✓ The radian

✓ Angular kinematics

<u>Torque</u> – Think of the kinematic analogies

$$a = \alpha$$

$$F = \tau$$

- tendency of a force to rotate an object about an axis.

$$\boxed{\tau = rF\sin\theta}$$

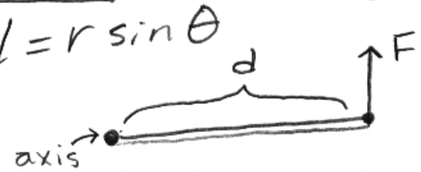
SI unit $(N \cdot m)$

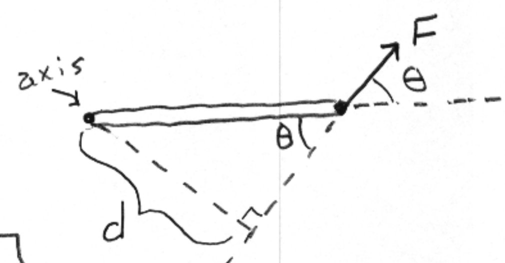
- only the component of force perpendicular to r, $(F\sin\theta)$ can cause rotation.

often written as cross product $\boxed{\tau = r \times F}$

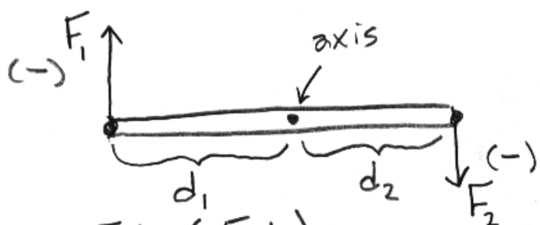
<u>Moment arm (lever arm)</u>

$$d = r\sin\theta$$





<u>2 or more forces</u>



$$\Sigma\tau = \tau_1 + \tau_2 \ldots$$

$$\Sigma\tau = -F_1 d_1 + (-F_2 d_2)$$

Net torque is important and is treated like force

Torque is called the <u>moment of force</u>

Direction of torque

⊙ out of page

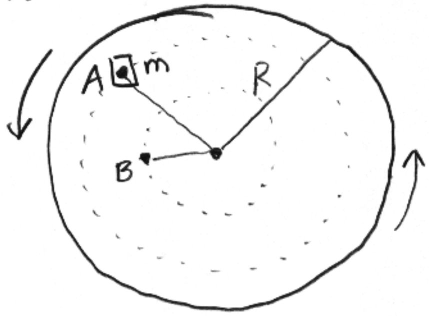
⊗ into page

Use right hand rule!

Rotational Inertia (Analogy for mass)

take a solid uniform disk

ω = constant



A and B have diff. linear velocities due to radial position

They must also have diff. kinetic energies if look at them as point masses.

We cannot expect to find the Rotational kinetic energy of the disk using $K=\frac{1}{2}mv^2$, because the point masses that make up the object are all at different velocities!

We would have to say $\sum K = \frac{1}{2} m_i v_i^2$

Analogy for Kinetic Energy

$v = r\omega$

$$\sum K = \sum \frac{1}{2} m (r\omega)^2$$

$$\sum \frac{1}{2} (mr^2) \omega^2$$

What does this mean?

Also called rotational inertia → This is moment of inertia and our analogy for mass

$$\boxed{I = mr^2}$$ ← this is for a point mass

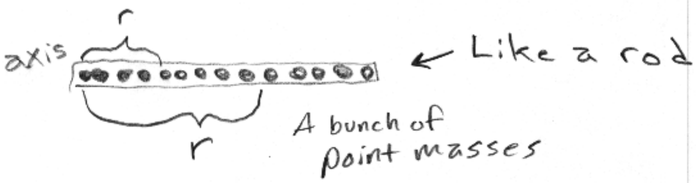
SI unit $kg \cdot m^2$

So the moment of inertia for a large object made of point masses is the sum of the rotational inertia of every mass.

$$I = \sum m_i r_i^2 \quad \text{-or-} \quad \boxed{I = \int r^2 dm}$$

Ex: Single mass

Many mass (solid object)



$$I = \sum m_i r_i^2$$

This concept can be used to find (I) for many solids

Analogy $(F = ma)$ $\quad a = r\alpha$

$$(r)\cdot F = m r \alpha \cdot (r)$$

$$Fr = m r^2 \alpha$$

$$\tau = I\alpha \longrightarrow F = ma$$

Rotational $\qquad$ Translational or linear

Same mass will not always equal the same moment of inertia

Demo: disk vs. hoop (Depends on the distribution of mass)

Mass density is important to find (I)

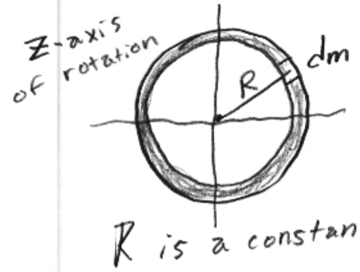
For 3D objects (solids) mass per unit volume $dm = \rho\, dV$

For large area of negligable thickness (plates and, shells) mass per unit area $dm = \sigma\, dA$

For long lenths of negligable thickness (rods and rings) mass per unit length $dm = \lambda\, dx$

Ex: Find (thin) Ring -or- hoop

$\boxed{\int dm = M}$



$I = \int r^2\, dm$

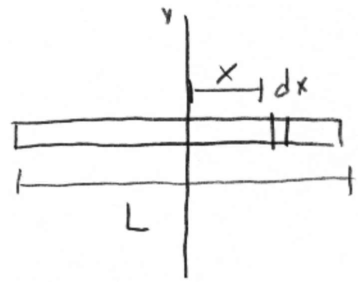
R is a constant $\quad I = R^2 \int dm$

$\boxed{I = MR^2}$

How to solve

1. Identify infinitesimal (dm)
2. Choose applicable density type
3. Write out moment of inertia
4. Choose limits based on object

Find Rod from side and middle.



$\lambda = \frac{M}{L}$

$dm = \lambda\, dx$

$\lambda = \frac{M}{L}$ linear

$\rho = \frac{M}{V}$ Volume

$\sigma = \frac{M}{A}$ Area

$I = \int r^2\, dm$

$I = \int r^2 \lambda\, dx$

Integrate entire length →

$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{M}{L}\, dx$

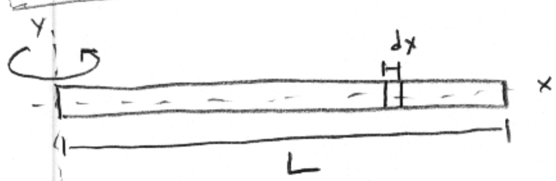
$I = \frac{M}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\, dx$

$I = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}}$

$I = \frac{M}{L}\left[\left(\frac{L^3/8}{3}\right) - \left(\frac{-L^3/8}{3}\right)\right]$

$I = \frac{M}{L}\left[\frac{L^3}{24} + \frac{L^3}{24}\right]$

$I = \frac{M}{\cancel{L}} \cdot \frac{2L^{\cancel{3}}}{24} = \boxed{\frac{1}{12} ML^2}$



$\lambda = \frac{M}{L}$

$dm = \lambda\, dx$

$I = \int r^2\, dm$

$I = \int r^2 \lambda\, dx$

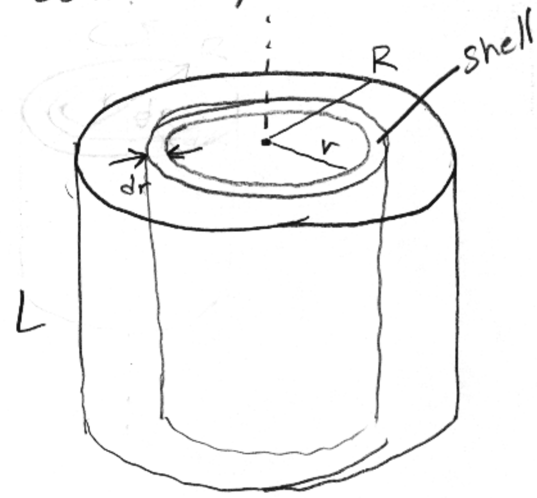
$I = \int r^2 \frac{M}{L}\, dx$

$I = \frac{M}{L} \int_0^L r^2\, dx$

$I = \frac{M}{L}\left[\frac{r^3}{3}\right]_0^L$

$I = \frac{M}{\cancel{L}} \cdot \frac{L^{\cancel{3}}}{3} = \boxed{\frac{1}{3} ML^2}$

Ex: Solid Cylinder



$\rho = \frac{M}{V}$ $\quad dm = \rho\, dV \quad$ $\rho = \frac{M}{\pi R^2 L}$

$dV = dA \cdot L$

$dV = (2\pi r\, dr) \cdot L$

cross area of shell

$$I = \int r^2\, dm$$

$$I = \int r^2 \rho\, dV$$

$$I = \int r^2 \rho\, 2\pi r L\, dr$$

$$I = \rho 2\pi L \int_0^R r^3\, dr$$

$$I = \rho 2\pi L \left[\frac{R^4}{4}\right]_0^R$$

$$I = \left[\frac{M}{\pi \cancel{R^2} \cancel{L}}\right] 2\cancel{\pi} \cancel{L} \cdot \frac{R^4}{4}$$

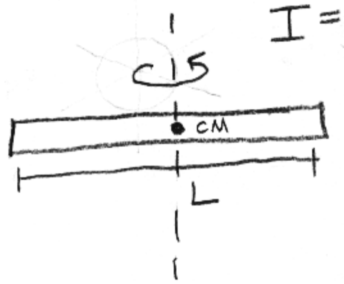
$$I = \frac{2}{4} M R^2$$

$$\boxed{I = \frac{1}{2} M R^2}$$

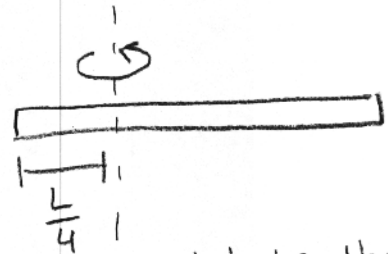
Parallel-axis Theorem

- Finding the moment of inertia about some strange axis is difficult.
- parallel-axis theorem allows us to simplify this.

Ex: $I = \frac{1}{12}ML^2$



$\rightarrow$ same rod "new" axis



Whats the new I?

Use parallel-axis theorem

$$\boxed{I = I_{cm} + MD^2}$$

$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{4}\right)^2$$

$$I = \frac{ML^2}{12} + \frac{ML^2}{16}$$

$$I = \frac{4ML^2}{48} + \frac{3ML^2}{48} = \boxed{\frac{7}{48}ML^2}$$

Yes!!