

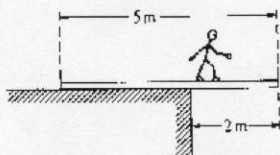
1) If the mass of a simple pendulum is doubled but its length remains constant, its period is multiplied by a factor of

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{\sqrt{2}}$
- (C) 1
- (D)  $\sqrt{2}$
- (E) 2

$T_p = 2\pi\sqrt{\frac{L}{g}}$   
 MC 1.25 pts each  
 MC 30 pts  
 FR 60 pts  
 total 90 pts  
 Students need 56 pts for 100%

2) Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?

- (A) The kinetic and potential energies are equal at all times.
- (B) The kinetic and potential energies are both constant.
- (C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
- (D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
- (E) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position.



3) A 5-meter uniform plank of mass 100 kilograms rests on the top of a building with 2 meters extended over the edge as shown above. How far can a 50-kilogram person venture past the edge of the building on the plank before the plank just begins to tip?

- (A)  $\frac{1}{2}m$
- (B) 1 m
- (C)  $\frac{2}{3}m$
- (D) 2 m
- (E) It is impossible to make the plank tip since the person would have to be more than 2 meters from the edge of the building.

$100g(0.5) = 50g(x)$

4) When a person stands on a rotating merry-go-round, the frictional force exerted on the person by the merry-go-round is

- (A) greater in magnitude than the frictional force exerted on the person by the merry-go-round
- (B) opposite in direction to the frictional force exerted on the merry-go-round by the person
- (C) directed away from the center of the merry-go-round
- (D) zero if the rate of rotation is constant
- (E) independent of the person's mass

5) Each of five satellites makes a circular orbit about an object that is much more massive than any of the satellites. The mass and orbital radius of each satellite are given below. Which satellite has the greatest speed?

Mass	Radius
(A) $\frac{1}{2}m$	$R$
<u>(B) <math>m</math></u>	<u><math>\frac{1}{2}R</math></u>
(C) $m$	$R$
(D) $m$	$2R$
(E) $2m$	$R$

6) When a mass is attached to a spring, the period of oscillation is approximately 2.0 seconds. When the mass attached to the spring is doubled, the period of oscillation is most nearly

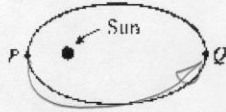
- (A) 0.5 s
- (B) 1.0 s
- (C) 1.4 s
- (D) 2.0 s
- (E) 2.8 s

$T_s = 2\pi\sqrt{\frac{m}{k}}$   
 $\sqrt{2}(2s)$

7) A ball attached to a string is whirled around in a horizontal circle having a radius  $r$ . If the radius of the circle is changed to  $4r$  and the same centripetal force is applied by the string, the new speed of the ball is which of the following?

- (A) One-quarter the original speed
- (B) One-half the original speed
- (C) The same as the original speed
- (D) Twice the original speed
- (E) Four times the original speed

$F_c = \frac{mv^2}{4r}$



8) An asteroid moves in an elliptical orbit with the Sun at one focus as shown above. Which of the following quantities increases as the asteroid moves from point  $P$  in its orbit to point  $Q$ ?

- (A) Speed
- (B) Angular momentum
- (C) Total energy
- (D) Kinetic energy
- (E) Potential energy

9) Two planets have the same size, but different masses, and no atmospheres. Which of the following would be the same for objects with equal mass on the surfaces of the two planets?

- I. The rate at which each would fall freely
- II. The amount of mass each would balance on an equal-arm balance
- III. The amount of momentum each would acquire when given a certain impulse

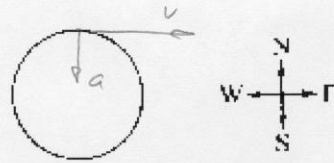
- (A) I only
- (B) III only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III



10) A person weighing 800 newtons on Earth travels to another planet with twice the mass and twice the radius of Earth. The person's weight on this other planet is most nearly

- (A) 400 N
- (B)  $\frac{800}{\sqrt{2}}$  N
- (C) 800 N
- (D)  $800\sqrt{2}$  N
- (E) 1,600 N

$$\frac{1}{2}g = G \frac{2M}{(2R)^2}$$



View of Track from Above

11) A racing car is moving around the circular track of radius 300 meters shown above. At the instant when the car's velocity is directed due east, its acceleration is directed due south and has a magnitude of 3 meters per second squared. When viewed from above, the car is moving

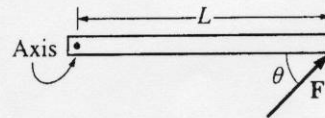
- (A) clockwise at 30 m/s
- (B) clockwise at 10 m/s
- (C) counterclockwise at 30 m/s
- (D) counterclockwise at 10 m/s
- (E) with constant velocity

$$a = \frac{v^2}{r}$$

$$3 = \frac{v^2}{300}$$

$$v^2 = 900$$

$$v = 30$$

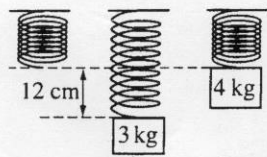


View from Above

12) A rod on a horizontal tabletop is pivoted at one end and is free to rotate without friction about a vertical axis, as shown above. A force  $F$  is applied at the other end, at an angle  $\theta$  to the rod. If  $F$  were to be applied perpendicular to the rod, at what distance from the axis should it be applied in order to produce the same torque?

- (A)  $L \sin \theta$
- (B)  $L \cos \theta$
- (C)  $L$
- (D)  $L \tan \theta$
- (E)  $\sqrt{2}L$

$$\tau = FL \sin \theta$$



13) A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown above. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown above, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?

- (A) 9 cm
- (B) 18 cm
- (C) 24 cm
- (D) 32 cm**
- (E) 48 cm

14) An object has a weight  $W$  when it is on the surface of a planet of radius  $R$ . What will be the gravitational force on the object after it has been moved to a distance of  $4R$  from the center of the planet?

- (A)  $16W$
- (B)  $4W$
- (C)  $W$
- (D)  $\frac{1}{4}W$
- (E)  $\frac{1}{16}W$**

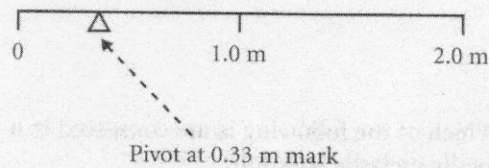
$$\frac{F_g = G \frac{m_1 m_2}{R^2}}{16} = \frac{G \frac{m_1 m_2}{(4R)^2}}$$

15) What is the kinetic energy of a satellite of mass  $m$  that orbits the Earth, of mass  $M$ , in a circular orbit of radius  $R$ ?

- (A) Zero
- (B)  $\frac{1}{2} \frac{GMm}{R}$**
- (C)  $\frac{1}{4} \frac{GMm}{R}$
- (D)  $\frac{1}{2} \frac{GMm}{R^2}$
- (E)  $\frac{GMm}{R^2}$

$$F_c = G \frac{Mm}{R^2} = \frac{mv^2}{R}$$

16) Which of the following arrangements of three masses (hanging off the horizontal rod) would result in a balanced mobile? Note:  $M_1 = M_2 = M_3$ .



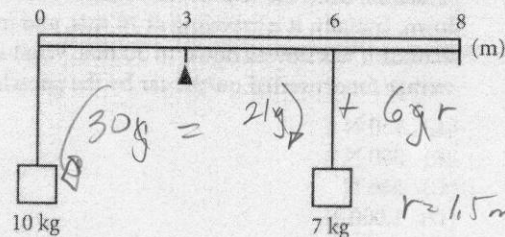
	Mass 1	Mass 2	Mass 3
(A)	$x = 0.5$ m	$x = 0.0$ m	$x = 1.5$ m
(B)	$x = 1.0$ m	$x = 0.0$ m	$x = 2.0$ m
(C)	$x = 2.0$ m	$x = 1.0$ m	$x = 0.0$ m
<b>(D)</b>	<b><math>x = 0.0</math> m</b>	<b><math>x = 1.0</math> m</b>	<b><math>x = 0.0</math> m</b>
(E)	$x = 0.5$ m	$x = 0.0$ m	$x = 2.0$ m

17) An amusement park ride has a diameter of 16 meters and revolves at 15 rpm (revolutions per minute). How fast is a rider moving if she sits at the outermost part of the ride?

- (A) 29.4 m/s
- (B) 1.07 m/s
- (C) 225 m/s
- (D) 12.56 m/s**
- (E) 9.8 m/s

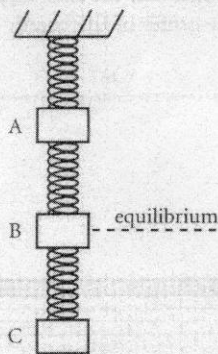
$$15 \frac{\text{revolutions}}{\text{minute}} \times \frac{2\pi (8\text{m})}{\text{rev}} \times \frac{1}{60} \frac{\text{min}}{\text{sec}}$$

18) In the picture below, where should a third mass (6 kg) be hung to produce rotational equilibrium?



- (A) 2 meters to the right of the fulcrum
- (B) 1.88 meters to the left of the fulcrum
- (C) 1.5 meters to the right of the fulcrum**
- (D) 0.17 meters to the left of the fulcrum
- (E) There is no place to put the mass so that it will balance.

- 19) The diagram shows three positions of a mass as it bounces on a spring. The spring is attached to a ceiling. What is true at point C, the lowest point of the oscillation?



- I. Speed = 0 m/s  
 II. Acceleration = 0 m/s<sup>2</sup>  
 III. Displacement = 0 m

- (A) I  
 (B) II  
 (C) III  
 (D) I and II  
 (E) II and III

- 20) What is the gravitational force between two runners standing 100 meters apart on a track if their masses are 60 kg and 75 kg respectively?

- (A) 588 N for the first runner and 735 N for the second runner  
 (B) 45 N  
 (C)  $3 \cdot 10^{-7}$  N  
 (D)  $3 \cdot 10^{-11}$  N  
 (E) 3.0 N

$$G = 6.67 \times 10^{-11} \frac{(60)(75)}{(100)^2}$$

- 21) A circular, rotating space station uses centripetal force to produce artificial gravity. If the radius of the space station is 450 m and it spins at a speed of 55 m/s, what is the apparent value of  $g$  on the surface of the station?

- (A) 1.8 m/s<sup>2</sup>  
 (B) 9.8 m/s<sup>2</sup>  
 (C) 13.1 m/s<sup>2</sup>  
 (D) 6.7 m/s<sup>2</sup>  
 (E) zero, because in space everything is weightless

$$a_c = \frac{v^2}{r} = \frac{(55)^2}{450}$$

- 22) On an unknown planet, a pendulum of length 1.5 m swings with a period of 3.3 s. What is the acceleration due to gravity on this planet?

- (A) 3.7 m/s<sup>2</sup>  
 (B) 12.7 m/s<sup>2</sup>  
 (C) 7.1 m/s<sup>2</sup>  
 (D) 5.4 m/s<sup>2</sup>  
 (E) 9.8 m/s<sup>2</sup>

$$T_p = 2\pi \sqrt{\frac{l}{g}}$$

$$3.3 = 2\pi \sqrt{\frac{1.5}{g}}$$

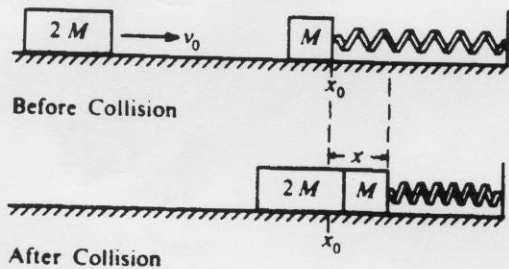
- 23) If the mass on an oscillating spring is decreased to  $\frac{1}{2}$  the original mass:

- (A) The period will be cut in half, but the frequency will remain the same.  
 (B) The period and the frequency will remain the same.  
 (C) The period will be increased by the square root of two.  
 (D) The period will be decreased by a factor of two.  
 (E) The frequency will be increased by a factor of the square root of two.

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

- 24) The moon's mass is about one-sixth of the earth's mass. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on the earth:

- (A) is one-sixth as much  
 (B) is one-half as much  
 (C) is the same  
 (D) is twice as much  
 (E) is six times as much



1983B (15 points).

A block of mass  $M$  is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant  $k$ . A second block of mass  $2M$  and initial speed  $v_0$ , collides with and sticks to the first block. Develop expressions for the following quantities in terms of  $M$ ,  $k$ , and  $v_0$ .

(4) a.  $v$ , the speed of the blocks immediately after impact

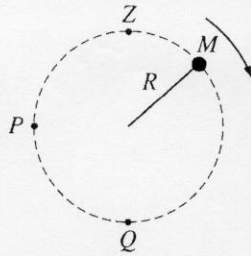
①  $p_1 = p_2$   
 ①  $2Mv_0 = 3Mv$   
 ①  $v = \frac{2}{3}v_0$

(8) b.  $x$ , the maximum distance the spring is compressed

Collision is not Elastic!  
 ①  $E_1 = E_2$   
 ①  $K_{\text{After impact}} = U_{s \text{ max compression}}$   
 ①  $K = \frac{1}{2}mv^2$   
 ①  $v = \frac{2}{3}v_0$   
 ①  $U_s = \frac{1}{2}kx^2$   
 $\frac{1}{2}(3M)\left(\frac{2}{3}v_0\right)^2 = \frac{1}{2}kx^2$   
 $3M \frac{4}{9}v_0^2 = kx^2$   
 $x^2 = \frac{4/3 M v_0^2}{k}$   
 ①  $x = \sqrt{\frac{4M}{3k}} v_0$

(3) c.  $T$ , the period of the subsequent simple harmonic motion

①  $T_s = 2\pi\sqrt{m/k}$        $m = 3M$   
 ①  $T_s = 2\pi\sqrt{\frac{3M}{k}}$



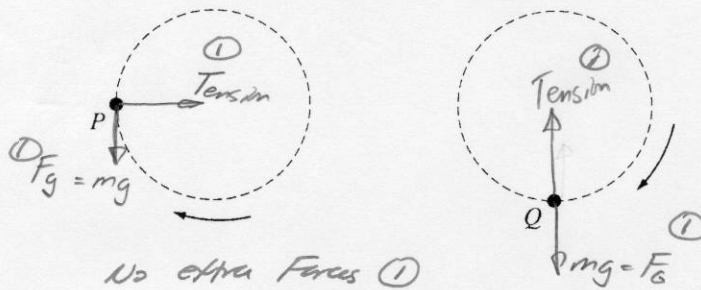
Side View

2001B1 (15 points)

A ball of mass  $M$  is attached to a string of length  $R$  and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point  $P$ , the string is horizontal. Point  $Q$  is at the bottom of the circle and point  $Z$  is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

- a. On the figures below, draw and label all the forces exerted on the ball when it is at points  $P$  and  $Q$ , respectively.

# 5



- b. Derive an expression for  $v_{\min}$  the minimum speed the ball can have at point  $Z$  without leaving the circular path.

4

$\Sigma F_c = ma_c$   
 $T + mg = m \frac{v^2}{R}$   
 at  $v_{\min}$   $T = 0$   
 $mg = m \frac{v^2}{R}$   
 $v = \sqrt{gR}$

Alt Solution  
 at  $v_{\min}$   
 $a_c = g$   
 $\frac{v^2}{R} = g$   
 $v = \sqrt{gR}$

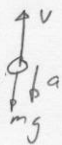
- c. The maximum tension the string can have without breaking is  $T_{\max}$ . Derive an expression for  $v_{\max}$ , the maximum speed the ball can have at point  $Q$  without breaking the string.

4

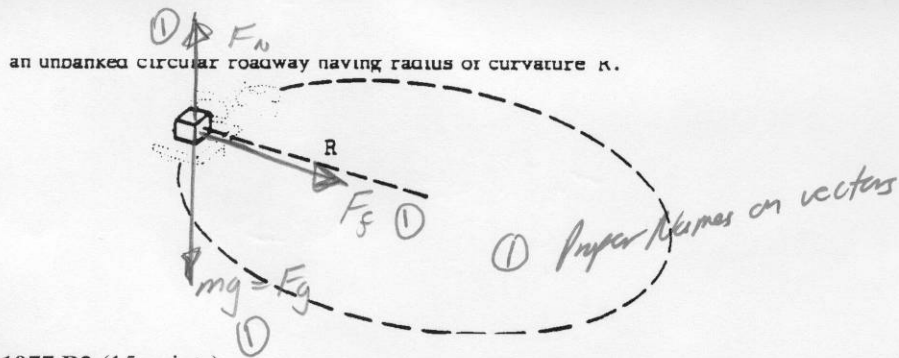
$\Sigma F_c = ma_c$   
 $T_{\max} - mg = m \frac{v_{\max}^2}{R}$   
 $\frac{T_{\max}}{m} - g = \frac{v_{\max}^2}{R}$   
 $v_{\max} = \sqrt{R \left[ \frac{T_{\max}}{m} - g \right]}$

- d. Suppose that the string breaks at the instant the ball is at point  $P$ . Describe the motion of the ball immediately after the string breaks.

2



Motion will be 1 dimensional with  $v_0$  up and  $a = g$  down. Ball will go up stop & then accelerate back down.



1977 B2 (15 points)

A box of mass  $M$ , held in place by friction, rides on the flatbed of a truck which is traveling with constant speed  $v$ . The truck is on an unbanked circular roadway having radius of curvature  $R$ .

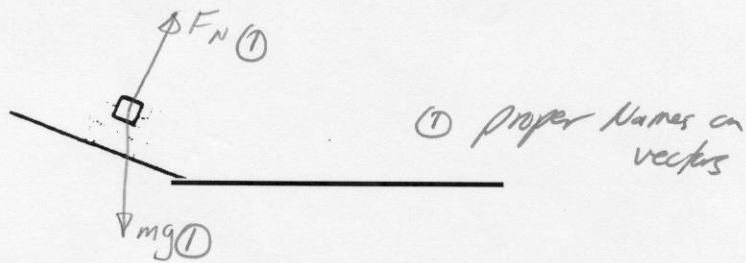
- ④ a. On the diagram provided above, indicate and clearly label all the force vectors acting on the box.
- ④ b. Find what condition must be satisfied by the coefficient of static friction  $\mu$  between the box and the truck bed. Express your answer in terms of  $v$ ,  $R$ , and  $g$ .

$$\begin{aligned} \textcircled{1} \Sigma F_c &= ma_c \\ F_f &= m \frac{v^2}{R} \\ \mu F_N &= m \frac{v^2}{R} \\ \mu mg &= m \frac{v^2}{R} \\ \textcircled{1} \mu &= \frac{v^2}{gR} \end{aligned}$$

$$\begin{aligned} \textcircled{1} F_c &= F_f \\ \textcircled{1} a_c &= \frac{v^2}{R} \\ \textcircled{1} F_f &= \mu F_N \\ \textcircled{1} F_N &= mg \end{aligned}$$

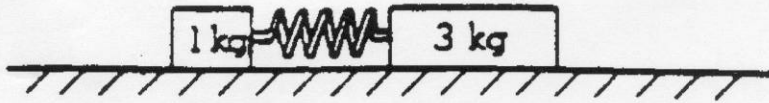
If the roadway is properly banked, the box will still remain in place on the truck for the same speed  $v$  even when the truck bed is frictionless.

- ③ c. On the diagram below indicate and clearly label the two forces acting on the box under these conditions



- ② d. Which, if either, of the two forces acting on the box is greater in magnitude?

②  $F_N$  is larger



1981B2 (15 points)

A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second.

- a. Determine the minimum work needed to compress the spring in this experiment.

⑦

②  $W = \Delta K$

Work Compressing spring =  $K$  of 3kg mass

$\frac{1}{2}mv^2$  ①

$= \frac{1}{2}(3\text{kg})(10)^2$  ① Substitution

$= 150\text{J}$  ②

$K$  of 3kg mass only ①

The spring is compressed again exactly as above, but this time both masses are released simultaneously.

- b. Determine the final velocity of each mass relative to the table after the masses are released.

⑧

①  $W_{\text{work}} = K_1 + K_2$

①  $150\text{J} = \frac{1}{2}1v_1^2 + \frac{1}{2}3v_3^2$

$150 = \frac{v_1^2}{2} + \frac{3}{2}v_3^2$

$300 = v_1^2 + 3v_3^2$  ①

①  $300 = (-3v_3)^2 + 3v_3^2$

$300 = 9v_3^2 + 3v_3^2$

$300 = 12v_3^2$

$v_3^2 = 25$

①  $v_3 = \pm 5\text{m/s}$

①  $p_1 = p_2$

①  $p = mv$

$0 = mv_1 + mv_3$

$0 = 1v_1 + 3v_3$

①  $v_1 = -3v_3$

$v_1 = \pm 15\text{m/s}$  ①

Substitute