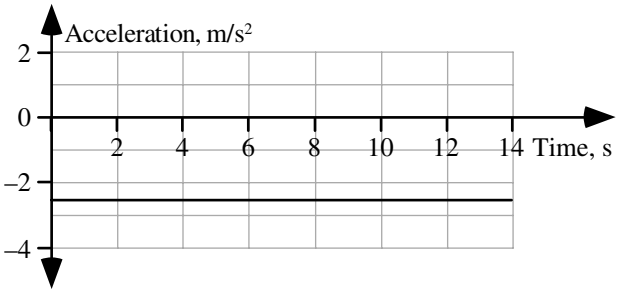
Please explain your reasoning.

Answer: Cesar is correct because the average speed is the displacement (here, the distance traveled) divided by the total time.

NT3C-WWT25: ACCELERATION VS. TIME GRAPH—FINAL VELOCITY

A student is given the acceleration versus time graph for a motorcyclist traveling along a straight level stretch of road. The student states:

> *"This motorcyclist was slowing down during the period up to 14 seconds because her acceleration was negative during this period."*



What, if anything, is wrong with this student's contention? If something is wrong, identify it, and explain how to correct it. If it is correct, explain why.

.Answer: The student cannot make this contention because he/she doesn't know whether the motorcyclist's initial velocity was positive or negative. If the cyclist started out with an initial negative velocity then his/her speed was increasing, not decreasing, during the interval graphed.

NT3C-QRT26: MOTION EQUATIONS IN ONE DIMENSION—LOCATION, VELOCITY, & ACCELERATION

An object moves along the x-axis according to this expression (with x in meters and t in seconds):

$$x = 10 - 4t + 2t^3$$

a) What is the object's location at t = 0?

Substituting 0 for the variable t gives x = 10 meters.

b) What is the object's location at t = 2 sec?

Substituting 2 seconds for the variable t gives x = 10m – 8m + 16m = 18 meters.

c) What is the object's displacement between t = 0 and t = 2 sec?

From the answers above, the object is at position x = 10m at t = 0, and at position x = 18m at t = 2 sec, so the displacement is 18m – 10m = 8m.

d) What is the object's velocity at t = 0?

The velocity as a function of time is found by taking the time derivative of the position equation: $v(t) = -4 + 6t^2$. Substituting 0 for time gives a velocity of –4 meters per second at t = 0.

e) What is the object's velocity at t = 2 sec?

The velocity as a function of time is found by taking the time derivative of the position equation: $v(t) = -4 + 6t^2$. Substituting 2 seconds for time gives a velocity of +20 meters per second at t = 0.

f) What is the object's average velocity between t = 0 and t = 2 sec?

The average velocity is the displacement for the first two seconds, 8m, divided by the time, or 4 m/s.

g) What is the object's acceleration at t = 0?

The acceleration as a function of time is found by taking the time derivative of the velocity equation: a(t) = 12t. Substituting 0 for time gives an acceleration of 0 at t = 0.

h) What is the object's acceleration at t = 2 sec?

The acceleration as a function of time is found by taking the time derivative of the velocity equation: a(t) = 12t. Substituting 2 seconds for time gives an acceleration of 24 m/s^2 at t = 0.

i) What is the object's average acceleration between t = 0 and t = 2 sec?

The average acceleration during this time interval is the change in velocity for the interval (20 m/s – (-4 m/s) divided by the time interval of 2 seconds, or 12 m/s^2.

NT3C-RT27: POSITION AS A FUNCTION OF TIME EQUATIONS—INSTANTANEOUS SPEED

The six equations below tell us the position in meters as a function of time in seconds for six objects that are moving along a straight line. As the equations show, these objects vary in their initial positions, initial velocities, and accelerations.

A. $x(t) = -7 + 9t - 2t^2$ **B.** $x(t) = +4 + 9t + t^2$ **C.** $x(t) = +3 - 7t - 2t^2$

D. $x(t) = -4 + 3t - 4t^2$ **E.** $x(t) = -1 - 9t - 2t^2$ **F.** $x(t) = -7 + t + 2t^2$

Rank these situations on the basis of the speed of the objects 2 seconds after the motions begin.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The speed at 2 seconds is the same but not zero for all these objects. ___

OR, The speed at 2 seconds is zero for all these objects. ___

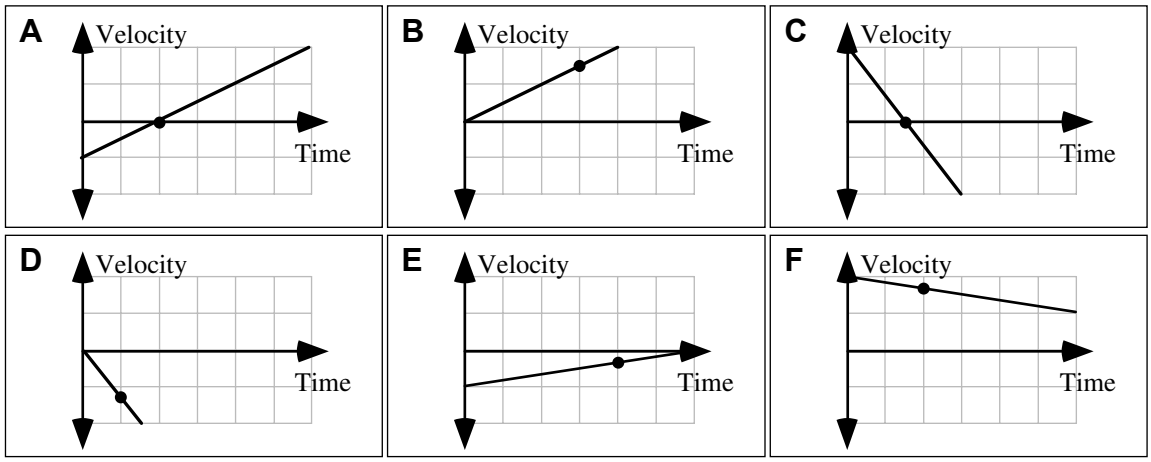
OR, We cannot determine the ranking for the speeds of these objects. ___

Please explain your reasoning.

Answer: The speed is the absolute value of the velocity, and the velocity for each object is given by the time derivative of the position-time equations above. When time 2 seconds is substituted into these equations, we get the velocity at this time. The speed is the absolute value of this quantity, which leads to a ranking E > C >B = D > F > A.

NT3C-RT28: VELOCITY VS. TIME GRAPHS—INSTANTANEOUS VELOCITY

The graphs below show the velocity versus time for six boats traveling along a narrow channel that runs east to west. The scales on both axes are the same for all of these graphs, and east is positive. In each graph, a point is marked with a dot.



Rank these situations on the basis of the velocity of the boat at the point indicated.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The velocity at the marked points is the same but not zero for all these boats. ___

OR, The velocity at the marked points is zero for all these boats. ___

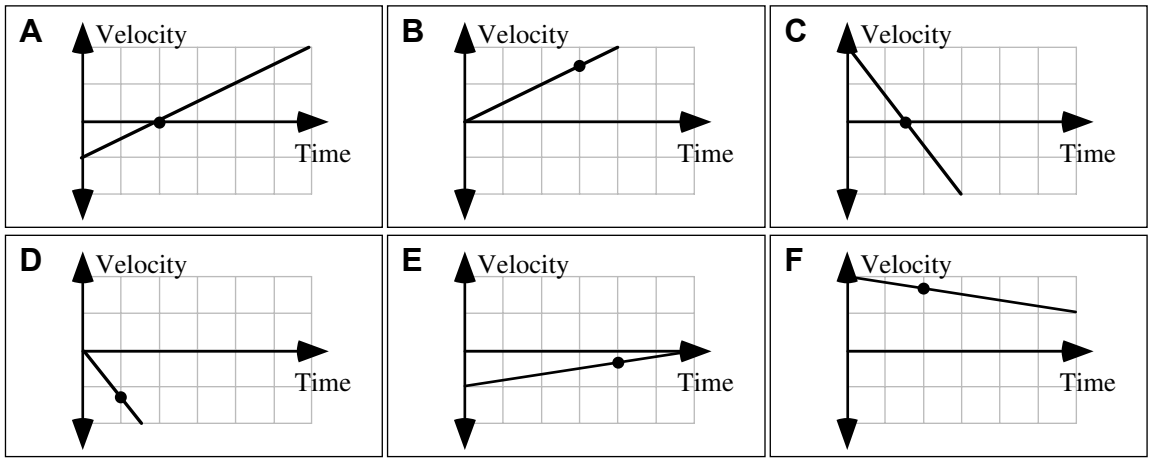
OR, We cannot determine the ranking for the velocity of these boats. ___

Please explain your reasoning.

Answer: E > C >B = D > F > A based on taking a derivative, substituting 2 seconds for the time, and ranking the absolute values.

NT3C-RT29: VELOCITY VS. TIME GRAPHS—ACCELERATION

The graphs below show the velocity versus time for six boats traveling along a narrow channel that runs east to west. The scales on both axes are the same for all of these graphs, and east is positive. In each graph, a point is marked with a dot.



Rank these situations on the basis of the acceleration of the boat at the point indicated.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The acceleration at the marked points is the same but not zero for all these boats. ___

OR, The acceleration at the marked points is zero for all these boats. ___

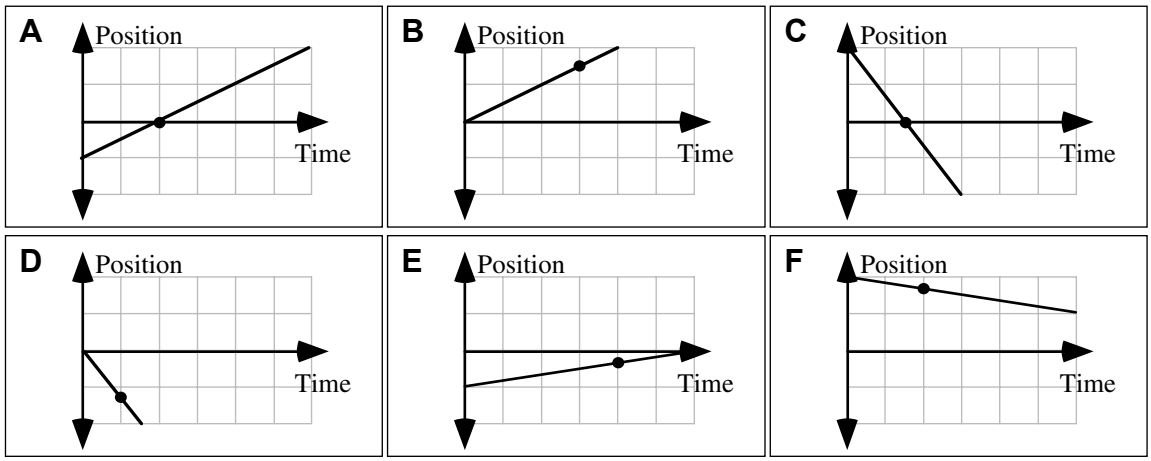
OR, We cannot determine the ranking for the accelerations of these boats. ___

Please explain your reasoning.

Answer: A = B > E > F > C = D; the instantaneous accelerations are determined by the slopes of the velocity-time graphs at the points on the graphs. Since these graphs are straight lines, the slope (and the acceleration) does not change as a function of time.

NT3C-RT30: POSITION VS. TIME GRAPHS—INSTANTANEOUS SPEED

The graphs below show the position versus time for six boats traveling along a narrow channel. The scales on both axes are the same for all of these graphs. In each graph, a point is marked with a dot.



Rank these situations on the basis of the speed of the boat at the point indicated.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The speed at the marked points is the same for all these boats. ___

OR, The speed at the marked points is zero for all these boats. ___

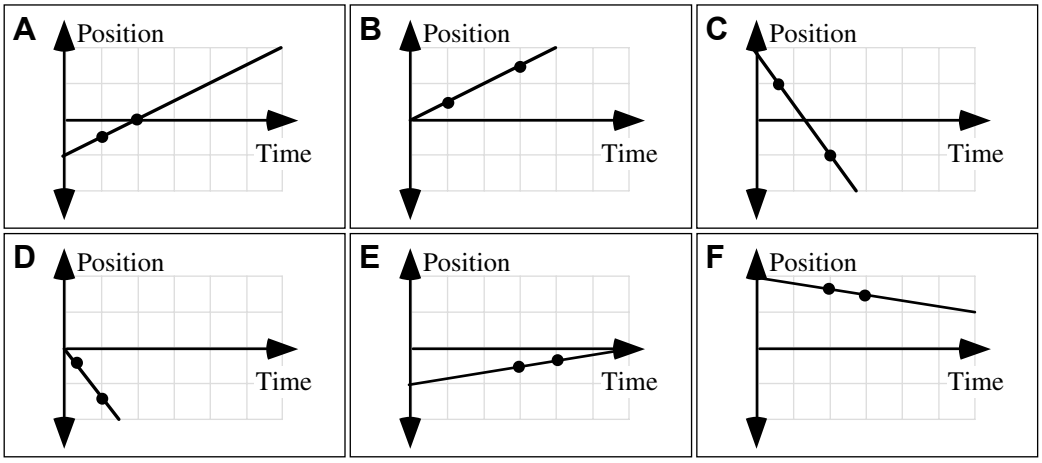
OR, We cannot determine the ranking for the speeds of these boats. ___

Please explain your reasoning.

Answer: C = D > A = B > E = F; the instantaneous speeds are determined by the magnitudes of the slopes of the position-time graphs.

NT3C-RT31: POSITION VS. TIME GRAPHS—DISPLACEMENT

The graphs below show the position versus time for six boats traveling along a narrow channel. The scales on both axes are the same for all of these graphs. In each graph, two points are marked with dots.



Rank these situations on the basis of the displacement between the two points.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The displacement between the points is the same but not zero for all the boats. ___

OR, The displacement between the points is zero for all the boats. ___

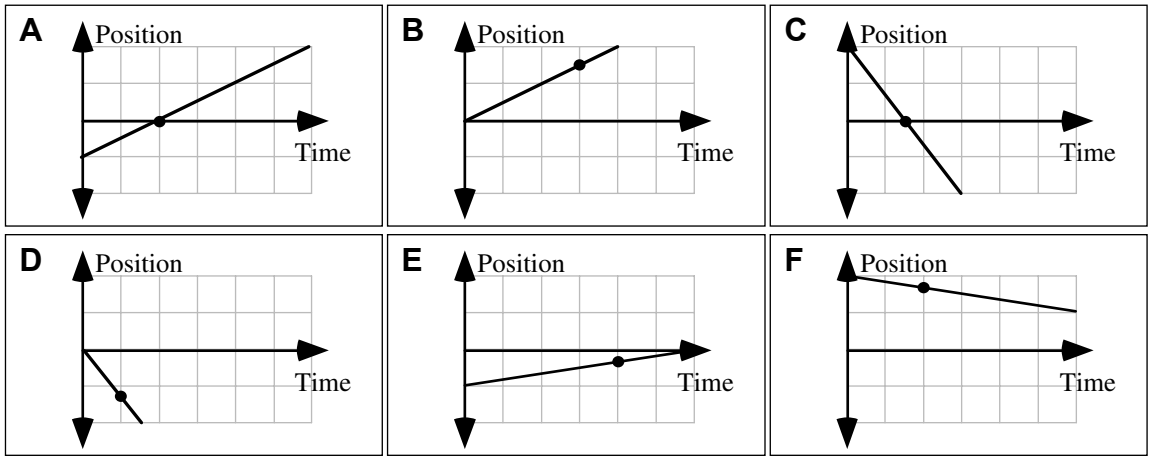
OR, We cannot determine the ranking for the displacements of these boats. ___

Please explain your reasoning.

Answer: B > A > E > F > D > C; the magnitudes of the displacements can be read as the distance between the positions on the vertical axes for the two points.

NT3C-RT32: POSITION VS. TIME GRAPHS—INSTANTANEOUS VELOCITY

The graphs below show the position versus time for six boats traveling along a narrow channel. The scales on both axes are the same for all of these graphs. In each graph, a point is marked with a dot.



Rank these situations on the basis of the velocity of the boat at the point indicated.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The velocity at the marked points is the same but not zero for all the boats. ___

OR, The velocity at the marked points is zero for all the boats. ___

OR, We cannot determine the ranking for the velocities of these boats. ___

Please explain your reasoning.

Answer: A = B > E > F > C = D; the instantaneous velocities are determined by the slopes of the position-time graphs.

NT3C-CCT33: POSITION TIME EQUATION—INSTANTANEOUS SPEED

Three students are discussing the motion of an object which is described by the following equation:

$$x(t) = -4\text{m} - (9\text{m/s})t + (2\text{m/s}^2)t^2$$

The students make the following contentions about the motion during the first 2 seconds:

Amadeo: *"I think this object's instantaneous speed will increase with time since the acceleration is positive."*

Barrett: *"No, the object will have a decreasing speed, since the acceleration is directed opposite to the initial velocity."*

Chinue: *"I don't think we can tell what will happen to the speed from this equation, since it tells us about the position as a function of time, not about speed or velocity."*

Which, if any, of these three students do you agree with?

Amadeo_____ Barrett _____ Chinue _____ None of them_____

Please explain your reasoning.

Answer: Barrett is correct since the velocity and acceleration terms have opposite signs and the velocity will not have decreased to zero in two seconds.

NT3C-QRT34: POSITION TIME EQUATION—VELOCITY AND ACCELERATION EQUATION

A student is told that the motion of an object is described by the following equation:

$$x(t) = -4\text{m} - (9\text{m/s})t + (2\text{m/s}^2)t^2$$

What are the equations that describe the object's velocity and acceleration at various times?

$v(t) =$

$a(t) =$

Answer: This equation describes motion with constant acceleration with a value of 4 m/s^2 and initial position of - 4m and initial velocity of – 9m/s.
Take the derivative of x(t) = – 4m – 9(m/s)t + 2 m/s^2 t^2 for the velocity and take another derivative to get the acceleration.
Thus the equations for v(t) = – 9(m/s) + 4 (m/s) t and for a(t) = 4 m/s^2

Is the initial acceleration in the positive-*x* or negative-*x* direction?

The acceleration is always positive, equal to 4 m/s^2 at all times, so it points in the +x-direction.

Is the initial velocity in the positive-*x* or negative-*x* direction?

Substituting t = 0 into the equation for velocity gives an initial velocity of -9 m/s, so the initial velocity is in the negative-x direction.

Is the initial position in the positive-*x* or negative-*x* direction from the origin?

Substituting t = 0 into the equation for position gives an initial position of –4m, so the initial position is in the negative-x direction.

After 3 seconds, is the acceleration in the positive-*x* or negative-*x* direction?

The acceleration is always positive, equal to 4 m/s^2 at all times, so it points in the +x-direction.

After 3 seconds, is the velocity in the positive-*x* or negative-*x* direction?

Substituting t = 3s into the equation for velocity gives a velocity of +3 m/s, so the velocity at this time is in the positive-x direction.

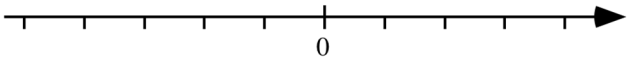
After 3 seconds, is the position in the positive-*x* or negative-*x* direction from the origin?

Substituting t = 3s into the equation for position gives a position of –13m, so the position at this time is in the negative-x direction.

NT3E-QRT35: POSITION, VELOCITY, & ACCELERATION SIGNS—POSITION, DIRECTION, & RATE

Eight possible combinations for the signs for the instantaneous position, velocity, and acceleration of an object are given in the table below. Above the table is a coordinate axis that shows the origin, marked 0, and that indicates that the positive direction is to the right. The three columns on the right-hand side of the table are to describe the location of the object (either left or right of the origin), the direction of the velocity of the object (either toward or away from the origin), and what is happening to the speed of the object (either speeding up or slowing down at the given instant). The appropriate descriptions for the first case are shown.

Complete the rest of the table for position, direction, and rate.

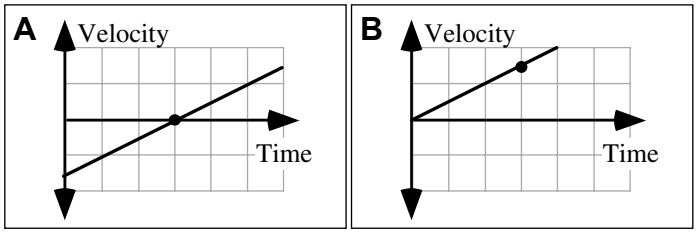


Position	**Velocity**	**Acceleration**	**Position** (Left or Right)	**Direction** (Toward or Away from)	**Rate** (Speeding up or Slowing down)
+	+	+	Right	Away from	Speeding up
+	+	–	*Right*	*Away from*	*Slowing down*
+	–	+	*Right*	*Toward*	*Slowing down*
+	–	–	*Right*	*Toward*	*Speeding up*
–	+	+	*Left*	*Toward*	*Speeding up*
–	+	–	*Left*	*Toward*	*Slowing down*
–	–	+	*Left*	*Away from*	*Slowing down*
–	–	–	*Left*	*Away from*	*Speeding up*

Answer: A positive position indicates that the object is to the right of the origin, and a negative position indicates that the object is to the left of the origin. When the position and the velocity have the same sign, the object is moving away from the origin, and when they have opposite sign the object is moving toward the origin. When the object's acceleration is the same sign as its velocity, it is speeding up, and when the signs are opposite, it is slowing down.

NT3E-WWT36: VELOCITY VS. TIME GRAPHS—ACCELERATION

The graphs show the velocity versus time for two boats traveling along a narrow channel that runs east to west. The scales on both axes are the same for the two graphs, and east is positive. In each graph, a point is marked with a dot. A student who is asked how the acceleration at the marked point for the object in graph A compares to the acceleration at the marked point for the object in graph B states:



> *"I think that at the marked point the boat in graph B has the larger acceleration because the boat in graph A is at rest and its acceleration is zero."*

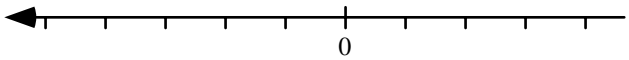
What, if anything, is wrong with this student's contention? If something is wrong, identify it, and explain how to correct it. If it is correct, explain why.

Answer: The student's contention is wrong. Since both graphs have the same slope, both boats experience the same acceleration for the intervals shown. The sailboat is changing direction at the point indicated in the graph in case A.

NT3E-QRT37: POSITION, VELOCITY, & ACCELERATION SIGNS II—POSITION, DIRECTION, & RATE

Eight possible signs combinations for the instantaneous position, velocity, and acceleration of an object are given in the table below. Above the table is a coordinate axis that shows the origin, marked 0, and that indicates that the positive direction is to the left. The three columns on the right-hand side of the table are to describe the location of the object (either left or right of the origin), the direction of the motion of the object (either toward or away from the origin), and what is happening to the speed of the object (either speeding up or slowing down at the given instant). The appropriate descriptions for the first case are shown.

Complete the table for the object's location relative to the origin, motion direction from the origin, and change in the speed. Please indicate any impossible combinations.

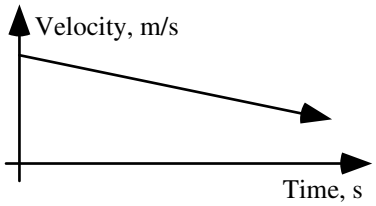


Position	**Velocity**	**Acceleration**	**Position** (Left or Right)	**Direction** (Toward or Away from)	**Rate** (Speeding up or Slowing down)
+	0	+	Right	0	Speeding up
+	0	–	*Right*	*0*	*Speeding up*
+	+	0	*Right*	*Away from*	*Constant*
+	–	0	*Right*	*Toward*	*Constant*
–	0	+	*Left*	*0*	*Speeding up*
–	0	–	*Left*	*0*	*Speeding up*
–	+	0	*Left*	*Toward*	*Constant*
–	–	0	*Left*	*Away from*	*Constant*

Answer: A positive position indicates that the object is to the right of the origin, and a negative position indicates that the object is to the left of the origin. When the acceleration is zero, the object will have a constant speed. When the position and the velocity have the same sign, the object is moving away from the origin, and when they have opposite sign the object is moving toward the origin. When the object has zero velocity, it is not moving, and so its direction is neither toward nor away from the origin.

NT3F-WWT38: BICYCLIST ON A HILL—VELOCITY VS. TIME GRAPH

A bicyclist moving at high speed on a straight road comes to a hill that slopes upward gradually. She decides to coast up the hill. A physics student observing the bicyclist plots the velocity-time graph for her trip up the hill as shown.

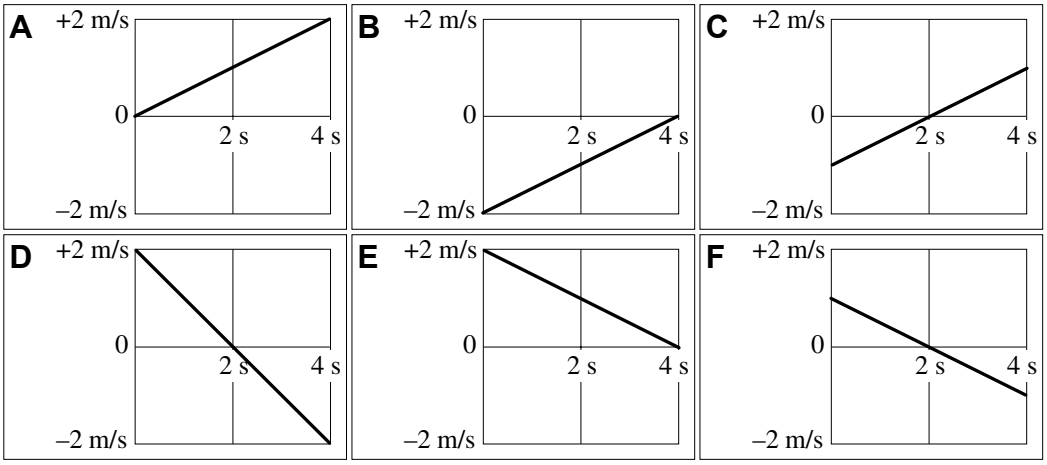


What, if anything, is wrong with this student's graph? If something is wrong, explain the error and how to correct it. If the graph is correct, explain why.

Answer: This is a correct graph. The bicyclist starts out with a large velocity that decreases continuously as she goes up the hill, so the graph has to be a straight line with a negative slope.

NT3F-RT39: VELOCITY VS. TIME GRAPHS—DISPLACEMENT OF IDENTICAL OBJECTS

Graphs of velocity versus time during 4 seconds for six identical objects are shown below. The objects move along a straight, horizontal surface under the action of a force exerted by an external agent.



Rank these situations on the basis of the displacement of the objects during each of these intervals.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The displacement is the same for all these situations. ___

OR, The displacement is zero for all these situations. ___

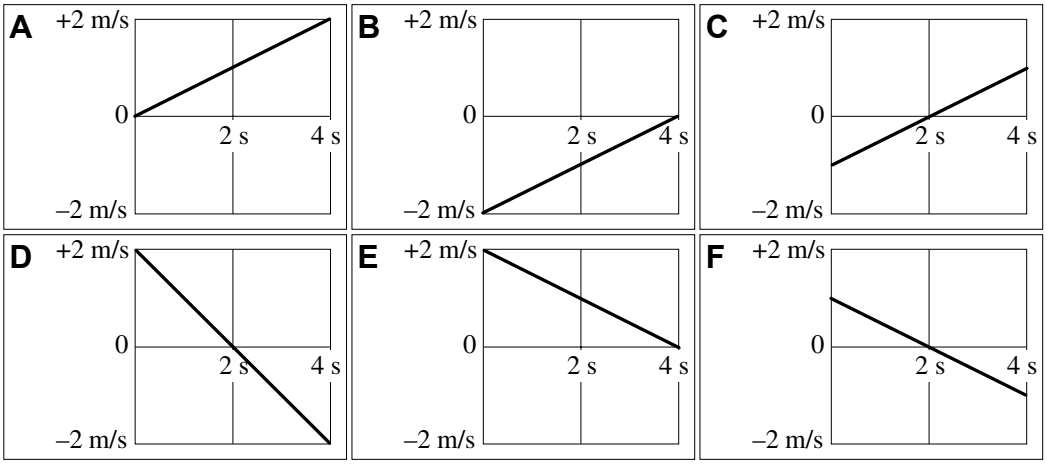
OR, We cannot determine the ranking for the displacement for these situations. ___

Please explain your reasoning.

Answer: A = E > C = D =F > B.
The displacement is equal to the positive or negative area under the graphs of velocity vs time.

NT3F-RT40: VELOCITY VS. TIME GRAPHS—AVERAGE VELOCITY OF IDENTICAL OBJECTS

Graphs of velocity versus time during 4 seconds for six identical objects are shown below. The objects move along a straight, horizontal surface under the action of a force exerted by an external agent.



Rank these situations on the basis of the average velocity of the objects during each of these intervals.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The average velocity is the same but not zero for all these situations. ___

OR, The average velocity is zero for all these situations. ___

OR, We cannot determine the ranking of the average velocity for these situations. ___

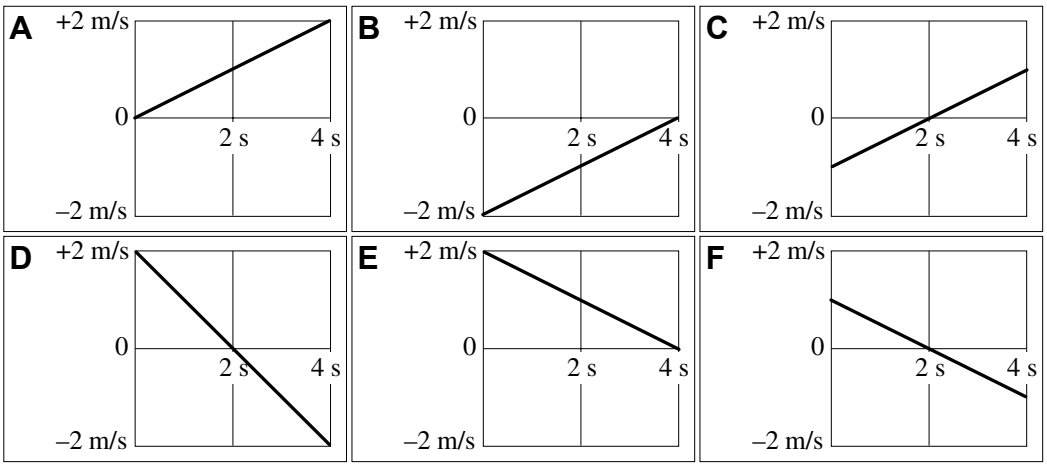
Please explain your reasoning.

Answer: A = E > C = D = F > B

The average velocity is equal to the displacement of the object divided by the time interval of the displacement. Since the time intervals are the same, it is enough to rank the displacement which is the area between the graph and the horizontal axis.

NT3F-RT41: VELOCITY VS. TIME GRAPHS—ACCELERATION OF IDENTICAL OBJECTS

Graphs of velocity versus time during 4 seconds for six identical objects are shown below. The objects move along a straight, horizontal surface under the action of a force exerted by an external agent.



Rank these situations on the basis of the acceleration of these objects during each of these intervals.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

OR, The acceleration is the same but not zero for all these situations. ___

OR, The acceleration is zero for all these situations. ___

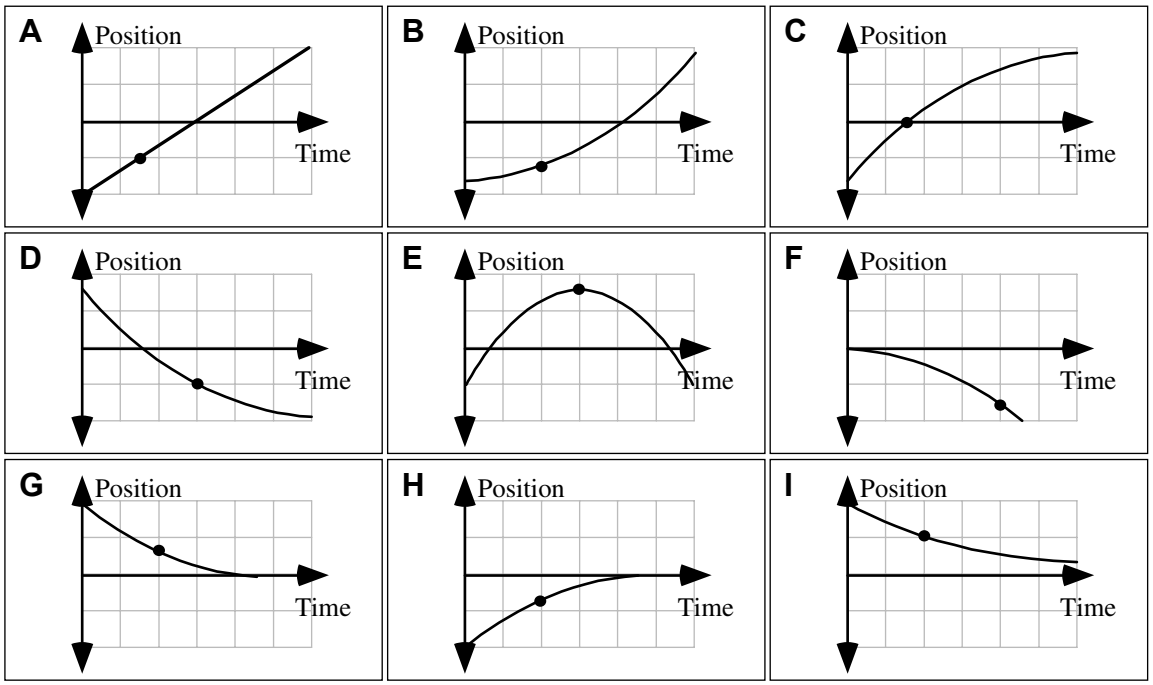
OR, We cannot determine the ranking of the acceleration for these situations. ___

Please explain your reasoning.

Answer: A = B = C > E = F > D.
The acceleration is equal to the positive or negative slope of the graph of velocity vs time.

NT3G-QRT42: POSITION VS. TIME GRAPHS—ACCELERATION AND VELOCITY

Position versus time graphs for boats traveling along a narrow channel are shown below. The scales on both axes are the same for all of these graphs. In each graph, a point is marked with a dot.



a) For which of these, if any, is the position zero at the indicated point?

Answer: The position is zero for the indicated point in graph C, which can be read directly off of the graph.

b) For which of these, if any, is the position negative at the indicated point?

Answer: The position is zero for case C, positive for E, G, and I, and negative for A, B, D, F, H.

c) For which of these, if any, is the velocity zero at the indicated point?

Answer: The velocity is zero if the slope of the graph is zero at the indicated point, which is that case only for graph E.

d) For which of these, if any, is the velocity negative at the indicated point?

Answer: The velocity is given by the slope of the position-time graph, and a negative velocity corresponds to a downward-sloping graph at the indicated point. Graphs D, F, G, and I have negative slopes at the indicated point.

e) For which of these, if any, is the acceleration zero at the indicated point?

Answer: A zero acceleration corresponds to a constant velocity, which in turn corresponds to a linear position-time graph. The acceleration is zero for graph A.

f) For which of these, if any, is the acceleration negative at the indicated point?

A negative acceleration corresponds to a velocity graph whose slope is negative, which in turn corresponds to a position-time graph that curves downward (i.e., the slope becomes less positive or more negative with time. The accelerations are negative for graphs C, E, F, and H